JEE Main April 2026

Question Paper With Text Solution

06 April | Shift-2

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN APRIL 2026 | 6TH APRIL SHIFT-2

SECTION – A

Question ID : 6911211201

1. Let $f : R \to R$ be defined as $f(x) = \frac{2x^2 - 3x + 2}{3x^2 + x + 3}$. Then f is :

(1) both one-one and onto
(2) one-one but not onto
(3) onto but not one-one
(4) neither one-one nor onto

Ans. Official answer NTA (4)

Sol. To check if the function is onto, we find its range. Let:

$$y = \frac{2x^2 - 3x + 2}{3x^2 + x + 3}$$

Cross-multiplying, we get:

$$3yx^2 + yx + 3y = 2x^2 - 3x + 2$$

$$(3y - 2)x^2 + (y + 3)x + (3y - 2) = 0$$

Since $x \in R$, the discriminant $D \geq 0$:

$$(y + 3)^2 - 4(3y - 2)(3y - 2) \geq 0$$

$$(y + 3)^2 - [2(3y - 2)]^2 \geq 0$$

Using $a^2 - b^2 = (a - b)(a + b)$:

$$(y + 3 - 6y + 4)(y + 3 + 6y - 4) \geq 0$$

$$(-5y + 7)(7y - 1) \geq 0$$

$$(5y - 7)(7y - 1) \leq 0$$

The range of the function is $\left[\frac{1}{7}, \frac{7}{5}\right]$. Since the codomain is R, and the Range $\neq$ Codomain, the function is not onto

To check if it is one-one, we observe that for any y in the interior of the range (1/7, 7/5), the quadratic in x will yield two distinct real roots. For example, if x_1 and x_2 are roots, $f(x_1) = f(x_2)$, making the function not one-one

Question ID : 6911211202

2. Consider the quadratic equation $(n^2 - 2n + 2)x^2 - 3x + (n^2 - 2n + 2)^2 = 0$, $n \in R$. Let α be the minimum value of the product of its roots and β be the maximum value of the sum of its roots. Then the sum of the first six terms of the G.P., whose first term is α and the common ratio is $\frac{\alpha}{\beta}$ is :

(1) $\frac{61}{37}$ (2) $\frac{121}{81}$ (3) $\frac{364}{243}$ (4) $\frac{1093}{729}$

Ans. (3)

Sol. Let $k = n^2 - 2n + 2 = (n - 1)^2 + 1$. Since $n \in R$, the minimum value of k is 1 (at n = 1). The equation is $kx^2 - 3x + k^2 = 0$

1. Sum of roots $= \frac{3}{k}$. Its maximum value $\beta = \frac{3}{k_{min}} = \frac{3}{1} = 3$

2. Product of roots $= \frac{k^2}{k} = k$. Its minimum value $\alpha = k_{min} = 1$

For the G.P.: First term $a = \alpha = 1$. Common ratio $r = \frac{\alpha}{\beta} = \frac{1}{3}$. Sum of first 6 terms ($S_6$):

$$S_6 = \frac{a(1-r^6)}{1-r} = \frac{1(1-(1/3)^6)}{1-1/3}$$

$$S_6 = \frac{1-1/729}{2/3} = \frac{728/729}{2/3} = \frac{728}{729} \times \frac{3}{2}$$

$$S_6 = \frac{364}{243}$$

Question ID : 6911211203

3. Let $S = \{z \epsilon C : z^2 + \sqrt{6}iz - 3 = 0\}$. Then $\sum_{z \in S} z^8$ is equal to :

(1) 162 (2) 184 (3) 262 (4) 324

Ans. (1)

Sol. The given equation is $z^2 + \sqrt{6}iz - 3 = 0$. Using the quadratic formula:

$$z = \frac{-\sqrt{6}i \pm \sqrt{(\sqrt{6}i)^2 - 4(1)(-3)}}{2}$$

$$z = \frac{-\sqrt{6}i \pm \sqrt{-6+12}}{2} = \frac{-\sqrt{6}i \pm \sqrt{6}}{2}$$

The roots are $z_1 = \frac{\sqrt{6}-\sqrt{6}i}{2}$ and $z_2 = \frac{-\sqrt{6}-\sqrt{6}i}{2}$

Squaring the roots:

$$z_1^2 = \left(\sqrt{\frac{6}{4}}(1-i)\right)^2 = \frac{3}{2}(1+i^2-2i) = \frac{3}{2}(-2i) = -3i$$

$$z_2^2 = \left(-\sqrt{\frac{6}{4}}(1+i)\right)^2 = \frac{3}{2}(1+i^2+2i) = \frac{3}{2}(2i) = 3i$$

Now find z^8:

$z_1^8 = (z_1^2)^4 = (-3i)^4 = 81i^4 = 81$

$z_2^8 = (z_2^2)^4 = (3i)^4 = 81i^4 = 81$

Sum = $z_1^8 + z_2^8 = 81 + 81 = 162$

Question ID : 6911211204

4. The sum of all possible values of $\theta \in [0, 2\pi]$, for which the system of equations :

$x\cos3\theta - 8y - 12z = 0$

$x\cos2\theta + 3y + 3z = 0$

$x + y + 3z = 0$

has a non-trivial solution, is equal to :

(1) π (2) 2π (3) 3π (4) 4π

Ans. (4)

Sol. For a non-trivial solution, the determinant of the coefficient matrix must be zero:

$$\begin{vmatrix} \cos3\theta & -8 & -12 \\ \cos2\theta & 3 & 3 \\ 1 & 1 & 3 \end{vmatrix} = 0$$

