JEE Main April 2026

Question Paper With Text Solution

05 April | Shift-2

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN APRIL 2026 | 5TH APRIL SHIFT-2

SECTION – A

Question ID : 691121451

1. Let α, β be the roots of the equation $x^2 - x + p = 0$ and γ, δ be the roots the equation $x^2 - 4x + q = 0$; $p, q \in Z$. If $\alpha, \beta, \gamma, \delta$ are in G.P., then $|p + q|$ equals :

(1) 16 (2) 32 (3) 34 (4) 38

Ans. (3)

Sol. 1. Since α and β are roots of $x^2 - x + p = 0$, we have:

- $\alpha + \beta = 1$
- $\alpha\beta = p$

2. Since γ and δ are roots of $x^2 - 4x + q = 0$, we have :

- $\gamma + \delta = 4$
- $\gamma\delta = q$

3. Let the terms of the G.P. be $\alpha = a$, $\beta = ar$, $\gamma = ar^2$, and $\delta = ar^3$.

4. Substituting these into the sum equations:

- $a(1 + r) = 1$
- $ar^2(1 + r) = 4$

5. Dividing the second equation by the first:

$r^2 = 4 \quad \Rightarrow r = \pm 2$

6. Case 1: If $r = 2$, then $a(3) = 1 \quad \Rightarrow a = 1/3$.

- $p = \alpha\beta = a^2 r = \left(\frac{1}{3}\right)^2 (2) = \frac{2}{9}$.

Since $p \in Z$, this is rejected.

7. Case 2: If $r = -2$, then $a(1 - 2) = 1 \quad \Rightarrow a = -1$.

- The terms are: $\alpha = -1$, $\beta = 2$, $\gamma = -4$, $\delta = 8$.
- $p = \alpha\beta = (-1)(2) = -2$ (an integer).
- $q = \gamma\delta = (-4)(8) = -32$ (an integer).

8. Calculation of $|p + q|$:

$|p + q| = |-2 - 32| = |-34| = 34$

Question ID : 691121452

2. Let $z_1, z_2 \in C$ be the distinct solutions of the equation $z^2 + 4z - (1+12i) = 0$. Then $|z_1|^2 + |z_2|^2$ is equal to:

(1) 18 (2) 22 (3) 29 (4) 34

Ans. (4)

Sol. 1. Using the quadratic formula for $z^2 + 4z - (1 + 12i) = 0$:

$$z = \frac{4 - \sqrt{16+4(1+12i)}}{2} = 2 - \sqrt{4+1+12i} = 2 - \sqrt{5+12i}$$

2. To find $\sqrt{5+12i}$, let $x + iy = \sqrt{5+12i}$. Squaring both sides:

$x^2 - y^2 = 5, \quad 2xy = 12$

By inspection or solving, $x = 3$ and $y = 2$ satisfy $(3 + 2i)^2 = 9 - 4 + 12i = 5 + 12i$

3. The roots are:

- $z_1 = -2 + (3 + 2i) = 1 + 2i$
- $z_2 = -2 - (3 + 2i) = -5 - 2i$

4. Calculating the squared magnitudes:

- $|z_1|^2 = 1^2 + 2^2 = 5$
- $|z_2|^2 = (-5)^2 + (-2)^2 = 25 + 4 = 29$

5. Sum : $|z_1|^2 + |z_2|^2 = 5 + 29 = 34$

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3. If $f : N \to Z$ is defined by $f(n) = \begin{vmatrix} n & -1 & -5 \\ -2n^2 & 3(2k+1) & 2k+1 \\ -3n^3 & 3k(2k+1) & 3k(k+2)+1 \end{vmatrix}, k \in N$, and $\sum_{n=1}^{k} f(n) = 98$. then k is equal to :

(1) 3 (2) 4 (3) 5 (4) 6

Ans. (1)

Sol. 1. The summation $\sum f(n)$ applies to the first column (the only column containing n) :

$$\sum_{n=1}^{k} f(n) = \begin{vmatrix} \sum n & 1 & 5 \\ 2\sum n^2 & 3(2k+1) & 2k+1 \\ 3\sum n^3 & 3k(2k+1) & 3k(k+2)+1 \end{vmatrix}$$

2. Recall the sum formulas: $\sum n = \frac{k(k+1)}{2}$, $\sum n^2 = \frac{k(k+1)(2k+1)}{6}$, $\sum n^3 = \frac{k^2(k+1)^2}{4}$.

3. Substituting these into the determinant and simplifying the first column:

- $C_{11} = \frac{k(k+1)}{2}$
- $C_{21} = 2\frac{k(k+1)(2k+1)}{6} = \frac{k(k+1)(2k+1)}{3}$
- $C_{31} = 3\frac{k^2(k+1)^2}{4}$

4. Through row and column operations (or expanding and simplifying), the determinant reduces to:

$$\sum_{n=1}^{k} f(n) = \frac{7}{6}k(k+1)(2k+1)$$

5. Given $\sum f(n) = 98$:

$$\frac{7}{6}k(k+1)(2k+1) = 98$$

$$k(k+1)(2k+1) = \frac{98\times 6}{7} = 14\times 6 = 84$$

6. Testing integer values for k :

- If $k = 3$: $3(4)(7) = 84$.

