JEE Main April 2026

Question Paper With Text Solution

06 April | Shift-1

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

Office : Piprali Road, Sikar (Raj.) | Ph. 01572-241911
Website : www.matrixedu.in ; Email : smd@matrixacademy.co.in

JEE MAIN APRIL 2026 | 6^{TH} APRIL SHIFT-1

SECTION – A

Question ID : 6952782136

1. Let [.] denote the greatest integer function. If the domain of the function $f(x) = \sin^{-1}\left(\frac{x+[x]}{3}\right)$ is $[\alpha, \beta)$, then $\alpha^2 + \beta^2$ is equal to :

(1) 2 (2) 5 (3) 10 (4) 13

Ans. (2)

Sol. For the function $f(x) = \sin^{-1}\left(\frac{x+[x]}{3}\right)$ to be defined, the argument of the $\sin^{-1}$ function must lie in the interval [–1, 1].

$$-1 \le \frac{x+[x]}{3} \le 1$$

Multiplying by 3:

$$-3 \le x + [x] \le 3$$

Let g(x) = x + [x]. We need to find the values of x such that $-3 \le g(x) \le 3$. Since g(x) is a non-decreasing function, we can check the boundaries :

1. Lower Bound $(g(x) \ge -3)$:

If x = –1, g(–1) = –1 + [–1] = –1 – 1 = –2 ≥ –3 (True).

If x = –1.5, g(–1.5) = –1.5 + [–1.5] = –1.5 – 2 = –3.5 < –3 (False).

At the jump point x = –1, the function value jumps from below –3 to –2. Thus, the minimum value is $\alpha = -1$.

2.Upper Bound $(g(x) \le 3)$:

If x = 2, g(2) = 2 + [2] = 2 + 2 = 4 > 3 (False).

If x is slightly less than 2 (e.g., x = 1.99), g(1.99) = 1.99 + [1.99] = 1.99 + 1 = 2.99 ≤ 3 (True).

As $x \to 2^-$, $g(x) \to 3$. Thus, the maximum value is $\beta = 2$ (exclusive).

The domain is [–1, 2), meaning $\alpha = -1$ and $\beta = 2$.

Calculations:

$\alpha^2 + \beta^2 = (-1)^2 + 2^2 = 1 + 4 = 5$

Question ID : 6952782137

2. Let one root of the quadratic equation in x : $(k^2 - 15k + 27)x^2 + 9(k - 1)x + 18 = 0$ be twice the other. Then the length of the latus rectum of the parabola $y^2 = 6kx$ is equal to :

(1) 4 (2) 6 (3) 8 (4) 12

Ans. (4)

Sol. Let the roots of the quadratic equation be m and 2m.

From the properties of quadratic equations:

1. Sum of roots :

$$m + 2m = 3m = \frac{-9(k-1)}{k^2 - 15k + 27} \Rightarrow m = \frac{-3(k-1)}{k^2 - 15k + 27}$$

2. Product of roots :

$$m \cdot 2m = 2m^2 = \frac{18}{k^2 - 15k + 27} \Rightarrow m^2 = \frac{9}{k^2 - 15k + 27}$$

Substituting the value of m from the first equation into the second :

$$\left(\frac{-3(k-1)}{k^2 - 15k + 27}\right)^2 = \frac{9}{k^2 - 15k + 27}$$

$$\frac{9(k-1)^2}{(k^2 - 15k + 27)^2} = \frac{9}{k^2 - 15k + 27}$$

Assuming $k^2 - 15k + 27 \neq 0$:

$(k - 1)^2 = k^2 - 15k + 27$

$k^2 - 2k + 1 = k^2 - 15k + 27$

$13k = 26 \Rightarrow k = 2$

The parabola is $y^2 = 6(2)x \Rightarrow y^2 = 12x$.

The length of the latus rectum for a parabola $y^2 = 4ax$ is 4a. Here, $4a = 12$.

Question ID : 6952782138

3. Let e_1 and e_2 be two distinct roots of the equation $x^2 - ax + 2 = 0$. Let the sets $\{a \in R : e_1$ and e_2 are the eccentricities of hyperbolas$\} = (\alpha, \beta)$, and $\{a \in R : e_1$ and e_2 are the eccentricities of an ellipse and a hyperbola, respectively$\} = (\gamma, \infty)$. Then $\alpha^2 + \beta^2 + \gamma^2$ is equal to :

(1) 18 (2) 22 (3) 26 (4) 34

Ans. (3)

Sol. Given equation: $x^2 - ax + 2 = 0$. Roots are e_1, e_2.

Sum of roots $e_1 + e_2 = a$; Product of roots $e_1 e_2 = 2$.

Since roots must be distinct and real, $D > 0$

$\Rightarrow a^2 - 8 > 0 \Rightarrow a > 2\sqrt{2}$ (as $e > 0$).

Case 1: Both are hyperbolas ($e_1 > 1$, $e_2 > 1$)

$e_1 + e_2 = a > 2$

$(e_1 - 1)(e_2 - 1) > 0 \Rightarrow e_1e_2 - (e_1 + e_2) + 1 > 0$

$\Rightarrow 2 - a + 1 > 0 \Rightarrow a < 3$.

Combining with $D > 0$, we get $a \in \left(2\sqrt{2}, 3\right)$.

Thus, $\alpha = 2\sqrt{2}$ and $\beta = 3$.