Expanding along the first column:

$\cos 3\theta(9-3)-\cos 2\theta(-24+12)+1(-24+36)=0$

$6\cos 3\theta + 12\cos 2\theta + 12 = 0$

$\cos 3\theta + 2\cos 2\theta + 2 = 0$

Using identities $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$ and $\cos 2\theta = 2\cos^2\theta - 1$:

$(4\cos^3\theta - 3\cos\theta) + 2(2\cos^2\theta - 1) + 2 = 0$

$4\cos^3\theta + 4\cos^2\theta - 3\cos\theta = 0$

$\cos\theta(4\cos^2\theta + 4\cos\theta - 3) = 0$

Factoring the quadratic:

$\cos\theta\,(2\cos\theta - 1)(2\cos\theta + 3) = 0$

Possible values:

1. $\cos\theta = 0 \rightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2}$

2. $\cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}, \frac{5\pi}{3}$

3. $\cos\theta = -\frac{3}{2}$ (No real solution)

$\text{Sum} = \frac{\pi}{2}+\frac{3\pi}{2}+\frac{\pi}{3}+\frac{5\pi}{3} = 2\pi + 2\pi = 4\pi$

Question ID : 6911211205

5. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix}$ and $B = [b_{ij}], 1 \le i, j \le 3$. If $B = A^{99} - I$, then the value of $\frac{b_{31} - b_{21}}{b_{32}}$ is :

(1) 99 (2) 199 (3) 149 (4) 159

Ans. (3)

Sol. Let $A = I + N$, where $N = \begin{bmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 9 & 3 & 0 \end{bmatrix}$

$$N^2 = \begin{bmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 9 & 3 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 9 & 3 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 9 & 0 & 0 \end{bmatrix}$$

$N^3 = O$ (Zero matrix).

Using Binomial Expansion for $A^{99} = (I + N)^{99}$:

$$A^{99} = I + 99N + \frac{99 \times 98}{2}N^2 + 0 + \ldots$$

$B = A^{99} - I = 99N + 4851N^2$

Substituting N and N^2 :

$$B = 99\begin{bmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 9 & 3 & 0 \end{bmatrix} + 4851\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 9 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 297 & 0 & 0 \\ 891 + 43659 & 297 & 0 \end{bmatrix}$$

Identify elements:

$b_{21} = 297$

$b_{32} = 297$

$b_{31} = 99(9) + 4851(9) = 9(99 + 4851) = 9(4950) = 44550$

Now calculate $\frac{b_{31} - b_{21}}{b_{32}}$:

$$\frac{44550 - 297}{297} = \frac{44550}{297} - 1 = 150 - 1 = 149$$

Question ID : 6911211206

6. The sum $1 + \frac{1}{2}(1^2 + 2^2) + \frac{1}{3}(1^2 + 2^2 + 3^2) + \ldots$ upto 10 terms is equal to :

(1) 130 (2) 155 (3) $\frac{315}{2}$ (4) $\frac{325}{2}$

Ans. (3)

Sol. The general term T_n of the series is given by:

$$T_n = \frac{1}{n}\sum_{k=1}^{n} k^2$$

Using the formula for the sum of squares of the first n natural numbers, $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$:

$$T_n = \frac{1}{n} \cdot \frac{n(n+1)(2n+1)}{6} = \frac{2n^2+3n+1}{6}$$

We need to find the sum of the first 10 terms (S_{10}):

$$S_{10} = \sum_{n=1}^{10} T_n = \frac{1}{6}\left[2\sum_{n=1}^{10} n^2 + 3\sum_{n=1}^{10} n + \sum_{n=1}^{10} 1\right]$$

Substitute the summation values for $n = 10$:

$$\sum n^2 = \frac{10 \times 11 \times 21}{6} = 385$$

$$\sum n = \frac{10 \times 11}{2} = 55$$

$$\sum 1 = 10$$

$$S_{10} = \frac{1}{6}[2(385) + 3(55) + 10]$$

$$S_{10} = \frac{1}{6}[770 + 165 + 10] = \frac{945}{6}$$

Dividing by 3:

$$S_{10} = \frac{315}{2}$$

Question ID : 6911211207

7. A building has ground floor and 10 more floors. Nine persons enter in a lift at the ground floor. The lift goes up to the 10^{th} floor. The number of ways, in which any 4 persons exit at a floor and the remaining 5 persons exit at a different floor, if the lift does not stop at the first and the second floors, is equal to :

(1) 2184 (2) 3064 (3) 7056 (4) 11340

Ans. (3)

Sol. 1. Identify available floors: The lift can stop at floors 3 through 10. Total available floors = 10 – 2 = 8

2. Select floors for exit: We need to choose 2 distinct floors from the 8 available floors where people will get off

Number of ways to choose 2 floors = $\binom{8}{2} = 28$

3. Group the persons: The 9 persons are divided into two groups: one of 4 people and one of 5 people

Number of ways to choose 4 people for the first group $= \binom{9}{4} = \frac{9\times8\times7\times6}{4\times3\times2\times1} = 126$

(The remaining 5 automatically form the second group)

4. Assign groups to floors: The two groups (group of 4 and group of 5) can be assigned to the 2 chosen floors in 2! ways

Total ways = 28 × 126 × 2 = 7056

Question ID : 6911211208

8. Let the mean and the variance of seven observations 2, 4, α, 8, β, 12, 14, $\alpha < \beta$, be 8 and 16 respectively. Then the quadratic equation whose roots are $3\alpha + 2$ and $2\beta + 1$ is :