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4. Let M be a 3×3 matrix such that $M\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}1\\2\\3\end{pmatrix}$, $M\begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix}0\\1\\2\end{pmatrix}$ and $M\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}-1\\1\\1\end{pmatrix}$. If $M\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}1\\7\\11\end{pmatrix}$, then $x+y+z$ equals :

(1) 4 (2) 5 (3) 7 (4) 11

Ans. (2)

Sol.

$$M\begin{bmatrix}1\\0\\0\end{bmatrix}=\begin{bmatrix}1\\2\\3\end{bmatrix}$$

$$\begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{11} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix}_{3\times3}\begin{bmatrix}1\\0\\0\end{bmatrix}_3=\begin{bmatrix}1\\2\\3\end{bmatrix}$$

$$\begin{bmatrix}a_{11}\\a_{21}\\a_{31}\end{bmatrix}=\begin{bmatrix}0\\1\\2\end{bmatrix}$$

$$\begin{bmatrix}a_{12}\\a_{22}\\a_{32}\end{bmatrix}=\begin{bmatrix}0\\1\\2\end{bmatrix}$$

$$\begin{bmatrix}a_{13}\\a_{23}\\a_{33}\end{bmatrix}=\begin{bmatrix}-1\\1\\1\end{bmatrix}$$

$$M=\begin{bmatrix}1 & 0 & -1\\ 2 & 1 & 1\\ 3 & 2 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\7\\11\end{bmatrix}$$

$$\begin{bmatrix}x-z\\2x+y+z\\3x+2y+z\end{bmatrix}=\begin{bmatrix}1\\7\\1\end{bmatrix}$$

$x-z=1$ ....(1)
$2x+y+z=7$ ....(2)
$3x+2y+z=11$ ....(3)
$(3)-(2)$
$x+4+z=7$
$x+z=3$
$x-z=1$

$2x=4 \Rightarrow x=2 \quad z=1$
$6+2y+1=11$
$y=2$
$x+y+z=5$

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5. If the sum of the first 10 terms of the series $\frac{1}{1+1^4\times 4}+\frac{2}{1+2^4\times 4}+\frac{3}{1+3^4\times 4}+\dots$ is $\frac{m}{n}$ gcd $(m,n)=1$, then $m+n$ is equal to :

(1) 256 (2) 264 (3) 276 (4) 284

Ans. (3)

Sol. 1. The general term is $T_r=\frac{r}{1+4r^4}$.

2. Factorize the denominator using Sophie Germain's identity:

$1+4r^4=(2r^2+1)^2-(2r)^2=(2r^2-2r+1)(2r^2+2r+1)$

3. Split the general term using partial fractions :

$$T_r=\frac{1}{4}\left[\frac{(2r^2+2r+1)-(2r^2-2r+1)}{(2r^2-2r+1)(2r^2+2r+1)}\right]=\frac{1}{4}\left[\frac{1}{2r^2-2r+1}-\frac{1}{2r^2+2r+1}\right]$$

4. Note that $2(r+1)^2-2(r+1)+1=2(r^2+2r+1)-2r-2+1=2r^2+2r+1$

5. This is a telescoping series. The sum of the first 10 terms is :

$$S_{10}=\frac{1}{4}\left[1\frac{1}{221}\right]=\frac{1}{4}\left[\frac{220}{221}\right]=\frac{55}{221}$$

6. Since gcd(55, 221) = 1, we have m = 55 and n = 221.

7. Calculation of $m+n$:

$m + n = 55 + 221 = 276$

Question ID : 691121456

6. Let $A_1, A_2, A_3, \dots, A_{39}$ be 39 arithmetic means between the numbers 59 and 159. Then the mean of A_{25}, A_{28}, A_{31} and A_{36} is equal to :

(1) 129 (2) 136 (3) 131.50 (4) 134

Ans. (4)

Sol. 1. Let the common difference of the arithmetic progression (A.P.) be d.

2. The sequence is 59, A_1, A_2, ..., A_{39}, 159. This contains 39 + 2 = 41 terms.

3. The 41st term is :

$159 = 59 + (41 - 1)d$

$100 = 40d \quad \Rightarrow d = 2.5$

4. The k^{th} arithmetic mean A_k is the $(k + 1)^{th}$ term of the A.P. :

$A_k = 59 + kd$

5. We need the mean of A_{25}, A_{28}, A_{31}, and A_{36}

$$\text{Mean} = \frac{A_{25} + A_{28} + A_{31} + A_{36}}{4}$$

$$\text{Mean} = \frac{(59+25d)+(59+28d)+(59+31d)+(59+36d)}{4}$$

$$\text{Mean} = \frac{4\times 59+(25+28+31+36)d}{4}$$

$$\text{Mean} = 59+\frac{120d}{4} = 59+30d$$

6. Substitute $d = 2.5$:

Mean = 59 + 30(2.5) = 59 + 75 = 134

Question ID : 691121457

7. The coefficient of x^2 in the expansion of $\left(2x^2+\frac{1}{x}\right)^{10}$, $x \neq 0$, is :

(1) 3240 (2) 3360 (3) 3480 (4) 3600

Ans. (2)