Case 2: One ellipse ($0 < e_1 < 1$) and one hyperbola ($e_2 > 1$)

Since $e_1 e_2 = 2$ and $e_1 < 1$, it follows that $e_2 = 2/e_1 > 2$.

The condition for one root to be in $(0, 1)$ and the other to be in $(2, \infty)$ for $f(x) = x^2 - ax + 2$ is $f(1) < 0$.

$1 - a + 2 < 0 \Rightarrow a > 3$.

Thus, $\gamma = 3$.

Calculation :

$\alpha^2 + \beta^2 + \gamma^2 = \left(2\sqrt{2}\right)^2 + 3^2 + 3^2 = 8 + 9 + 9 = 26$

Question ID : 6952782139

4. Let the set of all values of $k \in R$ such that the equation $z(\bar{z} + 2 + i) + k(2 + 3i) = 0$, $z \in C$, has at least one solution, be the interval $[\alpha, \beta]$. Then $9(\alpha + \beta)$ is equal to :

(1) –10 (2) –8 (3) $10\sqrt{13}$ (4) $8\sqrt{13}$

Ans. (1)

Sol. Let $z = x + iy$. Then $\bar{z} = x - iy$.

The equation is : $z\bar{z} + z(2 + i) + k(2 + 3i) = 0$

$(x^2 + y^2) + (x + iy)(2 + i) + 2k + 3ki = 0$

$(x^2 + y^2 + 2x - y + 2k) + i(x + 2y + 3k) = 0$

Equating real and imaginary parts to zero :

1. $x + 2y + 3k = 0 \Rightarrow x = -2y - 3k$

2. $x^2 + y^2 + 2x - y + 2k = 0$

Substitute x into the second equation :

$(-2y - 3k)^2 + y^2 + 2(-2y - 3k) - y + 2k = 0$

$(4y^2 + 9k^2 + 12ky) + y^2 - 4y - 6k - y + 2k = 0$

$5y^2 + y(12k - 5) + (9k^2 - 4k) = 0$

For at least one real solution for y, the discriminant $D \geq 0$:

$(12k - 5)^2 - 4(5)(9k^2 - 4k) \geq 0$

$144k^2 + 25 - 120k - 180k^2 + 80k \geq 0$

$-36k^2 - 40k + 25 \geq 0 \Rightarrow 36k^2 + 40k - 25 \leq 0$

The roots of $36k^2 + 40k - 25 = 0$ are :

$$k = \frac{-40 \pm \sqrt{1600 - 4(36)(-25)}}{72} = \frac{-40 \pm \sqrt{1600 + 3600}}{72}$$

$$= \frac{-40 \pm 20\sqrt{13}}{72} = \frac{-10 \pm 5\sqrt{13}}{18}$$

The interval is $[\alpha, \beta]$ where $a = \frac{-10 - 5\sqrt{13}}{18}$ and $\beta = \frac{-10 + 5\sqrt{13}}{18}$.

$$a + \beta = \frac{-20}{18} = -\frac{10}{9}$$

$$9(a + \beta) = 9\left(-\frac{10}{9}\right) = -10$$

Question ID : 6952782140

5. The value of $1^3 - 2^3 + 3^3 - .... + 15^3$ is :

(1) 1706 (2) 1856 (3) 1982 (4) 2403

Ans. (2)

Sol. We need to find the sum $S = 1^3 - 2^3 + 3^3 - 4^3 + ... + 15^3$.

This can be written as :

$S = (1^3 + 2^3 + 3^3 + ... + 15^3) - 2(2^3 + 4^3 + 6^3 + ... + 14^3)$

$$S = \sum_{r=1}^{15} r^3 - 2 \cdot 2^3 \sum_{r=1}^{7} r^3$$

$S = \sum_{r=1}^{15} r^3 - 16\sum_{r=1}^{7} r^3$

Using the formula $\sum n^3 = \left(\frac{n(n+1)}{2}\right)^2$:

$\sum_{r=1}^{15} r^3 = \left(\frac{15\times16}{2}\right)^2 = (120)^2 = 14400$

$\sum_{r=1}^{7} r^3 = \left(\frac{7\times8}{2}\right)^2 = (28)^2 = 784$

Now, calculate S :

S = 14400 – 16(784)

S = 14400 – 12544 = 1856

Question ID : 6952782141

6. The sum of the first ten terms of an A.P. is 160 and the sum of the first two terms of a G.P. is 8. If the first term of the A.P. is equal to the common ratio of the G.P. and the first term of the G.P. is equal to common difference of the A.P., then the sum of all possible values of the first term of the G.P. is :

(1) $\frac{34}{9}$ (2) $\frac{34}{13}$ (3) $\frac{32}{9}$ (4) $\frac{32}{13}$

Ans. (1)

Sol. Let the A.P. have first term a and common difference d. Let the G.P. have first term A and common ratio r. From the problem :

1. Sum of first 10 terms of A.P. : $S_{10} = \frac{10}{2}[2a+9d] = 160 \Rightarrow 2a+9d = 32$.

2. Sum of first 2 terms of G.P.: $A + Ar = 8 \Rightarrow A(1+r) = 8$.

3. Conditions : a = r and A = d.

Substitute a = r and A = d into the equations :

1. $2r + 9d = 32 \Rightarrow 2r = 32 - 9d \Rightarrow r = \frac{32-9d}{2}$.

2. d(1 + r) = 8.