(1) $x^2 - 35x + 306 = 0$ (2) $x^2 - 41x + 420 = 0$

(3) $x^2 - 45x + 506 = 0$ (4) $x^2 - 37x + 342 = 0$

Ans. (2)

Sol. Step 1: Use Mean to find $\alpha + \beta$

$$\text{Mean} = \frac{2+4+\alpha+8+\beta+12+14}{7} = 8$$

$40 + \alpha + \beta = 56 \Rightarrow \alpha + \beta = 16$

Step 2: Use Variance to find $\alpha^2 + \beta^2$

$$\text{Variance} = \frac{\sum x_i^2}{n} - (\bar{x})^2 = 16$$

$$\frac{2^2+4^2+\alpha^2+8^2+\beta^2+12^2+14^2}{7} - 8^2 = 16$$

$$\frac{4+16+\alpha^2+64+\beta^2+144+196}{7} - 64 = 16$$

$$\frac{424+\alpha^2+\beta^2}{7} = 80 \Rightarrow \alpha^2+\beta^2 = 560-424 = 136$$

Step 3: Solve for α and β

$(\alpha+\beta)^2 = \alpha^2+\beta^2+2\alpha\beta \Rightarrow 16^2 = 136+2\alpha\beta$

$256 = 136 + 2\alpha\beta \Rightarrow 2\alpha\beta = 120 \Rightarrow \alpha\beta = 60$

Roots of $t^2 - 16t + 60 = 0$ are 6 and 10. Since $\alpha < \beta$, $\alpha = 6$ and $\beta = 10$

Step 4: Form the new quadratic equation New roots: $R_1 = 3(6) + 2 = 20$ and $R_2 = 2(10) + 1 = 21$

Sum of roots = 20 + 21 = 41; Product of roots = 20 × 21 = 420

Equation: $x^2 - 41x + 420 = 0$

Question ID : 6911211209

9. A bag contains 6 blue and 6 green balls. Pairs of balls are drawn without replacement until the bag is empty. The probability that each drawn pair consists of one blue and one green ball is :

(1) $\frac{63}{925}$ (2) $\frac{17}{231}$ (3) $\frac{16}{231}$ (4) $\frac{64}{925}$

Ans. (3)

Sol. Total balls = 12. We are drawing 6 pairs in succession. Total ways to form 6 pairs from 12 balls:

$$N(S) = \frac{\binom{12}{2}\times\binom{10}{2}\times\binom{8}{2}\times\binom{6}{2}\times\binom{4}{2}\times\binom{2}{2}}{6!}$$

$$N(S) = \frac{12!}{(2!)^6 \cdot 6!}$$

Favorable ways: Each pair has 1 Blue (B) and 1 Green (G). Think of this as arranging 6 B's and matching each with one of the 6 G's. The number of ways to match 6 distinct blue balls with 6 distinct green balls is 6!. However, since we are drawing pairs of balls where the order within the pair doesn't matter, we can simply say:

For the 1st pair: Choose 1B out of 6 and 1G out of 6: $\binom{6}{1}\binom{6}{1}$ For the 2nd pair: Choose 1B out of 5 and 1G out of 5: $\binom{5}{1}\binom{5}{1}$ And so on.

$$N(E) = \frac{(6\times6)\times(5\times5)\times(4\times4)\times(3\times3)\times(2\times2)\times(1\times1)}{6!} = \frac{(6!)^2}{6!} = 6!$$

Probability $P(E) = \frac{N(E)}{N(S)} = \frac{6!}{\frac{12!}{2^6 \cdot 6!}} = \frac{(6!)^2 \cdot 2^6}{12!}$

$$P(E) = \frac{2^6}{\binom{12}{6}} = \frac{64}{924}$$

Simplifying by dividing by 4:

$P(E) = \frac{16}{231}$

Question ID : 6911211210

10. Let C be a circle having centre in the first quadrant and touching the x-axis at a distance of 3 units from the origin. If the circle C has an intercept of length $6\sqrt{3}$ on y-axis, then the length of the chord of the circle C on the line $x - y = 3$ is :

(1) 8 (2) 6 (3) $6\sqrt{2}$ (4) $8\sqrt{2}$

Ans. (3)

Sol. 1. Find the circle equation: Since it touches the x-axis at (3,0), the center is (3, r) and the radius is r

Equation: $(x - 3)^2 + (y - r)^2 = r^2$.

2. Use the y-intercept: Intercept length on y-axis $= 2\sqrt{r^2 - h^2}$, where h = 3

$2\sqrt{r^2 - 3^2} = 6\sqrt{3} \Rightarrow \sqrt{r^2 - 9} = 3\sqrt{3}$

$r^2 - 9 = 27 \Rightarrow r^2 = 36 \Rightarrow r = 6$

Circle: $(x - 3)^2 + (y - 6)^2 = 36$. Center O(3, 6), Radius R = 6

3. Find the chord length on $x - y - 3 = 0$:

Perpendicular distance (d) from (3, 6) to $x - y - 3 = 0$:

$d = \frac{|3 - 6 - 3|}{\sqrt{1^2 + (-1)^2}} = \frac{6}{\sqrt{2}} = 3\sqrt{2}$

Length of chord $= 2\sqrt{R^2 - d^2} = 2\sqrt{36 - (3\sqrt{2})^2} = 2\sqrt{36 - 18} = 2\sqrt{18}$

$= 2 \times 3\sqrt{2} = 6\sqrt{2}$

Question ID : 6911211211

11. The eccentricity of an ellipse E with centre at the origin O is $\frac{\sqrt{3}}{2}$ and its directrices are $x = \pm\frac{4\sqrt{6}}{3}$. Let $H : \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be a hyperbola whose eccentricity is equal to the length of semi-major axis of E, and whose length of latus rectum is equal to the length of minor axis of E. Then the distance between the foci of H is :