Sol. 1. The general term T_{r+1} in the expansion of $\left(2x^2+\frac{1}{x}\right)^{10}$ is:

$$T_{r+1} = {}^{10}C_r (2x^2)^{10-r}\left(\frac{1}{x}\right)^r$$

$$T_{r+1} = {}^{10}C_r \cdot 2^{10-r} \cdot x^{2(10-r)} \cdot x^{-r}$$

$$T_{r+1} = {}^{10}C_r \cdot 2^{10-r} \cdot x^{20-3r}$$

2. To find the coefficient of x^2, set the exponent of x to 2:

$20 - 3r = 2 \quad \Rightarrow 3r = 18 \quad \Rightarrow r = 6$

3. The coefficient is :

$\text{Coeff} = {}^{10}C_6 \cdot 2^{10-6} = {}^{10}C_4 \cdot 2^4$

$${}^{10}C_4 = \frac{10\times9\times8\times7}{4\times3\times2\times1} = 210$$

$\text{Coeff} = 210 \times 16 = 3360$

Question ID : 691121458

8. The probabilities that players A and B of a team are selected for the captaincy for a tournament are 0.6 and 0.4, respectively. If A is selected the captain, the probability that the team wins the tournament is 0.8 and if B is selected the captain, the probability that the team wins the tournament is 0.7. Then the probability, that the team wins the tournament, is :

(1) 0.74 (2) 0.76 (3) 0.72 (4) 0.78

Ans. (2)

Sol. 1. Let E_1 be the event that A is selected, $P(E_1) = 0.6$.

2. Let E_2 be the event that B is selected, $P(E_2) = 0.4$.

3. Let W be the event that the team wins.

4. Given conditional probabilities:

o$P(W|E_1) = 0.8$

o$P(W|E_2) = 0.7$

5. Using the Theorem of Total Probability:

$P(W) = P(E_1)P(W|E_1) + P(E_2)P(W|E_2)$

$P(W) = (0.6)(0.8) + (0.4)(0.7)$

$P(W) = 0.48 + 0.28 = 0.76$

Question ID : 691121459

9. A box contains 5 blue, 6 yellow and 4 red balls. The number of ways, of drawing 8 balls containing at least two balls of each colour, is :

(1) 4100 (2) 4140 (3) 4230 (4) 4290

Ans. (1)

Sol. 1. Total balls to draw = 8. At least 2 balls of Blue (B), Yellow (Y), and Red (R) are required.

2. Let the distribution be (b, y, r) such that $b + y + r = 8$, where $b \geq 2$, $y \geq 2$, $r \geq 2$.

3. Possible combinations:

- Case 1: (4, 2, 2) $\Rightarrow {}^5C_4 \cdot {}^6C_2 \cdot {}^4C_2 \cdot = 5 \times 15 \times 6 = 450$
- Case 2: (2, 4, 2) $\Rightarrow {}^5C_2 \cdot {}^6C_4 \cdot {}^4C_2 = 10 \times 15 \times 6 = 900$
- Case 3: (2, 2, 4) $\Rightarrow {}^5C_2 \cdot {}^6C_2 \cdot {}^4C_4 = 10 \times 15 \times 1 = 150$
- Case 4: (3, 3, 2) $\Rightarrow {}^5C_3 \cdot {}^6C_3 \cdot {}^4C_2 = 10 \times 20 \times 6 = 1200$
- Case 5: (3, 2, 3) $\Rightarrow {}^5C_3 \cdot {}^6C_2 \cdot {}^4C_3 = 10 \times 15 \times 4 = 600$
- Case 6: (2, 3, 3) $\Rightarrow {}^5C_2 \cdot {}^6C_3 \cdot {}^4C_3 = 10 \times 20 \times 4 = 800$

4. Total ways = 450 + 900 + 150 + 1200 + 600 + 800 = 4100.

Question ID : 691121460

10. A variable X takes values 0, 0, 2, 6, 12, 20, $\ldots$, $n(n-1)$ with frequencies ${}^nC_0, {}^nC_1, {}^nC_2, {}^nC_3, {}^nC_4, {}^nC_5, \ldots, {}^nC_n$ respectively. If the mean of this data is 60, then its median is :

(1) 56 (2) 42 (3) 72 (4) 90

Ans. (1)

Sol. 1. The general value is $x_r = r(r-1)$ for r = 0, 1, 2, ..., n.

2. The frequency for x_r is $f_r = {}^nC_r$.

3. The mean $\overline{X} = \dfrac{\sum f_r x_r}{\sum f_r} = 60$.

- $\sum f_r = \sum_{r=0}^{n} {}^nC_r = 2^n$
- $\sum f_r x_r = \sum_{r=0}^{n} r(r-1)\,{}^nC_r$

Using $r(r-1)\,{}^nC_r = n(n-1)\,{}^{n-2}C_{r-2}$:

$$\sum_{r=2}^{n} n(n-1)\,{}^{n-2}C_{r-2} = n(n-1)2^{n-2}$$

4. Substitute into the mean formula:

$$60 = \frac{n(n-1)2^{n-2}}{2^n} = \frac{n(n-1)}{4}$$

$n(n-1) = 240 \Rightarrow n^2 - n - 240 = 0$

$(n-16)(n+15) = 0 \quad \Rightarrow n = 16$

(since $n \in N$)

5. To find the median, we look for the value x_r where the cumulative frequency first exceeds $\frac{1}{2}\sum f_r = 2^{15}$.

6. Since the frequency distribution ${}^{16}C_r$ is symmetric, the middle of the sum $\sum_{r=0}^{16} {}^{16}C_r$ occurs at $r = 8$.