Substitute r into the second equation:

$$d\left(1+\frac{32-9d}{2}\right)=8$$

$$d\left(\frac{2+32-9d}{2}\right)=8$$

$$d(34-9d)=16 \Rightarrow 34d-9d^2=16$$

$9d^2 - 34d + 16 = 0$

The first term of the G.P. is A = d. The possible values of d are the roots of the quadratic equation $9d^2 - 34d + 16 = 0$.

The sum of all possible values of d (sum of roots) is :

$$\text{Sum} = -\frac{-34}{9} = \frac{34}{9}$$

Question ID : 6952782142

7. The number of 4-letter words, with or without meaning, each consisting of two vowels and two consonants that can be formed from the letters of the word INCONSEQUENTIAL, without repeating any letter, is :

(1) 2670 (2) 2840 (3) 2920 (4) 3600

Ans. (4)

Sol. First, identify the unique vowels and consonants in "INCONSEQUENTIAL".

Vowels: I, O, E, U, A. (5 unique types).

Consonants: N, C, S, Q, T, L. (6 unique types).

Since we must form a word "without repeating any letter", we choose from the distinct types available.

1. Select 2 vowels from 5 : $\binom{5}{2} = \frac{5\times4}{2} = 10$.

2. Select 2 consonants from 6 : $\binom{6}{2} = \frac{6\times5}{2} = 15$.

3. Arrange the 4 chosen letters: 4! [cite$_s$tart] = 24.

Total number of words :

Total = 10 × 15 × 24 = 150 × 24 = 3600

Question ID : 6952782143

8. If the coefficients of the middle terms in the binomial expansions of $(1+\alpha x)^{26}$ and $(1-\alpha x)^{28}$, $\alpha \neq 0$, are equal, then the value of α is :

(1) 1 (2) $\frac{14}{13}$ (3) $\frac{27}{7}$ (4) $\frac{7}{27}$

Ans. (4)

Sol. For $(1+ax)^{26}$, n = 26 (even). The middle term is $T_{\frac{26}{2}+1} = T_{14}$.

$$\text{Coefficient of } T_{14} = \binom{26}{13} a^{13}.$$

For $(1-ax)^{28}$, n = 28 (even). The middle term is $T_{\frac{28}{2}+1} = T_{15}$.

$$\text{Coefficient of } T_{15} = \binom{28}{14}(-a)^{14} = \binom{28}{14} a^{14}.$$

According to the question :

$$\binom{26}{13} a^{13} = \binom{28}{14} a^{14}$$

$$a = \frac{\binom{26}{13}}{\binom{28}{14}}$$

$$a = \frac{26!}{13!\cdot 13!} \times \frac{14!\cdot 14!}{28!}$$

$$a = \frac{26!}{28!} \times \frac{14\cdot 13!}{13!} \times \frac{14\cdot 13!}{13!}$$

$$a = \frac{1}{28\times 27} \times 14 \times 14 = \frac{196}{756}$$

$$a = \frac{14\times 14}{2\times 14\times 27} = \frac{14}{2\times 27} = \frac{7}{27}$$

Question ID : 6952782144

9. A data consists of 20 observations $x_1, x_2, \ldots\ldots\ldots, x_{20}$. If $\sum_{i=1}^{20}(x_i+5)^2 = 2500$ and $\sum_{i=1}^{20}(x_i-5)^2 = 100$, then the ratio of mean to standard deviation of this data is :

(1) 2 : 1 (2) 3 : 1 (3) 3 : 2 (4) 4 : 1

Ans. (2)

Sol. Expand the given equations:

1. $\sum(x_i^2+10x_i+25)=2500 \Rightarrow \sum x_i^2+10\sum x_i+500=2500$

$\Rightarrow \sum x_i^2+10\sum x_i=2000$.

2. $\sum(x_i^2-10x_i+25)=100 \Rightarrow \sum x_i^2-10\sum x_i+500=100$

$\Rightarrow \sum x_i^2-10\sum x_i=-400$.

Subtract (2) from (1) :

$20\sum x_i=2400 \Rightarrow \sum x_i=120$

Mean $(\bar{x})=\frac{\sum x_i}{20}=\frac{120}{20}=6$.

Add (1) and (2) :

$2\sum x_i^2=1600 \Rightarrow \sum x_i^2=800$

Variance $(\sigma^2)=\frac{\sum x_i^2}{n}-(\bar{x})^2=\frac{800}{20}-(6)^2=40-36=4$.

Standard Deviation $(\sigma)=\sqrt{4}=2$.

Ratio of mean to standard deviation :

$\frac{\bar{x}}{\sigma}=\frac{6}{2}=\frac{3}{1}$

Question ID : 6952782145

10. A bag contains (N + 1) coins–N fair coins, and one coin with 'Head' on both sides. A coin is selected at random and tossed. If the probability of getting 'Head' is $\frac{9}{16}$, then N is equal to :

(1) 5 (2) 7 (3) 8 (4) 9

Ans. (2)

Sol. Total coins = N + 1. Let E_1 be the event of selecting a fair coin, and E_2 be the event of selecting the two-headed coin. $P(E_1)=\frac{N}{N+1}$ and $P(E_2)=\frac{1}{N+1}$.

Let H be the event of getting a 'Head'. $P(H\,|\,E_1)=\frac{1}{2}$ (fair coin). $P(H|E_2)=1$ (two-headed coin).