(1) $\frac{4\sqrt{2}}{\sqrt{7}}$ (2) $\frac{4\sqrt{2}}{7}$ (3) $\frac{4}{\sqrt{7}}$ (4) $\frac{8}{7}$

Ans. (4)

Sol. Step 1: Determine the parameters of Ellipse E

The eccentricity $e_E = \frac{\sqrt{3}}{2}$

The directrices are $x = \pm\frac{a_E}{e_E} = \pm\frac{4\sqrt{6}}{3}$

$$\frac{a_E}{\sqrt{3}/2} = \frac{4\sqrt{6}}{3} \Rightarrow a_E = \frac{4\sqrt{6}}{3}\times\frac{\sqrt{3}}{2} = \frac{4\sqrt{18}}{6} = \frac{4\times3\sqrt{2}}{6} = 2\sqrt{2}$$

The semi-major axis length is $a_E = 2\sqrt{2}$

The length of the semi-minor axis b_E is given by $b_E^2 = a_E^2(1-e_E^2)$:

$$b_E^2 = (2\sqrt{2})^2\left(1-\frac{3}{4}\right) = 8\times\frac{1}{4} = 2 \Rightarrow b_E = \sqrt{2}$$

The length of the minor axis is $2b_E = 2\sqrt{2}$

Step 2: Determine the parameters of Hyperbola H. Eccentricity of H (e_H) is the semi-major axis of E :

$$e_H = a_E = 2\sqrt{2}$$

Length of latus rectum of H is the minor axis of E : $\frac{2b_H^2}{a_H} = 2b_E = 2\sqrt{2}$. Thus, $b_H^2 = \sqrt{2}a_H$. We use the relation

$b_H^2 = a_H^2(e_H^2-1)$:

$$\sqrt{2}a_H = a_H^2((2\sqrt{2})^2-1) \Rightarrow \sqrt{2}a_H = a_H^2(8-1) = 7a_H^2$$

Since $a_H \neq 0$, $a_H = \frac{\sqrt{2}}{7}$

Step 3: Calculate the distance between the foci of H

The distance between foci $= 2a_H e_H$:

$$2\times\left(\frac{\sqrt{2}}{7}\right)\times(2\sqrt{2}) = \frac{4\times2}{7} = \frac{8}{7}$$

Question ID : 6911211212

12. Let $x = 9$ be a directrix of an ellipse E, whose centre is at the origin and eccentricity is $\frac{1}{3}$. Let $P(\alpha, 0)$, $\alpha > 0$, be a focus of E and AB be a chord passing through P. Then the locus of the mid point of AB is :

(1) $9y^2 = 8x(1 - x)$ (2) $3y^2 = 4x(1 - x)$ (3) $9y^2 = 8x(x - 1)$ (4) $3y^2 = 4x(x - 1)$

Ans. (1)

Sol. Find the equation of the ellipse. Given directrix $x = \frac{a}{e} = 9$ and $e = \frac{1}{3}$

$a = 9e = 9 \times \frac{1}{3} = 3$

$b^2 = a^2(1 - e^2) = 3^2(1 - 1/9) = 9 \times \frac{8}{9} = 8$

Ellipse E : $\frac{x^2}{9} + \frac{y^2}{8} = 1$

Step 2: Identify the focus P. Focus $P = (ae, 0) = \left(3 \times \frac{1}{3}, 0\right) = (1, 0)$

Step 3: Find the locus of the midpoint (h, k) of chord AB

The equation of a chord of an ellipse with midpoint (h, k) is $T = S_1$:

$\frac{xh}{a^2} + \frac{yk}{b^2} = \frac{h^2}{a^2} + \frac{k^2}{b^2} \Rightarrow \frac{xh}{9} + \frac{yk}{8} = \frac{h^2}{9} + \frac{k^2}{8}$

Since the chord passes through P(1, 0)

$\frac{(1)h}{9} + \frac{(0)k}{8} = \frac{h^2}{9} + \frac{k^2}{8} \Rightarrow \frac{h}{9} = \frac{h^2}{9} + \frac{k^2}{8}$

Multiplying by 72:

$8h = 8h^2 + 9k^2 \Rightarrow 9k^2 = 8h - 8h^2 = 8h(1 - h)$

Replacing (h, k) with (x, y), the locus is $9y^2 = 8x(1 - x)$

Question ID : 6911211213

13. If $\sin\left(\tan^{-1}\left(x\sqrt{2}\right)\right) = \cot\left(\sin^{-1}\sqrt{1 - x^2}\right)$, $x \in (0, 1)$, then the value of x is :

(1) $\frac{1}{2}$ (2) $\frac{1}{3}$ (3) $\frac{2}{3}$ (4) $\frac{5}{8}$

Ans. (1)

Sol. Let $\theta = \tan^{-1}(x\sqrt{2}) \Rightarrow \tan\theta = \frac{x\sqrt{2}}{1}$

Then $\sin\theta = \dfrac{x\sqrt{2}}{\sqrt{(x\sqrt{2})^2+1^2}} = \dfrac{x\sqrt{2}}{\sqrt{2x^2+1}}$

Let $\phi = \sin^{-1}\sqrt{1-x^2} \Rightarrow \sin\phi = \dfrac{\sqrt{1-x^2}}{1}$

Then $\cos\phi = \sqrt{1-(\sqrt{1-x^2})^2} = \sqrt{1-1+x^2} = x$ (since $x \in (0, 1)$)