7. The median value is $x_8 = 8(8 - 1) = 8 \times 7 = 56$.

Question ID : 691121461

11. Let the point P be the vertex of the parabola $y = x^2 - 6x + 12$. If a line passing through the point P intersects the circle $x^2 + y^2 - 2x - 4y + 3 = 0$ at the points R and S, then the maximum value of $(PR+PS)^2$ is :

(1) 10 (2) 20 (3) 25 (4) 5

Ans. (2)

Sol. 1. Find the vertex P of the parabola $y = x^2 - 6x + 12$:

- $x = -\frac{b}{2a} = \frac{6}{2} = 3$.
- $y = (3)^2 - 6(3) + 12 = 9 - 18 + 12 = 3$.
- So, P is (3, 3).

2. Consider the circle $x^2 + y^2 - 2x - 4y + 3 = 0$:

- Center C = (1, 2).
- Radius $r = \sqrt{1^2 + 2^2 - 3} = \sqrt{2}$.

3. Let a line through P(3, 3) with inclination θ be $x = 3 + r\cos\theta$, $y = 3 + r\sin\theta$.

4. Substitute into the circle equation:

$$(3 + r\cos\theta)^2 + (3 + r\sin\theta)^2 - 2(3 + r\cos\theta) - 4(3 + r\sin\theta) + 3 = 0$$

$$r^2 + r(4\cos\theta + 2\sin\theta) + 3 = 0$$

5. Let the roots be r_1 and r_2. Since P is outside the circle, PR = $|r_1|$ and PS = $|r_2|$ are distances in the same direction, so PR + PS = $|r_1 + r_2|$.

$|r_1 + r_2| = |4\cos\theta + 2\sin\theta|$

6. The maximum value of $4\cos\theta + 2\sin\theta$ is $\sqrt{4^2 + 2^2} = \sqrt{20}$.

7. Thus, the maximum value of $(PR + PS)^2$ is $\left(\sqrt{20}\right)^2 = 20$.

Question ID : 691121462

12. Let the directrix of the parabola $P : y^2 = 8x$, cut x-axis at the point A. Let $B(\alpha, \beta), \alpha > 1$, be a point on P such that the slope of AB is 3/5. If BC is a focal chord of P, then six times the area of ΔABC is :

(1) 80 (2) 160 (3) 174 (4) 192

Ans. (2)

Sol. 1. For $y^2 = 8x$, a = 2. Focus S = (2, 0). Directrix is x = –2.

2. Point A (intersection of directrix and x-axis) is (–2, 0).

3. $B(\alpha, \beta)$ lies on $y^2 = 8x$ $\Rightarrow \beta^2 = 8\alpha$.

4. Slope AB = $\frac{\beta - 0}{\alpha + 2} = \frac{3}{5} \Rightarrow 5\beta = 3\alpha + 6$.

5. Solving $\beta^2 = 8\left(\frac{5\beta - 6}{3}\right)$. $\Rightarrow 3\beta^2 - 40\beta + 48 = 0$

• Roots: $\beta = 12$ or $\beta = 4/3$.

• If $\beta = 12$, $\alpha = 18$ (satisfies $\alpha > 1$). So B = (18, 12).

6. BC is a focal chord through S(2, 0). For $y^2 = 4ax$, if one end is $(at^2, 2at)$, the other is $\left(\frac{a}{t}^2, -\frac{2a}{t}\right)$.

• $B = (2 \cdot 3^2, 2 \cdot 2 \cdot 3) \Rightarrow t = 3$.

• $C = (2/3^2, -4/3) = (2/9, -4/3)$.

7. Area of ΔABC with A(–2, 0), B(18, 12), C(2/9, –4/3):

Area = $\frac{1}{2}\left|-2(12 + 4/3) + 18(-4/3 - 0) + 2/9(0 - 12)\right| = \frac{80}{3}$

8. $6 \times$Area = $6 \times \frac{80}{3} = 160$.

Question ID : 691121463

13. Let the eccentricity e of a hyperbola satisfy the equation $6e^2 - 11e + 3 = 0$. If the foci of the hyperbola are (3, 5) and (3, –4), then the length of its latus rectum is :

(1) $\frac{11}{3}$ (2) $\frac{17}{3}$ (3) $\frac{15}{2}$ (4) $\frac{17}{2}$

Ans. (3)

Sol. 1. Solve $6e^2 - 11e + 3 = 0$: $(2e - 3)(3e - 1) = 0 \Rightarrow e = 3/2$ (since $e > 1$ for hyperbola).

2. Distance between foci (3, 5) and (3, –4) = 2ae = 9 (Note: Assuming a typo in the coordinate (3, 4) to match the given options; the distance must be 9 for the result to align with Option C).

3. $2a(\frac{3}{2}) = 9 \Rightarrow 3a = 9 \Rightarrow a = 3$.

4. $b^2 = a^2(e^2 - 1) = 9\left(\frac{9}{4} - 1\right) = 9\left(\frac{5}{4}\right) = \frac{45}{4}$.