Using the Law of Total Probability :

$$P(H) = P(E_1)P(H|E_1) + P(E_2)P(H|E_2)$$

$$\frac{9}{16} = \left(\frac{N}{N+1}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{N+1}\right)(1)$$

$$\frac{9}{16} = \frac{N}{2(N+1)} + \frac{2}{2(N+1)} = \frac{N+2}{2(N+1)}$$

$18(N + 1) = 16(N + 2)$

$18N + 18 = 16N + 32$

$2N = 14 \Rightarrow N = 7$

Question ID : 6952782146

11. If the eccentricity e of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ passing through $(6, 4\sqrt{3})$ satisfies $15(e^2 + 1) = 34e$, then the length of the latus rectum of the hyperbola $\frac{x^2}{b^2} - \frac{y^2}{2(a^2+1)} = 1$ is :

(1) 10 (2) 20 (3) 25 (4) 30

Ans. (1)

Sol. First, we find the eccentricity e from the given equation $15e^2 - 34e + 15 = 0$:

$$e = \frac{34 \pm \sqrt{34^2 - 4(15)(15)}}{30} = \frac{34 \pm \sqrt{1156 - 900}}{30} = \frac{34 \pm 16}{30}$$

The values are $e = \frac{50}{30} = \frac{5}{3}$ or $e = \frac{18}{30} = \frac{3}{5}$. Since it is a hyperbola, $e > 1$, so $e = \frac{5}{3}$.

The relation between a, b, and e is $b^2 = a^2(e^2 - 1)$:

$$b^2 = a^2\left(\frac{25}{9} - 1\right) = \frac{16a^2}{9} \Rightarrow \frac{1}{b^2} = \frac{9}{16a^2}$$

The hyperbola passes through $\left(6, 4\sqrt{3}\right)$:

$$\frac{36}{a^2} - \frac{48}{b^2} = 1 \Rightarrow \frac{36}{a^2} - 48\left(\frac{9}{16a^2}\right) = 1 \Rightarrow \frac{36-27}{a^2} = 1 \Rightarrow a^2 = 9$$

Thus, $b^2 = \frac{16}{9}(9) = 16$.

The second hyperbola equation is $\frac{x^2}{b^2}-\frac{y^2}{2(a^2+1)}=1$:

$$\frac{x^2}{16}-\frac{y^2}{2(9+1)}=1 \Rightarrow \frac{x^2}{16}-\frac{y^2}{20}=1$$

Length of the latus rectum $LR=\frac{2\cdot(\text{conjugate axis})^2}{\text{transverse axis}}=\frac{2\cdot 20}{\sqrt{16}}=\frac{40}{4}=10$.

Question ID : 6952782147

12. Let chord PQ of length $3\sqrt{13}$ of the parabola $y^2 = 12x$ be such that the ordinates of points P and Q are in the ratio 1 : 2. If the chord PQ subtends an angle α at the focus of the parabola, then $\sin\alpha$ is equal to :

(1) $\frac{3}{5}$ (2) $\frac{4}{5}$ (3) $\frac{5}{13}$ (4) $\frac{12}{13}$

Ans. (1)

Sol. For $y^2 = 12x$, $a = 3$. Let $P(at_1^2, 2at_1)$ and $Q(at_2^2, 2at_2)$.

Given $y_Q = 2y_P \Rightarrow 6t_2 = 2(6t_1) \Rightarrow t_2 = 2t_1$.

The length $PQ=\sqrt{(3t_2^2-3t_1^2)^2+(6t_2-6t_1)^2}=3\sqrt{13}$:

$$[3(4t_1^2-t_1^2)]^2+[6(2t_1-t_1)]^2=\left(3\sqrt{13}\right)^2 \Rightarrow (9t_1^2)^2+(6t_1)^2=117$$

$$81t_1^4+36t_1^2-117=0 \Rightarrow 9t_1^4+4t_1^2-13=0$$

$$\Rightarrow (9t_1^2+13)(t_1^2-1)=0$$

Since $t_1^2 \geq 0, t_1^2=1 \Rightarrow t_1=1$ and $t_2 = 2$.

Points are P(3, 6) and Q(12, 12). Focus S is (3, 0).

Vectors : SP = (0, 6) and SQ = (9, 12).

$$\cos\alpha=\frac{\vec{SP}\cdot\vec{SQ}}{|\vec{SP}||\vec{SQ}|}=\frac{0(9)+6(12)}{6\cdot\sqrt{9^2+12^2}}=\frac{72}{6\cdot 15}=\frac{4}{5}$$

$$\sin\alpha=\sqrt{1-\left(\frac{4}{5}\right)^2}=\frac{3}{5}$$

Question ID : 6952782148

13. Let $0<\alpha<1, \beta=\frac{1}{3\alpha}$ and $\tan^{-1}(1-\alpha)+\tan^{-1}(1-\beta)=\frac{\pi}{4}$. Then $6(\alpha+\beta)$ is equal to :

(1) 6 (2) 7 (3) 8 (4) 9

Ans. (2)

Sol. Using the identity $\tan^{-1}x+\tan^{-1}y=\tan^{-1}\frac{x+y}{1-xy}$:

$$\frac{(1-\alpha)+(1-\beta)}{1-(1-\alpha)(1-\beta)}=\tan\left(\frac{\pi}{4}\right)=1$$

$$\frac{2-(\alpha+\beta)}{1-(1-\alpha-\beta+\alpha\beta)}=1 \Rightarrow \frac{2-(\alpha+\beta)}{\alpha+\beta-\alpha\beta}=1$$

$$2-\alpha-\beta=\alpha+\beta-\alpha\beta \Rightarrow 2(\alpha+\beta)-\alpha\beta=2$$

Substitute $\beta=\frac{1}{3\alpha}$:

$$2\left(\alpha+\frac{1}{3\alpha}\right)-\frac{1}{3}=2 \Rightarrow 2\alpha+\frac{2}{3\alpha}=\frac{7}{3}$$

Multiplying by 3α :

$$6\alpha^2+2=7\alpha \Rightarrow 6\alpha^2-7\alpha+2=0$$

$$\Rightarrow (2\alpha-1)(3\alpha-2)=0$$

The roots are $\alpha=\frac{1}{2}$ or $\alpha=\frac{2}{3}$.