$\cot\phi = \dfrac{\cos\phi}{\sin\phi} = \dfrac{x}{\sqrt{1-x^2}}$

The given equation becomes:

$$\frac{x\sqrt{2}}{\sqrt{2x^2+1}} = \frac{x}{\sqrt{1-x^2}}$$

Since $x \in (0, 1), x \neq 0$

$$\frac{\sqrt{2}}{\sqrt{2x^2+1}} = \frac{1}{\sqrt{1-x^2}}$$

Squaring both sides:

$$\frac{2}{2x^2+1} = \frac{1}{1-x^2} \Rightarrow 2-2x^2 = 2x^2+1$$

$$4x^2 = 1 \Rightarrow x^2 = \frac{1}{4} \Rightarrow x = \frac{1}{2}$$

Question ID : 6911211214

14. The shortest distance between the lines $\dfrac{x-4}{1} = \dfrac{y-3}{2} = \dfrac{z-2}{-3}$ and $\dfrac{x+2}{2} = \dfrac{y-6}{4} = \dfrac{z-5}{-5}$ is :

(1) $\dfrac{5\sqrt{6}}{6}$ (2) $2\sqrt{5}$ (3) $3\sqrt{5}$ (4) $4\sqrt{5}$

Ans. (3)

Sol. Line 1 passes through A(4, 3, 2) with direction $\vec{d_1} = \hat{i}+2\hat{j}-3\hat{k}$

Line 2 passes through B(–2, 6, 5) with direction $\vec{d_2} = 2\hat{i}+4\hat{j}-5\hat{k}$. The shortest distance

$$S.D. = \frac{|(\vec{AB})\cdot(\vec{d_1}\times\vec{d_2})|}{|\vec{d_1}\times\vec{d_2}|}$$

Step 1: Calculate $\vec{AB}$

$\overrightarrow{AB} = (-2-4)\hat{i} + (6-3)\hat{j} + (5-2)\hat{k} = -6\hat{i} + 3\hat{j} + 3\hat{k}$

Step 2: Calculate $\vec{d_1} \times \vec{d_2}$

$$\vec{d_1} \times \vec{d_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix} = \hat{i}(-10+12) - \hat{j}(-5+6) + \hat{k}(4-4) = 2\hat{i} - \hat{j} + 0\hat{k}$$

Magnitude $|\vec{d_1} \times \vec{d_2}| = \sqrt{2^2 + (-1)^2 + 0^2} = \sqrt{5}$

Step 3: Calculate the Shortest Distance

$$\text{S.D.} = \frac{|(-6)(2) + (3)(-1) + (3)(0)|}{\sqrt{5}} = \frac{|-12-3|}{\sqrt{5}} = \frac{15}{\sqrt{5}} = 3\sqrt{5}$$

Question ID : 6911211215

15. Let $\vec{a} = 2\hat{i} + 3\hat{j} + 3\hat{k}$ and $\vec{b} = 6\hat{i} + 3\hat{j} + 3\hat{k}$. Then the square of the area of the triangle with adjacent sides determined by the vectors $(2\vec{a} + 3\vec{b})$ and $(\vec{a} - \vec{b})$:

(1) 450 (2) 900 (3) 1800 (4) 2400

Ans. (3)

Sol. The area of a triangle with adjacent sides $\vec{u}$ and $\vec{v}$ is given by $\frac{1}{2}|\vec{u} \times \vec{v}|$

Here, $\vec{u} = 2\vec{a} + 3\vec{b}$ and $\vec{v} = \vec{a} - \vec{b}$

$\vec{u} \times \vec{v} = (2\vec{a} + 3\vec{b}) \times (\vec{a} - \vec{b})$

Using the distributive property and the fact that $\vec{x} \times \vec{x} = 0$:

$\vec{u} \times \vec{v} = 2(\vec{a} \times \vec{a}) - 2(\vec{a} \times \vec{b}) + 3(\vec{b} \times \vec{a}) - 3(\vec{b} \times \vec{b})$

$\vec{u} \times \vec{v} = 0 - 2(\vec{a} \times \vec{b}) - 3(\vec{a} \times \vec{b}) - 0 = -5(\vec{a} \times \vec{b})$

Step 1: Calculate $\vec{a} \times \vec{b}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 3 \\ 6 & 3 & 3 \end{vmatrix} = \hat{i}(9-9) - \hat{j}(6-18) + \hat{k}(6-18) = 12\hat{j} - 12\hat{k}$$

Step 2: Calculate Area squared

$|\vec{u} \times \vec{v}| = |-5(12\hat{j} - 12\hat{k})| = |60\hat{k} - 60\hat{j}| = \sqrt{60^2 + (-60)^2} = 60\sqrt{2}$

$\text{Area} = \frac{1}{2} \times 60\sqrt{2} = 30\sqrt{2}$

$\text{Area}^2 = (30\sqrt{2})^2 = 900 \times 2 = 1800$

Question ID : 6911211216

16. Let $\lim\limits_{x\to 2} \frac{(\tan(x-2))(rx^2+(p-2)x-2p)}{(x-2)^2} = 5$ for some r, p $\in$ R. If the set of all possible values of q, such that the roots of the equation $rx^2 - px + q = 0$ lie in (0, 2), be the interval $(\alpha, \beta]$, then $4(\alpha + \beta)$ equals :

(1) 11 (2) 13 (3) 17 (4) 21

Ans. (3)

Sol. We start by simplifying the limit. Given:

$$\lim_{x\to 2}\left[\frac{\tan(x-2)}{x-2} \cdot \frac{rx^2+(p-2)x-2p}{x-2}\right] = 5$$