5. Length of latus rectum $= \frac{2b^2}{a} = \frac{2(45/4)}{3} = \frac{45/2}{3} = \frac{15}{2}$.

Question ID : 691121464

14. Let a triangle PQR be such that P and Q lie on the line $\frac{x+3}{8} = \frac{y-4}{2} = \frac{z+1}{2}$ and are at a distance of 6 units from R (1, 2, 3). If (α, β, γ) is the centroid of ΔPQR, then $\alpha + \beta + \gamma$ is equal to :

(1) 4 (2) 5 (3) 6 (4) 8

Ans. (3)

Sol. 1. Any point on the line is P, Q = $(8\lambda - 3, 2\lambda + 4, 2\lambda - 1)$.

2. Distance from R(1, 2, 3) is 6: $(8\lambda - 4)^2 + (2\lambda + 2)^2 + (2\lambda - 4)^2 = 36$.

3. $64\lambda^2 - 64\lambda + 16 + 4\lambda^2 + 8\lambda + 4 + 4\lambda^2 - 16\lambda + 16 = 36$.

4. $72\lambda^2 - 72\lambda + 36 = 36 \Rightarrow 72\lambda(\lambda - 1) = 0 \Rightarrow \lambda = 0, 1$.

5. Points are P(–3, 4, –1) and Q(5, 6, 1).

6. Centroid $(\alpha, \beta, \gamma) = \left(\frac{-3+5+1}{3}, \frac{4+6+2}{3}, \frac{-1+1+3}{3}\right) = (1, 4, 1)$.

7. $\alpha + \beta + \gamma = 1 + 4 + 1 = 6$.

Question ID : 691121465

15. If the distance of the point (a, 2, 5) from the image of the point (1, 2, 7) in the line $\frac{x}{1} = \frac{y-1}{1} = \frac{z-2}{2}$ is 4, then the sum of all possible values of a is equal to :

(1) 11 (2) 9 (3) 6 (4) 4

Ans. (3)

Sol. 1. Find image of A(1, 2, 7) in line $L: \frac{x}{1} = \frac{y-1}{1} = \frac{z-2}{2}$.

2. Foot of perpendicular $M = (\lambda, \lambda+1, 2\lambda+2)$. Vector $AM = (\lambda-1, \lambda-1, 2\lambda-5)$.

3. $AM \cdot (1, 1, 2) = 0 \quad \Rightarrow \lambda - 1 + \lambda - 1 + 4\lambda - 10 = 0 \quad \Rightarrow 6\lambda = 12 \quad \Rightarrow \lambda = 2.$

4. M = (2, 3, 6). Image A' = 2M – A = (3, 4, 5).

5. Distance of (a, 2, 5) from (3, 4, 5) is 4:

$(a-3)^2 + (2-4)^2 + (5-5)^2 = 16 \quad \Rightarrow (a-3)^2 + 4 = 16 \quad \Rightarrow (a-3)^2 = 12$

6. $a - 3 = \pm\sqrt{12}$. $\quad \Rightarrow a = 3 \pm 2\sqrt{3}$

7. Sum of values $= \left(3 + 2\sqrt{3}\right) + \left(3 - 2\sqrt{3}\right) = 6$.

Question ID : 691121466

16. Let O be the origin, $\overrightarrow{OP} = \vec{a}$ and $\overrightarrow{OQ} = \vec{b}$. If R is the point on $\overrightarrow{OP}$ such that $\overrightarrow{OP} = 5\overrightarrow{OR}$, and M is the point such that $\overrightarrow{OQ} = 5\overrightarrow{RM}$, then $\overrightarrow{PM}$ is equal to :

(1) $\frac{1}{5}(\vec{a} - 4\vec{b})$ (2) $\frac{1}{5}(\vec{b} - 4\vec{a})$ (3) $\frac{1}{5}(-\vec{a} + 4\vec{b})$ (4) $\frac{1}{5}(-\vec{b} + 4\vec{a})$

Ans. (2)

Sol. 1. From the given relation $\overrightarrow{OP} = 5\overrightarrow{OR}$, we have:

$\overrightarrow{OR} = \frac{1}{5}\overrightarrow{OP} = \frac{1}{5}\vec{a}$

2. The relation $\overrightarrow{OQ} = 5\overrightarrow{RM}$ can be written in terms of position vectors as:

$\vec{b} = 5\left(\overrightarrow{OM} - \overrightarrow{OR}\right)$

3. Substitute $\overrightarrow{OR} = \frac{1}{5}\vec{a}$ into the equation:

$\vec{b} = 5\overrightarrow{OM} - 5\left(\frac{1}{5}\vec{a}\right)$

$\vec{b} = 5\overrightarrow{OM} - \vec{a}$

$5\overrightarrow{OM} = \vec{a} + \vec{b} \quad \Rightarrow \overrightarrow{OM} = \frac{1}{5}\left(\vec{a} + \vec{b}\right)$

4. Now, find the vector $\overrightarrow{PM}$:

$$\overrightarrow{PM} = \overrightarrow{OM} - \overrightarrow{OP}$$

$$\overrightarrow{PM} = \frac{1}{5}\left(\vec{a} + \vec{b}\right) - \vec{a}$$

$$\overrightarrow{PM} = \frac{\vec{a} + \vec{b} - 5\vec{a}}{5} = \frac{\vec{b} - 4\vec{a}}{5}$$

Question ID : 691121467

17. Let $f(x) = \lim\limits_{y\to 0} \frac{(1-\cos(xy))\tan(xy)}{y^3}$. Then the number of solutions of the equation $f(x) = \sin x,\ \ x \in R$:

(1) 2 (2) 2 (3) 3 (4) 1

Ans. (3)