If $\alpha=\frac{1}{2}, \beta=\frac{2}{3}$. If $\alpha=\frac{2}{3}, \beta=\frac{1}{2}$.

In both cases, $\alpha+\beta=\frac{1}{2}+\frac{2}{3}=\frac{7}{6}$.

$$6(\alpha+\beta)=6\cdot\frac{7}{6}=7$$

Question ID : 6952782149

14. Let $S=\{\theta\in(-2\pi,2\pi):\cos\theta+1=\sqrt{3}\sin\theta\}$. Then $\sum_{\theta\in S}\theta$ is equal to :

(1) $-\frac{2\pi}{3}$ (2) $-\frac{4\pi}{3}$ (3) $\frac{2\pi}{3}$ (4) $\frac{4\pi}{3}$

Ans. (2)

Sol. The equation is $\sqrt{3}\sin\theta - \cos\theta = 1$. Dividing by 2 :

$$\frac{\sqrt{3}}{2}\sin\theta - \frac{1}{2}\cos\theta = \frac{1}{2} \Rightarrow \sin\left(\theta - \frac{\pi}{6}\right) = \frac{1}{2}$$

For $\theta \in (-2\pi, 2\pi), \theta - \frac{\pi}{6} \in \left(-\frac{13\pi}{6}, \frac{11\pi}{6}\right)$:

1. $\theta - \frac{\pi}{6} = \frac{\pi}{6} \Rightarrow \theta = \frac{\pi}{3}$

2. $\theta - \frac{\pi}{6} = \frac{5\pi}{6} \Rightarrow \theta = \pi$

3. $\theta - \frac{\pi}{6} = -\frac{7\pi}{6} \Rightarrow \theta = -\pi$

4. $\theta - \frac{\pi}{6} = -\frac{11\pi}{6} \Rightarrow \theta = -\frac{10\pi}{6} = -\frac{5\pi}{3}$

Sum of solutions : $\frac{\pi}{3} + \pi - \pi - \frac{5\pi}{3} = -\frac{4\pi}{3}$.

Question ID : 6952782150

15. Let the image of the point P(1, 6, a) in the line $L: \frac{x}{1} = \frac{y-1}{2} = \frac{z-a+1}{b}$, $b > 0$ be $Q\left(\frac{a}{3}, 0, a+c\right)$. If $S(\alpha, \beta, \gamma)$, $\alpha > 0$ is the point on L such that the distance of S from the foot of perpendicular from the point P on L is $2\sqrt{14}$, then $\alpha + \beta + \gamma$ is equal to :

(1) 19 (2) 20 (3) 21 (4) 22

Ans. (3)

Sol. The midpoint M of PQ lies on L.M $= \left(\frac{1+a/3}{2}, \frac{6+0}{2}, \frac{2a+c}{2}\right) = \left(\frac{3+a}{6}, 3, a+\frac{c}{2}\right)$.

Since M is on L, $\frac{3+a}{6} = \frac{3-1}{2} = 1 \Rightarrow a = 3$.

M is (1, 3, 3 + c/2). L is $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{b}$.

Substituting M coordinates into L : $\frac{1}{1} = \frac{3-1}{2} = \frac{3+c/2-2}{b} \Rightarrow 1 = \frac{1+c/2}{b} \Rightarrow b = 1+\frac{c}{2}$.

$PQ \perp L$: direction vector of PQ is (1 – 1, –6, c) = (0, –6, c). Direction of L is (1, 2, b).

$0(1) - 6(2) + bc = 0 \Rightarrow bc = 12.$

$(1 + c/2)c = 12 \Rightarrow c^2 + 2c - 24 = 0 \Rightarrow c = 4(b = 3).$

The foot of the perpendicular F is M(1, 3, 5).

S on L is $(\lambda, 2\lambda + 1, 3\lambda + 2)$. Distance $SF = 2\sqrt{14}$:

$$(\lambda-1)^2 + (2\lambda-2)^2 + (3\lambda-3)^2 = \left(2\sqrt{14}\right)^2 \Rightarrow 14(\lambda-1)^2 = 56.$$

$(\lambda - 1)^2 = 4 \Rightarrow \lambda = 3$ or -1.

If $\lambda = 3$, $\alpha = 3$ (> 0), so S is (3, 7, 11).

