JEE Main April 2026

Question Paper With Text Solution

04 April | Shift-2

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN APRIL 2026 | 4TH APRIL SHIFT-2

SECTION – A

Question ID : 695278376

1. For the function $f : [1, \infty) \to [1, \infty)$ defined by $f(x) = (x-1)^4+1$ among the two statements:

(I) The set $S = \{x \in [1, \infty) : f(x) = f^{-1}(x)\}$ contains exactly two elements, and

(II) The set $S = \{x \in [1, \infty) : f(x) = f^{-1}(x+1)\}$ is an empty set,

(1) only (I) is TRUE
(2) only (II) is TRUE
(3) both (I) and (II) are TRUE
(4) neither (I) nor (II) is TRUE

Ans. (1)

Sol. (I) $f(x) = f^{-1}(x) = x$

$(x-1)^4 + 1 = x \Rightarrow x = 1, 2$

(II) $f(x) = (x-1)^4 + 1$

$y = (x-1)^4 + 1$

$x = (y-1)^{1/4} + 1$

$f^{-1}(x) = (x-1)^{1/4} + 1$

$f(x) = f^{-1}(x+1)$

$(x-1)^4 + 1 = x^{1/4} + 1$

$(x-1)^4 = x^{1/4} \Rightarrow$ There will be one Sol. so only "I" is true

Question ID : 695278377

2. Let $S = \{z \in C\} : z^2 + 4z + 16 = 0\}$. Then $\sum_{z \in S} |z + \sqrt{3}i|^2$ is equal to :

(1) 42
(2) 23
(3) 27
(4) 38

Ans. (4)

Sol. $z^2 + 4z + 16 = 0$

$(z - 4\omega)(z - 4\omega^2) = 0$

$z = 4\omega$ or $4\omega^2$

$$z = 4\left(\frac{-1}{2} + \frac{i\sqrt{3}}{2}\right) \text{ or } 4\left(\frac{-1}{2} - \frac{i\sqrt{3}}{2}\right)$$

$z = -2 + 2\sqrt{3}i$ or $-2 - 2\sqrt{3}i$

$z+\sqrt{3}i=-2+3\sqrt{3}i$ or $-2-\sqrt{3}i$

$\left|z+\sqrt{3}i\right|^2 = 31$ or 7

$\Sigma^* z+\sqrt{3}i|^2 = 38$

Question ID : 695278378

3. If the system of equations: $x+y+z=5$ $x+2y+3z=9$ $x+3y+\lambda z=\mu$ has infinitely many solutions, then the value of $\lambda+\mu$ is :

(1) 16 (2) 18 (3) 19 (4) 21

Ans. (2)

Sol.
$$\Delta=\begin{vmatrix}1 & 1 & 1\\ 1 & 2 & 3\\ 1 & 3 & \lambda\end{vmatrix}=0$$

$\lambda=5$

$$\Delta_z=\begin{vmatrix}1 & 1 & 5\\ 1 & 2 & 9\\ 1 & 3 & \mu\end{vmatrix}=0$$

$\mu=13 \Rightarrow \lambda+\mu=18$

Question ID : 695278379

4. If $\alpha=1$ and $\beta=1+i\sqrt{2}$, where $i=\sqrt{-1}$ are two roots of the equation $x^3+ax^2+bx+c=0$, $a, b, c \in R$ then $\int_{-1}^{1}(x^3+ax^2+bx+c)dx$ is equal to :

(1) –2 (2) 4 (3) –8 (4) 10

Ans. (3)

Sol. Since the coefficients a, b, c are real, complex roots must occur in conjugate pairs. Given roots: $x_1=1$ and $x_2=1+i\sqrt{2}$. Therefore, the third root is $x_3=1-i\sqrt{2}$. The polynomial is $f(x)=(x-x_1)(x-x_2)(x-x_3)$:

$f(x)=(x-1)[(x-1-i\sqrt{2})(x-1+i\sqrt{2})]$

$f(x)=x^3-3x^2+5x-3$

Now, evaluate the integral:

$\int_{-1}^{1}(x^3-3x^2+5x-3)dx$

Using properties of even and odd functions: $\int_{-a}^{a}(\text{odd})dx=0$ and $\int_{-a}^{a}(\text{even})dx=2\int_{0}^{a}(\text{even})dx$.