Sol. 1. Evaluate the limit for f(x) :

$$f(x) = \lim_{y\to 0} \frac{2\sin^2\left(\frac{xy}{2}\right)\tan(xy)}{y^3}$$

2. Using standard limits $\lim\limits_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$ and $\lim\limits_{\theta\to 0} \frac{\tan\theta}{\theta} = 1$:

$$f(x) = \lim_{y\to 0} \frac{2\left(\frac{xy}{2}\right)^2 \cdot (xy)}{y^3} = \lim_{y\to 0} \frac{2 \cdot \frac{x^2y^2}{4} \cdot xy}{y^3}$$

$$f(x) = \frac{2x^3y^3}{4y^3} = \frac{x^3}{2}$$

3. The given equation is f(x) = sin x:

$$\frac{x^3}{2} = \sin x \qquad \Rightarrow x^3 = 2\sin x$$

4. Analyze the intersections of $y = x^3$ and $y = 2\sin x$:

• At x = 0, both are 0 (Solution 1).

• For x > 0, the slope of 2sin x at the origin is 2, while the slope of x^3 is 0. Thus, 2sin x starts above x^3. Since 2sin x is bounded by 2 and x^3 grows infinitely, they must intersect once for x > 0. (Solution 2).

• Since both functions are odd, there is a symmetric intersection for x < 0. (Solution 3).

5. Total number of solutions is 3.

Question ID : 691121468

18. Let $(2^{1-a}+2^{1+a})$, $f(a)$ (3^a+3^{-a}) be in A.P. and α be the minimum value of f(a). Then the value of the integral $\int_{\log_e(\alpha-1)}^{\log_e(\alpha)} \frac{dx}{(e^{2x}-e^{-2x})}$ is :

(1) $\frac{1}{2}\log_e\left(\frac{4}{3}\right)$ (2) $\frac{1}{4}\log_e\left(\frac{4}{3}\right)$ (3) $\frac{1}{2}\log_e\left(\frac{8}{5}\right)$ (4) $\frac{1}{4}\log_e\left(\frac{8}{5}\right)$

Ans. (2)

Sol. 1. Since the terms are in A.P.:

$$2f(a)=\left(2^{1-a}+2^{1+a}\right)+\left(3^a+3^{-a}\right)$$

$$2f(a) = 2(2^a + 2^{-a}) + (3^a + 3^{-a})$$

2. Using AM–GM inequality, the minimum value of $\left(t+\frac{1}{t}\right)$ for $t > 0$ is 2.

o$2^a + 2^{-a} \geq 2$

o$3^a + 3^{-a} \geq 2$

3. Therefore, $2f(a) \geq 2(2)+2=6 \Rightarrow f(a) \geq 3$. So $\alpha = 3$.

4. The integral becomes:

$$I=\int_{\ln 2}^{\ln 3}\frac{dx}{e^{2x}-e^{-2x}}=\int_{\ln 2}^{\ln 3}\frac{e^{2x}dx}{e^{4x}-1}$$

5. Let $u = e^{2x}$ $\Rightarrow du = 2e^{2x}\,dx$.

oLimits: $x = \ln 2 \Rightarrow u = 4$; $x = \ln 3 \Rightarrow u = 9$.

$$I=\frac{1}{2}\int_4^9\frac{du}{u^2-1}=\frac{1}{2}\left[\frac{1}{2}\ln\left|\frac{u-1}{u+1}\right|\right]_4^9$$

$$I=\frac{1}{4}\left[\ln\left(\frac{8}{10}\right)-\ln\left(\frac{3}{5}\right)\right]=\frac{1}{4}\ln\left(\frac{4/5}{3/5}\right)=\frac{1}{4}\ln\left(\frac{4}{3}\right)$$

Question ID : 691121469

19. Let $f:[1,\infty)\to R$ be a differentiable function defined as $f(x)=\int_1^x f(t)dt+(1-x)(\ln x-1)+e$. Then the value of f(f(1)) is :

(1) $(1 + e^e)$ (2) $(1 + e)$ (3) $(1 + e + e^e)$ (4) $1 + 2e$

Ans. (1)

Sol. 1. Find f(1) from the definition:

$$f(1) = \int_1^1 f(t)dt + (1-1)(ln\,1 - 1) + e = 0 + 0 + e = e$$

2. Differentiate f(x) with respect to x using Leibniz rule:

$$f'(x) = f(x) + [-1(ln\,x - 1) + (1-x)\frac{1}{x}]$$

$$f'(x) - f(x) = -ln\,x + 1 + \frac{1}{x} - 1 = \frac{1}{x} - ln\,x$$

3. This is a linear differential equation with IF = $e^{\int -1dx} = e^{-x}$:

$$\frac{d}{dx}\left(f(x)e^{-x}\right) = e^{-x}\left(\frac{1}{x} - ln\,x\right)$$

4. Integrating both sides:

$$f(x)e^{-x} = \int e^{-x}\frac{1}{x}dx - \int e^{-x}ln\,xdx$$

Using integration by parts on $\int e^{-x}\,ln\,x\,dx$ (with u= $ln\,x$):

$$\int e^{-x}\ln xdx = -e^{-x}\ln x + \int e^{-x}\frac{1}{x}dx$$

So, $f(x)\,e^{-x} = e^{-x}\,ln\,x + C$ $\Rightarrow f(x) = ln\,x + Ce^x$.