$\alpha + \beta + \gamma = 3 + 7 + 11 = 21.$

Question ID : 6952782151

16. Let a line L be perpendicular to both the lines $L_1 : \frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $L_2 : \frac{x-2}{1} = \frac{y-4}{4} = \frac{z-6}{7}$. If θ is the acute angle between the lines L and $L_3 : \frac{x-\frac{8}{7}}{2} = \frac{y-\frac{4}{7}}{1} = \frac{z}{2}$ then $\tan\theta$ is equal to :

(1) $\frac{3}{2}\sqrt{2}$ (2) $\frac{5}{2}\sqrt{2}$ (3) $\frac{5}{3}\sqrt{2}$ (4) $\frac{4}{3}\sqrt{2}$

Ans. (2)

Sol. The direction ratios of L_1 and L_2 are $v_1 = (3, 5, 7)$ and $v_2 = (1, 4, 7)$. Since L is perpendicular to both, its direction vector v is given by their cross product :

$$\vec{v} = \vec{v}_1 \times \vec{v}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 5 & 7 \\ 1 & 4 & 7 \end{vmatrix}$$

$$\vec{v} = \hat{i}(35-28) - \hat{j}(21-7) + \hat{k}(12-5) = 7\hat{i} - 14\hat{j} + 7\hat{k}$$

Simplifying the direction ratios of L: (1, –2, 1).

The direction ratios of L_3 are $v_3 = (2, 1, 2)$. The angle θ between L and L_3 is given by:

$$\cos\theta = \frac{|\vec{v}\cdot\vec{v}_3|}{|\vec{v}||\vec{v}_3|} = \frac{|(1)(2)+(-2)(1)+(1)(2)|}{\sqrt{1^2+(-2)^2+1^2}\sqrt{2^2+1^2+2^2}}$$

$$\cos\theta = \frac{|2-2+2|}{\sqrt{6}\cdot 3} = \frac{2}{3\sqrt{6}}$$

To find $\tan\theta$, we use the identity $\tan^2\theta = \sec^2\theta - 1$:

$$\sec\theta = \frac{3\sqrt{6}}{2} \Rightarrow \sec^2\theta = \frac{9\cdot 6}{4} = \frac{54}{4} = \frac{27}{2}$$

$$\tan^2\theta = \frac{27}{2} - 1 = \frac{25}{2}$$

$$\tan\theta = \frac{5}{\sqrt{2}} = \frac{5\sqrt{2}}{2} = \frac{5}{2}\sqrt{2}$$

Question ID : 6952782152

17. The value of $\lim_{x\to 0}\left(\frac{x^2\sin^2 x}{x^2 - \sin^2 x}\right)$ is :

(1) 2 (2) 3 (3) 4 (4) 6

Ans. (2)

Sol. We use the Taylor series expansion for $\sin x$ near $x = 0$:

$$\sin x = x - \frac{x^3}{6} + O(x^5)$$

Then, $\sin^2 x = \left(x - \frac{x^3}{6}\right)^2 \approx x^2 - \frac{x^4}{3}$.

Substitute this into the limit expression :

$$\lim_{x\to 0}\frac{x^2\left(x^2 - \frac{x^4}{3}\right)}{x^2 - \left(x^2 - \frac{x^4}{3}\right)} = \lim_{x\to 0}\frac{x^4 - \frac{x^6}{3}}{\frac{x^4}{3}}$$

Divide numerator and denominator by x^4 :

$$\lim_{x\to 0}\frac{1 - \frac{x^2}{3}}{\frac{1}{3}} = \frac{1}{1/3} = 3$$

Question ID : 6952782153

18. The value of the integral $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\left(\frac{32\cos^4 x}{1+e^{\sin x}}\right)dx$ is :

(1) $4\pi + 2$ (2) $3\pi + 8$ (3) $3\pi + 4$ (4) $4\pi + 3$

Ans. (2)

Sol. Let $I=\int_{-\pi/4}^{\pi/4}\frac{32\cos^4 x}{1+e^{\sin x}}dx$. Using the property $\int_{-a}^{a} f(x)dx=\int_0^a\left[f(x)+f(-x)\right]dx$:

$$I=\int_0^{\pi/4}\left(\frac{32\cos^4 x}{1+e^{\sin x}}+\frac{32\cos^4(-x)}{1+e^{\sin(-x)}}\right)dx$$

$$I=\int_0^{\pi/4}32\cos^4 x\left(\frac{1}{1+e^{\sin x}}+\frac{1}{1+e^{-\sin x}}\right)dx$$

Since $\frac{1}{1+e^k}+\frac{1}{1+e^{-k}}=1$:

$$I=\int_0^{\pi/4}32\cos^4 x dx=32\int_0^{\pi/4}\left(\frac{1+\cos 2x}{2}\right)^2 dx$$

$$I=8\int_0^{\pi/4}(1+2\cos 2x+\cos^2 2x)dx=8\int_0^{\pi/4}(1+2\cos 2x+\frac{1+\cos 4x}{2})dx$$

$$I=4\int_0^{\pi/4}(3+4\cos 2x+\cos 4x)dx$$

$$I=4\left[3x+2\sin 2x+\frac{\sin 4x}{4}\right]_0^{\pi/4}=4\left[\frac{3\pi}{4}+2\sin(\pi/2)+\frac{\sin\pi}{4}\right]$$

$$I=4\left[\frac{3\pi}{4}+2\right]=3\pi+8$$

Question ID : 6952782154

19. The area of the region $\{(x,y):0\le y\le 6-x, y^2\ge 4x-3, x\ge 0\}$ is :

(1) 8 (2) 9 (3) 12 (4) 15

Ans. (2)

Sol. The region is bounded by $x = 0$, $y = 0$, the line $x = 6 - y$, and the parabola $x=\frac{y^2+3}{4}$.

The inequality $y^2\ge 4x-3$ implies $x\le\frac{y^2+3}{4}$.

Find the intersection of $x = 6 - y$ and $x = \frac{y^2+3}{4}$:

$$6 - y = \frac{y^2+3}{4} \Rightarrow 24 - 4y = y^2 + 3$$

$$\Rightarrow y^2 + 4y - 21 = 0 \Rightarrow (y+7)(y-3) = 0$$

For $y \geq 0$, the intersection is at $y = 3$.