Since $\lim\limits_{x\to 2} \frac{\tan(x-2)}{x-2} = 1$, the expression reduces to:

$$\lim_{x\to 2} \frac{rx^2+(p-2)x-2p}{x-2} = 5$$

For the limit to exist, the numerator must be zero at x = 2:

$r(2)^2 + (p-2)(2) - 2p = 0 \Rightarrow 4r + 2p - 4 - 2p = 0 \Rightarrow 4r = 4 \Rightarrow r = 1$

Substituting r =1 back into the limit:

$$\lim_{x\to 2} \frac{x^2+(p-2)x-2p}{x-2} = \lim_{x\to 2} \frac{(x-2)(x+p)}{x-2} = \lim_{x\to 2}(x+p) = 2+p$$

Set $2 + p = 5 \Rightarrow p = 3$

The quadratic equation is $rx^2 - px + q = 0 \Rightarrow x^2 - 3x + q = 0$. For its roots to lie in (0, 2)

1. Discriminant: $D \geq 0 \Rightarrow 9 - 4q \geq 0 \Rightarrow q \leq 2.25$

2. Interval bounds: Let $f(x) = x^2 - 3x + q$

$f(0) > 0 \Rightarrow q > 0$

$f(2) > 0 \Rightarrow 4 - 6 + q > 0 \Rightarrow q > 2$

Vertex: $0 < \frac{-b}{2a} < 2 \Rightarrow 0 < 1.5 < 2$ (True)

Thus, $q \in (2, 2.25]$, where $\alpha = 2$ and $\beta = 2.25$

$4(\alpha + \beta) = 4(2 + 2.25) = 4(4.25) = 17$

Question ID : 6911211217

17. Let $A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 1 & \alpha \\ 0 & 1 & -1 \end{bmatrix}$ be a singular matrix. Let $f(x) = \int_0^x (t^2 + 2t + 3)dt, x \in [1, \alpha]$. If M and m are respectively the maximum and the minimum values of f in [1, α], then 3(M – m) is equal to :

(1) 64 (2) 68 (3) 72 (4) 76

Ans. (2)

Sol. Since A is a singular matrix, its determinant is zero (|A| = 0):

$$\begin{vmatrix} 1 & 3 & -1 \\ 2 & 1 & \alpha \\ 0 & 1 & -1 \end{vmatrix} = 1(-1-\alpha) - 3(-2) - 1(2) = 0 \Rightarrow -1 - \alpha + 6 - 2 = 0 \Rightarrow \alpha = 3$$

The function f(x) is defined on $x \in [1, 3]$

$$f(x) = \int_0^x (t^2 + 2t + 3)dt = \left[\frac{t^3}{3} + t^2 + 3t\right]_0^x = \frac{x^3}{3} + x^2 + 3x$$

Differentiating to check for extrema: $f'(x) = x^2 + 2x + 3$. Since the discriminant of f′(x) is negative ($2^2 - 4(3) = -8$), f′(x) is always positive. Thus, f(x) is strictly increasing on [1, 3]

Minimum (m) at x = 1 : $m = f(1) = \frac{1}{3} + 1 + 3 = \frac{13}{3}$

Maximum (M) at x = 3 : $M = f(3) = \frac{27}{3} + 9 + 9 = 27$

$$3(M - m) = 3\left(27 - \frac{13}{3}\right) = 81 - 13 = 68$$

Question ID : 6911211218

18. Let $f : R \to R$ be such that f(xy) = f(x) f(y), for all $x, y \in R$ and $f(0) \neq 0$. Let $g : [1, \infty) \to R$ be a differentiable function such that $x^2 g(x) = \int_1^x (t^2 f(t) - t g(t))dt$. Then g(2) is equal to :

(1) $\frac{13}{8}$ (2) $\frac{11}{16}$ (3) $\frac{15}{32}$ (4) $\frac{17}{64}$

Ans. (3)

Sol. First, we find f(x). From f(xy) = f(x)f(y), setting x = 0 gives f(0) = f(0)f(y). Since f(0) ≠ 0, we have f(y) = 1 for all y ∈ R. Now, the integral equation becomes:

$$x^2g(x) = \int_1^x (t^2(1) - tg(t))dt$$

Differentiating both sides with respect to x

$$2xg(x) + x^2g'(x) = x^2 - xg(x) \Rightarrow x^2g'(x) + 3xg(x) = x^2$$

Dividing by x (for x ≥ 1): x g'(x) + 3 g(x) = x $\Rightarrow g'(x) + \frac{3}{x}g(x) = 1$

This is a linear differential equation with $\text{I.F.} = e^{\int \frac{3}{x}dx} = x^3$

$$g(x)\cdot x^3 = \int (x^3 \cdot 1)dx = \frac{x^4}{4} + C$$

$$\text{At } x = 1,\ 1^2g(1) = \int_1^1 \ldots = 0 \Rightarrow g(1) = 0$$

$$0 = \frac{1}{4} + C \Rightarrow C = -1/4$$

$$g(x) = \frac{x}{4} - \frac{1}{4x^3} \Rightarrow g(2) = \frac{2}{4} - \frac{1}{4(2^3)} = \frac{1}{2} - \frac{1}{32} = \frac{15}{32}$$

Question ID : 6911211219

19. The area of the region $\{(x, y) : x^2 - 8x \le y \le -x\}$ is :

(1) $\frac{343}{6}$ (2) $\frac{637}{6}$ (3) $\frac{437}{6}$ (4) $\frac{523}{6}$

Ans. (1)