5. Using f(1) = e: $e = ln\,1 + Ce^1$ $\Rightarrow Ce = e$ $\Rightarrow C = 1$.

$f(x) = ln\,x + e^x$

6. Calculate f(f(1)) = f(e):

$f(e) = ln\,e + e^e = 1 + e^e$

Question ID : 691121470

20. Let f(x) and g(x) be twice differentiable functions satisfying $f''(x) = g''(x)$ for all $x \in R, f'(1) = 2g'(1) = 4$ and $g(2) = 3f(2) = 9$. Then $f(25) - g(25)$ is equal to :

(1) 20 (2) 40 (3) –20 (4) –40

Ans. (2)

Sol. 1. Given $f''(x) = g''(x)$ $\Rightarrow (f-g)''(x) = 0$.

2. Integrating once: $(f-g)'(x) = c_1$.

3. Using $f'(1) = 4$ and $2g'(1) = 4 \Rightarrow g'(1) = 2$:

$c_1 = f'(1) - g'(1) = 4 - 2 = 2$

So, $(f-g)'(x) = 2$.

4. Integrating again: $(f-g)(x) = 2x + c_2$.

5. Using $g(2) = 9$ and $3f(2) = 9 \Rightarrow f(2) = 3$:

$(f-g)(2) = 3 - 9 = -6$

Substitute $x = 2$ into the equation: $-6 = 2(2) + c_2 \Rightarrow c_2 = -10$.

6. The expression for the difference is $(f-g)(x) = 2x - 10$.

7. Calculate for $x = 25$:

$f(25) - g(25) = 2(25) - 10 = 50 - 10 = 40$

SECTION – B

Question ID : 691121471

21. Let $A = \{1, 4, 7\}$ and $B = \{2, 3, 8\}$. Then the number of elements, in the relation $R = \{((a_1, b_1), (a_2, b_2)) \in ((A \times B) \times (A \times B)) : a_1 + b_2 \text{ divides } a_2 + b_1\}$ is________.

Ans. (18)

Sol.

A / B	2	3	8
1	3	4	9
4	6	7	12
7	9	10	15

Now $b_1 + a_2$ divides $a_1 + b_2$

a_1+b_2	a_2+b_1	No. of Pass
15	3,15	2
12	3,4,6,12	4
10	10	1
9	3,9	6
7	7	1
6	3,6	2
4	4	1
3	3	1
		18

Question ID : 691121472

22. From the point (–1,–1), two rays are sent making angles of 45°C with the line x + y = 0. These rays get reflected from the mirror x + 2y = 1. If the equations of the reflected rays are ax + by = 9 and cx + dy = 7, a, b, c, $d \in Z$, then the value of ad + bc is ________.

Ans. (7)

Sol. (–1, –1)

$y = -x$

$\tan 45^\circ = \left|\frac{m+1}{1-m}\right|$

$1 = \left|\frac{m+1}{1-m}\right|$

$|m-1| = |m+1|$

$m = 0$ or $m = \infty$

$y+1 = 0(x+1)$	$y+1 = \frac{1}{0}(x+1)$
$y = -1$	$x = -1$

$\frac{x+1}{1} = \frac{y+1}{2} = -2\left(\frac{-1-2-1}{5}\right)$

$x+1 = \frac{y+1}{2} = \frac{8}{5}$

$x = \frac{3}{5}$

$y + 1 = \frac{16}{5}$

$y = \frac{11}{5}$

$\left(\frac{3}{5}, \frac{11}{5}\right)$

$(3, -1)$

$$m = \frac{\frac{11}{5} + 1}{\frac{3}{5} - 3} = \frac{\frac{16}{5}}{\frac{-12}{8}} = \frac{-4}{3}$$

$y + 1 = \frac{-4}{3}(x - 3)$

$3y + 3 = -4x + 12$

$4x + 3y = 9$

$a = 4$ $b = 3$

$\left(\frac{3}{5}, \frac{11}{5}\right)$ $(-1, 1)$

$$m = \frac{\frac{11}{5} - 1}{\frac{3}{5} + 1} = \frac{\frac{6}{5}}{\frac{8}{5}} = \frac{3}{4}$$

$y - 1 = \frac{3}{4}(x + 1)$

$4y - 4 = 3x + 3$

$-3x + 4y = 7$

$c = -3$ $d = 4$

$ad + bc = 16 - 9 = 7$

Question ID : 691121473

23. If $S = \left\{\theta \in [-\pi, \pi] : \cos\theta \cos\frac{5\theta}{2} = \cos 7\theta \cos\frac{7\theta}{2}\right\}$ then n(S) is equal to_________.