$\int_{-1}^{1}x^3dx=0,\quad \int_{-1}^{1}5xdx=0$

Integral $=2\int_{0}^{1}(-3x^2-3)dx=2[-x^3-3x]_0^1=2(-1-3)=-8$

Question ID : 695278380

5. If the quadratic equation $(\lambda+2)x^2-3\lambda x+4\lambda=0, \lambda\neq-2$, has two positive roots, then the number of possible integral values of λ is :

(1) 1 (2) 2 (3) 3 (4) 4

Ans. (2)

Sol. For the quadratic $Ax^2+Bx+C=0$ to have two positive roots:

1. Discriminant $D\geq 0$:

$(-3\lambda)^2-4(\lambda+2)(4\lambda)\geq 0$

$9\lambda^2-16\lambda^2-32\lambda\geq 0\Rightarrow -7\lambda^2-32\lambda\geq 0$

$7\lambda^2+32\lambda\leq 0\Rightarrow \lambda(7\lambda+32)\leq 0$

$\lambda\in\left[-\frac{32}{7},0\right]$

2. Product of roots $P>0$:

$\frac{4\lambda}{\lambda+2}>0\Rightarrow \lambda\in(-\infty,-2)\cup(0,\infty)$

3. Sum of roots $S>0$:

$\frac{3\lambda}{\lambda+2}>0\Rightarrow \lambda\in(-\infty,-2)\cup(0,\infty)$

Intersection of conditions:

The intersection of $\left[-\frac{32}{7},0\right]$ and $((-\infty,-2)\cup(0,\infty))$ is $\left[-\frac{32}{7},-2\right)$

Total number of integral values = 2

Question ID : 695278381

6. Let $A = \begin{bmatrix} 1 & 2 & 7 \\ 4 & -2 & 8 \\ 3 & 8 & -7 \end{bmatrix}$ and $\det(A - \alpha I) = 0$, where a is a real number. If the largest possible value of a is p, then the circle $(x - p)^2 + (y - 2p)^2 = 320$, intersects the co-ordinate axes at :

(1) 1 point (2) 2 points (3) 3 points (4) 4 points

Ans. (3)

Sol.

$$|A - \alpha I| = \begin{vmatrix} 1-\alpha & 2 & 7 \\ 4 & -2-\alpha & 8 \\ 3 & 8 & -7-\alpha \end{vmatrix} = 0$$

$R_1 \to R_1 + R_2 + R_3$

$$(8-\alpha)\begin{vmatrix} 1 & 1 & 1 \\ 4 & -2-\alpha & 8 \\ 3 & 8 & -7-\alpha \end{vmatrix} = 0$$

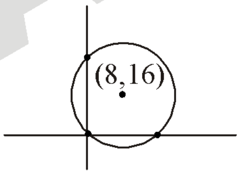
$$(8-\alpha)\left(\alpha^2 + 16\alpha + 40\right) = 0$$

$\alpha = 8, -8 \pm \sqrt{24}$

So P = 8

C : $(x - 8)^2 + (y - 16)^2 = 320$

Circle intersects coordinate ones at 3 points



Question ID : 695278382

7. Let $a = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots 00$ and $\beta = \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots 00$. Then the value of $(0.2)^{\log_{\sqrt{5}}(a)} + (0.04)^{\log_5(\beta)}$ is equal to :

(1) 4 (2) 5 (3) 8 (4) 25

Ans. (3)

Sol.

$$\alpha = \frac{\frac{1}{4}}{1 - \frac{1}{2}} = \frac{1}{2}$$

$$\beta = \frac{\frac{1}{3}}{1-\frac{1}{3}} = \frac{1}{2}$$

$$(0.2)^{\log_{\sqrt{5}}\alpha} + (0.04)^{\log_5 \beta} = (0.2)^{\log_5 \alpha^2} + \left((0.2)^2\right)^{\log_5 \beta}$$

$$= (0.2)^{\log_5 \alpha^2} + (0.2)^{\log_5 \beta^2}$$

$$= 2\cdot\left(\frac{1}{5}\right)^{\log_5 1/4}$$

$$= 2 \times 4 = 8$$

Question ID : 695278383

8. For 10 observations $x_1, x_2, ..., x_{10}$, if $\sum_{i=1}^{10}(x_i+2)^2 = 180$ and $\sum_{i=1}^{10}(x_i-1)^2 = 90$. then their standard deviation is :

(1) 2 (2) $\sqrt{3}$ (3) $2\sqrt{2}$ (4) 3

Ans. (4)

Sol. $\sum x_i^2 + 4\sum x_i + 40 = 180$

$\sum x_i^2 - 2\sum x_i + 10 = 90$

$\sum x_i^2 = 100$

$\sum x_i = 10$

$$\sigma^2 = \frac{\sum x_i^2}{10} - \left(\frac{\sum x_i}{10}\right)^2$$

$\sigma^2 = 10 - 1^2 = 9$

$\sigma = 3$

Question ID : 695278384

9. In the expansion of of $\left(9x - \frac{1}{3\sqrt{x}}\right)^{18}$, $x > 0$ if the term independent of x is (221)k, then k is equal to :

(1) 84 (2) 78 (3) 168 (4) 198

Ans. (1)