Sol. The region is bounded by the parabola y = x² – 8x and the line y = –x. Find the points of intersection:

$$x^2 - 8x = -x \Rightarrow x^2 - 7x = 0 \Rightarrow x(x - 7) = 0 \Rightarrow x = 0, 7$$

The area A is given by:

$$A = \int_0^7 [(-x) - (x^2 - 8x)]dx = \int_0^7 (7x - x^2)dx$$

$$A = \left[\frac{7x^2}{2} - \frac{x^3}{3}\right]_0^7 = \frac{7(49)}{2} - \frac{343}{3} = \frac{343}{2} - \frac{343}{3}$$

$$A = 343\left(\frac{1}{2}-\frac{1}{3}\right) = \frac{343}{6}$$

Question ID : 6911211220

20. The value of the integral $\int_{-1}^{1}\left(\frac{x^3+|x|+1}{x^2+2|x|+1}\right)dx$ is equal to :

(1) $3\log_e 2$ (2) $2\log_e 2$ (3) $5\log_e 3$ (4) $3\log_e 3$

Ans. (2)

Sol. Let $I = \int_{-1}^{1}\frac{x^3+|x|+[cite_{s}tart]1}{(|x|+1)^2}dx$

We can split the numerator:

$$I = \int_{-1}^{1}\frac{x^3}{(|x|+1)^2}dx + \int_{-1}^{1}\frac{|x|+1}{(|x|+1)^2}dx$$

For the first part, let $h(x) = \frac{x^3}{(|x|+1)^2}$. Since $h(-x) = -h(x)$, it is an odd function and its integral from -1 to 1 is 0

For the second part, $g(x) = \frac{1}{|x|+1}$ is an even function:

$$I = 0 + 2\int_{0}^{1}\frac{1}{x+1}dx = 2[\ln(x+1)]_0^1 = 2(\ln 2 - \ln 1) = 2\ln 2$$

SECTION – B

Question ID : 6911211221

21. Let R = { (x, y) $\in N \times N : \log_e(x+y) \le 2\}$. Then the minimum number of elements, required to be added in R to make it a transitive relation, is _________.

Ans. (15)

Sol. The condition $\log_e(x+y) \le 2 \Rightarrow x+y \le e^2$. Given $e \approx 2.718, e^2 \approx 7.389$. Since $x, y \in N$, the condition simplifies to:

$x + y \le 7$

The possible values for x and y are from the set A = {1, 2, 3, 4, 5, 6}. The relation R consists of the following 21 elements:

For x = 1: (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)

For x = 2: (2, 1), (2, 2), (2, 3), (2, 4), (2, 5)

For x = 3: (3, 1), (3, 2), (3, 3), (3, 4)

For x = 4: (4, 1), (4, 2), (4, 3)

For x = 5: (5, 1), (5, 2)

For x = 6: (6, 1)

For transitivity, if $(x, y) \in R$ and $(y, z) \in R$, then (x, z) must be in R

Since $(x, 1) \in R$ for all $x \in A$ and $(1, z) \in R$ for all $z \in A$, transitivity requires $(x, z) \in R$ for all $x, z \in A$

Thus, the transitive closure must be the set $A \times A$

Number of elements in $A \times A = 6 \times 6 = 36$

Number of elements to be added = 36 – 21 = 15

Question ID : 6911211222

22. If $(1-x^3)^{10} = \sum_{r=0}^{10} a_r x^r (1-x)^{30-2r}$, then $\frac{9a_9}{a_{10}}$ is equal to: ________.

Ans. (30)

Sol. The given identity is:

$$(1-x^3)^{10} = \sum_{r=0}^{10} a_r x^r (1-x)^{30-2r}$$

Divide both sides by $(1-x)^{30}$:

$$\frac{((1-x)(1+x+x^2))^{10}}{(1-x)^{30}} = \sum_{r=0}^{10} a_r \frac{x^r}{(1-x)^{2r}}$$

$$\left(\frac{1+x+x^2}{(1-x)^2}\right)^{10} = \sum_{r=0}^{10} a_r \left(\frac{x}{(1-x)^2}\right)^r$$

Let $y = \frac{x}{(1-x)^2}$. Note that $\frac{1+x+x^2}{(1-x)^2} = \frac{(x^2-2x+1)+3x}{(1-x)^2} = \frac{(1-x)^2+3x}{(1-x)^2} = 1+3y$

The equation becomes:

$(1+3y)^{10} = \sum_{r=0}^{10} a_r y^r$

Comparing this to the binomial expansion $(1+3y)^{10} = \sum_{r=0}^{10} \binom{10}{r}(3y)^r$:

$a_r = \binom{10}{r}3^r$

Calculate the required ratio:

$a_9 = \binom{10}{9}3^9 = 10 \cdot 3^9$

$a_{10} = \binom{10}{10}3^{10} = 3^{10}$

$\frac{9a_9}{a_{10}} = \frac{9 \cdot 10 \cdot 3^9}{3^{10}} = \frac{10 \cdot 3^{11}}{3^{10}}$

$\frac{9a_9}{a_{10}} = \frac{3^2 \cdot 10 \cdot 3^9}{3^{10}} = \frac{10 \cdot 3^{11}}{3^{10}} = 10 \times 3 = 30$

Question ID : 6911211223

23. Let the line $x - y = 4$ intersect the circle $C : (x - 4)^2 + (y + 3)^2 = 9$ at the points Q and R. If $P(\alpha, \beta)$ is a point on C such that PQ = PR, then $(6\alpha + 8\beta)^2$ is equal to _________.