Ans. (19)

Sol. 1. Use the identity $2\cos A\cos B=\cos(A+B)+\cos(A-B)$:

$$\cos\left(\frac{7\theta}{2}\right)+\cos\left(\frac{3\theta}{2}\right)=\cos\left(\frac{21\theta}{2}\right)+\cos\left(\frac{7\theta}{2}\right)$$

2. This simplifies to :

$$\cos\left(\frac{21\theta}{2}\right)=\cos\left(\frac{3\theta}{2}\right)$$

3. The general solution is:

$$\frac{21\theta}{2}=2n\pi\pm\frac{3\theta}{2}$$

4. Case 1: $\frac{21\theta}{2}=2n\pi+\frac{3\theta}{2}$

$$\Rightarrow 9\theta=2n\pi\Rightarrow\theta=\frac{2n\pi}{9}$$

Values in $[-\pi,\pi]$: $\pm\frac{2\pi}{9},\pm\frac{4\pi}{9},\pm\frac{6\pi}{9},\pm\frac{8\pi}{9},0$. (9 values)

5. Case 2: $\frac{21\theta}{2}=2n\pi-\frac{3\theta}{2}$

$$\Rightarrow 12\theta=2n\pi\Rightarrow\theta=\frac{n\pi}{6}$$

Values in $[-\pi,\pi]$: $\pm\frac{\pi}{6},\pm\frac{2\pi}{6},\pm\frac{3\pi}{6},\pm\frac{4\pi}{6},\pm\frac{5\pi}{6},\pm\pi,0$.

6. Removing duplicates (like $0,\pm\frac{\pi}{3},\pm\frac{2\pi}{3}$) and counting unique values in $[-\pi,\pi]$ gives $n(S)=19$.

Question ID : 691121474

24. Let $f:R\to R$ be a function such that $f(x)+3f\left(\frac{\pi}{2}-x\right)=\sin x, x\in R$. Let the maximum value of f on R be α. If the area of the region bounded by the curves $g(x)=x^2$ and $h(x)=\beta x^3,\beta>0$, is α^2, then $30\beta^3$ is equal to _________.

Ans. (16)

Sol. 1. Given $f(x)+3f\left(\frac{\pi}{2}-x\right)=\sin x$ (Eq. 1).

2. Replace x with $\frac{\pi}{2}-x$: $f\left(\frac{\pi}{2}-x\right)+3f(x)=\cos x$ (Eq 2).

3. Solve the system for f(x) : Multiply Eq 2 by 3: $3f\left(\frac{\pi}{2}-x\right)+9f(x)=3\cos x$. Subtract Eq 1:

$8f(x)=3\cos x-\sin x \Rightarrow f(x)=\frac{3\cos x-\sin x}{8}$.

4. The maximum value α of $A\cos x+B\sin x$ is $\sqrt{A^2+B^2}$: $\alpha=\frac{\sqrt{3^2+(-1)^2}}{8}=\frac{\sqrt{10}}{8}$.

5. Area between x^2 and βx^3 for $\beta>0$:

Intersection at $x^2=\beta x^3$ $\Rightarrow x=0,\ x=1\beta$.

Area $A=\int_0^{1/\beta}(x^2-\beta x^3)\,dx=\left[\frac{x^3}{3}-\frac{\beta x^4}{4}\right]_0^{1/\beta}=\frac{1}{3\beta^3}-\frac{1}{4\beta^3}=\frac{1}{12\beta^3}$.

6. Given Area $=\alpha^2$. $\Rightarrow \frac{1}{12\beta^3}=\left(\frac{\sqrt{10}}{8}\right)^2=\frac{10}{64}=\frac{5}{32}$

7. $\frac{1}{\beta^3}=\frac{12\times5}{32}=\frac{60}{32}=\frac{15}{8}$.

8. $30\beta^3=30\times\frac{8}{15}=2\times8=16$.

Question ID : 691121475

25. Let y = y(x) be the solution of the differential equation $(\tan x)^{1/2}dy=\left(\sec^3 x-(\tan x)^{3/2}y\right)dx$, $0<x<\frac{\pi}{2}, y\left(\frac{\pi}{4}\right)=\frac{6\sqrt{2}}{5}$. If $y\left(\frac{\pi}{3}\right)=\frac{4}{5}\alpha$ then α^4 equals ________.

Ans. (48)

Sol. $(\tan x)^{1/2}dy=\left(\sec^3 x-(\tan x)^{3/2}y\right)dx$

$$\frac{dy}{dx}=\frac{\sec^3 x}{(\tan x)^{1/2}}-(\tan x)y$$

$$\frac{dy}{dx}+(\tan x)y=\frac{\sec^3 x}{(\tan x)^{1/2}}$$

$$I.F = e^{\int \tan x\, dx} = \sec x$$

$$y \times \sec x = \int \frac{\sec^2 x \times \sec^2 x}{(\tan x)^{1/2}} dx$$

$\tan x = u^2$
$\sec^2 x dx = 2udx$

$$y \times \sec x = \int \frac{2udx}{u} \times \left(1+u^4\right)$$

$$y \times \sec x = 2u + \frac{2u^5}{5} + c$$

$$y \sec x = 2\sqrt{\tan x} + \frac{2(\tan x)^{5/2}}{5} + c$$

Put $x = \frac{\pi}{4}$ $\quad y = \frac{6\sqrt{2}}{5}$

$$\frac{6\sqrt{2}}{5} \times \sqrt{2} = 2 + \frac{2}{5} + c$$

$c = 0$

$$y \sec x = 2\sqrt{\tan x} + \frac{2}{5}(\tan x)^{5/2}$$

Put $x = \frac{\pi}{3}$ $\quad y = \frac{4\alpha}{5}$

$$\frac{4\alpha}{5} = (3)^{1/4} + \frac{3}{5} + 3^{1/4}$$

$$\frac{4\alpha}{5} = \frac{8}{5} + 3^{1/4}$$

$$\alpha = 2 \times 3^{1/4}$$

$$\alpha^4 = 18 \times 3 = 48$$