We integrate with respect to y :

$$\text{Area } = \int_0^3 \left(\frac{y^2+3}{4}\right)dy + \int_3^6 (6-y)dy$$

$$\text{Area } = \frac{1}{4}\left[\frac{y^3}{3} + 3y\right]_0^3 + \left[6y - \frac{y^2}{2}\right]_3^6$$

$$\text{Area } = \frac{1}{4}[9+9] + [(36-18)-(18-4.5)]$$

$$= \frac{18}{4} + [18 - 13.5] = 4.5 + 4.5 = 9$$

Question ID : 6952782155

20. Let e be the base of natural logarithm and let $f:\{1, 2, 3, 4\} \to \{1, e, e^2, e^3\}$ and $g:\{1, e, e^2, e^3\} \to \left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right\}$ be two bijective functions such that f is strictly decreasing and g is strictly increasing. If $\phi(x) = \left[f^{-1}\left\{g^{-1}\left(\frac{1}{2}\right)\right\}\right]^x$, then the area of the region $R = \{(x, y) : x^2 \leq y \leq \phi(x), 0 \leq x \leq 1\}$ is :

(1) $\frac{3-\log_e(2)}{3\log_e(2)}$ (2) $\frac{1}{3\log_e(2)}$ (3) $3+\log_e(2)$ (4) $\frac{3+\log_e(2)}{2+\log_e(3)}$

Ans. (1)

Sol. 1. Find f^{-1} and g^{-1} :

f is strictly decreasing: $f(1) = e^3$, $f(2) = e^2$, $f(3) = e$, $f(4) = 1$.

g is strictly increasing: $g(1) = 1/4$, $g(e) = 1/3$, $g(e^2) = 1/2$, $g(e^3) = 1$.

2. Find $\phi(x)$:

$g^{-1}(1/2) = e^2$.

$f^{-1}(e^2) = 2$.

$\phi(x) = 2^x$.

3. Calculate the area :

$$\text{Area} = \int_0^1 (2^x - x^2)dx = \left[\frac{2^x}{\log_e 2} - \frac{x^3}{3}\right]_0^1$$

$$\text{Area} = \left(\frac{2}{\log_e 2} - \frac{1}{3}\right) - \left(\frac{1}{\log_e 2} - 0\right) = \frac{1}{\log_e 2} - \frac{1}{3} = \frac{3 - \log_e 2}{3\log_e 2}$$

SECTION – B

Question ID : 6952782156

21. Let $A = \begin{bmatrix} -1 & 1 & -1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$, satisfy $A^2 + \alpha(\text{adj}(\text{adj}(A))) + \beta(\text{adj}(A)(\text{adj}(\text{adj}(A)))) = \begin{bmatrix} 2 & -2 & 2 \\ -2 & 0 & -1 \\ 0 & 0 & -1 \end{bmatrix}$ for some $\alpha, \beta \in \mathbb{R}$. Then $(\alpha - \beta)^2$ is equal to ________.

Ans. (4)

Sol. First, calculate the determinant of A :

|A| [cite$_s$ tart]$= -1(0-0) - 1(1-0) - 1(0-0) = -1$

Using matrix properties for an $n \times n$ matrix :

1. $\text{adj}(\text{adj}(A)) = |A|^{n-2} A = |A|^{3-2} A = |A|A = -A$.

2. $\text{adj}(A)\text{adj}(\text{adj}(A)) = \text{adj}(A.\text{adj}(A)) = \text{adj}(|A|I) = |A|^{n-1} I = (-1)^2 I = I$.

Now, calculate A^2 :

$$A^2 = \begin{bmatrix} -1 & 1 & -1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} -1 & 1 & -1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Substitute these into the given equation :

$$\begin{bmatrix} 2 & -1 & 1 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \alpha(A) + \beta(I) = \begin{bmatrix} 2 & -2 & 2 \\ -2 & 0 & -1 \\ 0 & 0 & -1 \end{bmatrix}$$

Comparing corresponding elements:

Element (1, 1) : $2 + \alpha + \beta = 2 \Rightarrow \alpha + \beta = 0$.

Element (1, 2): $-1 - \alpha = -2 \Rightarrow \alpha = 1$.

Thus, $\beta = -1$.

$(\alpha - \beta)^2 = (1 - (-1))^2 = 2^2 = 4$

Question ID : 6952782157

22. Let the centre of the circle $x^2 + y^2 + 2gx + 2fy + 25 = 0$ be in the first quadrant and lie on the line $2x - y = 4$. Let the area of an equilateral triangle inscribed in the circle be $27\sqrt{3}$. Then the square of the length of the chord of the circle on the line $x = 1$ is _________.

Ans. (80)

Sol. For a circle with radius R, the area of an inscribed equilateral triangle is $A = \frac{3\sqrt{3}}{4}R^2$.

$$27\sqrt{3} = \frac{3\sqrt{3}}{4}R^2 \Rightarrow R^2 = 36 \Rightarrow R = 6$$

The radius formula is $R^2 = g^2 + f^2 - c = g^2 + f^2 - 25 = 36 \Rightarrow g^2 + f^2 = 61$.

The center $(-g, -f)$ is in the 1st quadrant, so $g < 0$ and $f < 0$.

The center lies on $2x - y = 4 \Rightarrow 2(-g) - (-f) = 4$

$\Rightarrow f = 2g + 4$.