Ans. (18)

Sol. The circle C has center O(4, –3) and radius $r = 3$. Since P is on the circle and equidistant from Q and R (PQ = PR), P must lie on the perpendicular bisector of the chord QR. The perpendicular bisector of any chord passes through the center of the circle.

1. Find the perpendicular bisector: The slope of the line $x - y = 4$ is $m = 1$. The slope of the perpendicular bisector is $m' = -1$

Equation through (4, –3): $y - (-3) = -1(x - 4) \Rightarrow x + y = 1$

2. Find coordinates of $P(\alpha, \beta)$: P is the intersection of $x + y = 1$ and the circle

Substitute $y = 1 - x$ into $(x - 4)^2 + (y + 3)^2 = 9$:

$(x - 4)^2 + (1 - x + 3)^2 = 9 \Rightarrow (x - 4)^2 + (4 - x)^2 = 9 \Rightarrow 2(x - 4)^2 = 9$

$(x-4)^2 = 4.5 \Rightarrow x-4 = \pm\sqrt{4.5} = \pm\frac{3}{\sqrt{2}}$

$\alpha = 4 \pm \frac{3}{\sqrt{2}}$ and $\beta = 1-\alpha = -3 \mp \frac{3}{\sqrt{2}}$

3. Calculate $(6\alpha + 8\beta)^2$:

$6\alpha + 8\beta = 6\alpha + 8(1-\alpha) = 8 - 2\alpha$

Substituting $\alpha = 4 \pm \frac{3}{\sqrt{2}}$

$8 - 2\left(4 \pm \frac{3}{\sqrt{2}}\right) = 8 - 8 \mp \frac{6}{\sqrt{2}} = \mp 3\sqrt{2}$

$(6\alpha + 8\beta)^2 = (\mp 3\sqrt{2})^2 = 9 \times 2 = 18$

Question ID : 6911211224

24. Let the image of the point P(0, –5, 0) in the line $\frac{x-1}{2} = \frac{y}{1} = \frac{z+1}{-2}$ be the point R and the image of the point $Q\left(0, \frac{-1}{2}, 0\right)$ in the line $\frac{x-1}{-1} = \frac{y+9}{4} = \frac{z+1}{1}$ be the point S. Then the square of the area of the parallelogram PQRS is _________.

Ans. (162)

Sol. 1. Find Image R: Foot of perpendicular M of P(0, –5, 0) on $L_1(1+2\lambda, \lambda, -1-2\lambda)$:

Vector $\overrightarrow{PM} = (1+2\lambda, \lambda+5, -1-2\lambda)$. Dot with direction (2, 1, –2):

$2(1+2\lambda) + 1(\lambda+5) - 2(-1-2\lambda) = 0 \Rightarrow 9\lambda + 9 = 0 \Rightarrow \lambda = -1$

M = (–1, –1, 1). Image R = 2M – P = (–2, –2, 2) – (0, –5, 0) = (–2, 3, 2)

2. Find Image S: Foot of perpendicular N of Q(0, –0.5, 0 on $L_2(1-\mu, -9+4\mu, -1+\mu)$:

Vector $\overrightarrow{QN} = (1-\mu, -8.5+4\mu, -1+\mu)$. Dot with direction (–1, 4, 1):

$-1(1-\mu) + 4(-8.5+4\mu) + 1(-1+\mu) = 0 \Rightarrow 18\mu - 36 = 0 \Rightarrow \mu = 2$

N = (–1, –1, 1). Image S = 2N – Q = (–2, –2, 2) – (0, –0.5, 0) = (–2, –1.5, 2)

3. Calculate Area: Points are P(0, –5, 0), Q(0, –0.5, 0), R(–2, 3, 2), S(–2, –1.5, 2)

$\overrightarrow{PQ} = (0, 4.5, 0)$ and $\overrightarrow{PS} = (-2, 3.5, 2)$

Area Vector $\vec{A} = \overrightarrow{PQ} \times \overrightarrow{PS} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 4.5 & 0 \\ -2 & 3.5 & 2 \end{vmatrix} = 9\hat{i} + 9\hat{k}$

$\text{Area}^2 = |\vec{A}|^2 = 9^2 + 9^2 = 162$

Question ID : 6911211225

25. Let $f(x) = \begin{cases} x^3 + 8; & x < 0, \\ x^2 - 4; & x \geq 0, \end{cases}$ and $g(x) = \begin{cases} (x-8)^{\frac{1}{3}}; & x < 0, \\ (x+4)^{\frac{1}{2}}; & x \geq 0. \end{cases}$ Then the number of points, where the function gof is discontinuous, is ________.

Ans. (3)

Sol. We check the continuity of $h(x) = g(f(x))$

1. At $x = 0$: $f(x)$ is discontinuous ($f(0^-) = 8$, $f(0^+) = -4$)

$h(0^+) = g(-4) = (-12)^{1/3}$

$h(0^-) = g(8^-) = \sqrt{12}$

Since $h(0^+) \neq h(0^-)$, $h(x)$ is discontinuous at $x = 0$

2. Where $f(x)$ passes through 0 (discontinuity of g):

For $x < 0$, $f(x) = x^3 + 8 = 0 \Rightarrow x = -2$

$h(-2^+) = g(0^+) = 2$

$h(-2^-) = g(0^-) = -2$

Discontinuous at $x = -2$

For $x \geq 0$, $f(x) = x^2 - 4 = 0 \Rightarrow x = 2$

$h(2^+) = g(0^+) = 2$

$h(2^-) = g(0^-) = -2$

Discontinuous at $x = 2$

Total points of discontinuity = 3