Substitute f into $g^2 + f^2 = 61$:

$g^2 + (2g + 4)^2 = 61 \Rightarrow 5g^2 + 16g - 45 = 0 \Rightarrow (5g - 9)(g + 5) = 0$

Since $g < 0$, we have $g = -5$ and $f = 2(-5) + 4 = -6$.

The center is (5, 6) and $R^2 = 36$.

The distance d from the center (5, 6) to the line $x = 1$ is $d = |5 - 1| = 4$.

Length of the chord $L = 2\sqrt{R^2 - d^2} = 2\sqrt{36 - 16} = 2\sqrt{20}$.

Square of the length: $L^2 = \left(2\sqrt{20}\right)^2 = 80$.

Question ID : 6952782158

23. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=\hat{j}-\hat{k}$ and $\vec{c}$ be three vectors such that $\vec{a}\times\vec{c}=\vec{b}$ and $\vec{a}\cdot\vec{c}=3$ then $\vec{c}\cdot(\vec{a}-2\vec{b})$ is equal to ________.

Ans. (3)

Sol. We need to calculate c.(a – 2b) = c .a – 2(c . b).

1. We are given a . c = 3.

2. From the vector cross product a × c = b, we know that the resulting vector b is perpendicular to both a and c.

Therefore, c . b = 0.

Substituting these values :

c . (a – 2b) = 3 – 2(0) = 3

Question ID : 6952782159

24. For the functions $f(\theta)=\alpha\tan^2\theta+\beta\cot^2\theta$ and $g(\theta)=\alpha\sin^2\theta+\beta\cos^2\theta, \alpha>\beta>0$, let $\min\limits_{0<\theta<\frac{\pi}{2}} f(\theta)=\max\limits_{0<\theta<\pi} g(\theta)$. If the first term of a G.P. is $\left(\frac{\alpha}{2\beta}\right)$, its common ratio is $\left(\frac{2\beta}{\alpha}\right)$ and the sum of its first 10 terms is $\frac{m}{n}$, $\gcd(m,n)=1$, then $m+n$ is equal to ________.

Ans. (1279)

Sol. For f(θ), using AM–GM: $\frac{\alpha\tan^2\theta+\beta\cot^2\theta}{2}\geq\sqrt{\alpha\beta\tan^2\theta\cot^2\theta}\Rightarrow f(\theta)\geq 2\sqrt{\alpha\beta}$.

For $g(\theta)=\alpha\sin^2\theta+\beta\cos^2\theta$, since $\alpha>\beta$, the maximum occurs when $\sin^2\theta=1$, so max $g(\theta)=\alpha$.

Given $\min f(\theta)=\max g(\theta)\Rightarrow 2\sqrt{\alpha\beta}=\alpha\Rightarrow 4\alpha\beta=\alpha^2\Rightarrow\alpha=4\beta$.

For the G.P. :

First term $a=\frac{\alpha}{2\beta}=\frac{4\beta}{2\beta}=2$.

Common ratio $r=\frac{2\beta}{\alpha}=\frac{2\beta}{4\beta}=\frac{1}{2}$.

Sum of first 10 terms:

$$S_{10}=\frac{a(1-r^{10})}{1-r}=\frac{2(1-(1/2)^{10})}{1-1/2}=4\left(1-\frac{1}{1024}\right)=4\left(\frac{1023}{1024}\right)=\frac{1023}{256}$$

Here m = 1023, n = 256, and gcd(1023, 256) = 1.

m + n = 1023 + 256 = 1279

Question ID : 6952782160

25. Let $y = y(x)$ be the solution of the differential equation $(x^2 - x\sqrt{x^2-1})dy + (y(x - \sqrt{x^2-1}) - x)dx = 0, x \geq 1$.

If $y(1) = 1$ then the greatest integer less than $y\left(\sqrt{5}\right)$ is ________.

Ans. (3)

Sol. Rearranging the differential equation :

$$\frac{dy}{dx} = \frac{x - y(x - \sqrt{x^2-1})}{x(x-\sqrt{x^2-1})} = \frac{1}{x - \sqrt{x^2-1}} - \frac{y}{x}$$

Using rationalization, $\frac{1}{x-\sqrt{x^2-1}} = x + \sqrt{x^2-1}$.

$$\frac{dy}{dx} + \frac{y}{x} = x + \sqrt{x^2-1}$$

This is a linear differential equation with Integrating Factor IF = $e^{\int (1/x)dx} = x$.

$$y \cdot x = \int x(x + \sqrt{x^2-1})dx = \int (x^2 + x\sqrt{x^2-1})dx$$

$$= \frac{x^3}{3} + \frac{1}{3}(x^2-1)^{3/2} + C$$

Using $y(1) = 1 \Rightarrow 1 = \frac{1}{3} + 0 + C \Rightarrow C = \frac{2}{3}$.

$$yx = \frac{x^3 + (x^2-1)^{3/2} + 2}{3}$$

At $x = \sqrt{5}$:

$$y\sqrt{5} = \frac{\left(\sqrt{5}\right)^3 + (5-1)^{3/2} + 2}{3} = \frac{5\sqrt{5} + 8 + 2}{3} = \frac{5\sqrt{5} + 10}{3}$$

$$y\left(\sqrt{5}\right) = \frac{5 + 2\sqrt{5}}{3} \approx \frac{5 + 4.472}{3} \approx 3.157$$

The greatest integer less than 3.157 is 3.