Sol. $T_{r+1} = {}^{18}C_r (9x)^{18-r}\left(\frac{-1}{3\sqrt{x}}\right)^r$

$T_{r+1} = {}^{18}C_r 9^{18-r}\left(\frac{-1}{3}\right)^r x^{18-3r/2}$

$18-\frac{3r}{2}=0$

r = 12

$T_{13} = {}^{18}C_{12}\, 9^6\left(\frac{-1}{3}\right)^{12} = 221K$

${}^{18}C_{12} = 221K$

$K = \frac{{}^{18}C_{12}}{221} = 84$

Question ID : 695278385

10. Let P (3cos a, 2sin a), $\alpha \neq 0$, be a point on the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$, Q be a point on the circle $x^2+y^2-14x-14y+82=0$ and R be a point on the line x + y = 5 such that the centroid of the triangle PQR is $(2+\cos\alpha, 3+\frac{2}{3}\sin\alpha)$. Then the sum of the ordinates of all possible points R is :

(1) 6 (2) 2 (3) 4 (4) 8

Ans. (4)

Sol. $E : \frac{x^2}{9}+\frac{y^2}{4}=1 \Rightarrow P=(3\cos\alpha, 2\sin\alpha)$

$C : (x-7)^2+(y-7)^2 = 16 \Rightarrow Q = (7+4\cos\theta, 7+4\sin\theta)$

$L : x + y = 5 \Rightarrow R = (t, 5-t)$

$\text{Centroid} = \left(\frac{3\cos\alpha+7+4\cos\theta+t}{3}, \frac{2\sin\alpha+7+4\sin\theta+5-t}{3}\right)$

$= \left(2+\cos\alpha, 3+\frac{2}{3}\sin\alpha\right)$

$\Rightarrow 7 + 4\cos\theta + t = 6$

$12 + 4\sin\theta - t = 9$

$4\cos\theta = -1 - t$

$4\sin\theta = -3 + t$

$(t + 1)^2 + (t - 3)^2 = 16$

$t^2 - 2t - 3 = 0 \Rightarrow t = -1, 3$

$y_R = 5 - t = 6, 2$

Sum = 8

Question ID : 695278386

11. Let $H: \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be a hyperbola such that the distance between its foci is 6 and the distance between its directrices is $\frac{8}{3}$ If the line $x = \alpha$ intersects the hyperbola H at the points A and B such that the area of the triangle AOB is $4\sqrt{15}$ where O is the origin, then α^2 equals :

(1) 12 (2) 16 (3) 24 (4) 25

Ans. (2)

Sol.

$2ae = 6 \Rightarrow ae = 3$

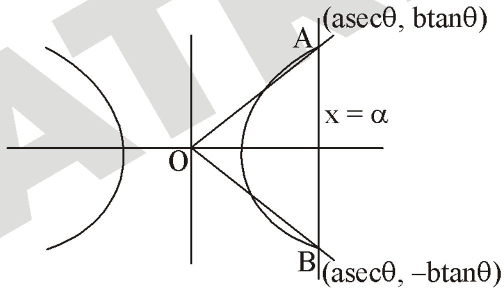
$2\frac{a}{e} = \frac{8}{3} \Rightarrow \frac{a}{e} = \frac{4}{3}$

$a^2 = 4$

$a^2e^2 = 9$

$a^2 + b^2 = 9$

$b^2 = 5$



$H: \frac{x^2}{4} - \frac{y^2}{5} = 1$

$\Delta_{AOB} = \frac{1}{2} \times 2b\tan\theta \times a\sec\theta = 4\sqrt{15}$

$ab\frac{\sin\theta}{\cos^2\theta} = 4\sqrt{15}$

$2\sqrt{5}\frac{\sin\theta}{\cos^2\theta} = 4\sqrt{5}$

$2\sqrt{3}\cos^2\theta = \sin\theta$

$2\sqrt{3} - 2\sqrt{3}\sin^2\theta = \sin\theta$

$(2\sin\theta - \sqrt{3})(\sqrt{3}\sin\theta + 2) = 0$

$\sin\theta = \frac{\sqrt{3}}{2}$ or $\frac{-2}{\sqrt{3}}$ (Rejected)

$\alpha^2 = a^2\sec^2\theta = \frac{4}{1 - \sin^2\theta} = 16$

Question ID : 695278387

12. $\max_{0\le x\le \pi}\left(16\sin\left(\frac{x}{2}\right)\cos^3\left(\frac{x}{2}\right)\right)$ is equal to :

(1) $\frac{3\sqrt{3}}{2}$ (2) $3\sqrt{3}$ (3) $4\sqrt{3}$ (4) $6\sqrt{3}$

Ans. (2)

Sol. $k = 16\sin\frac{x}{2}\cos^3\frac{x}{2}$

$k = 4(2\sin\frac{x}{2}\cos\frac{x}{2})\left(2\cos^2\frac{x}{2}\right)$

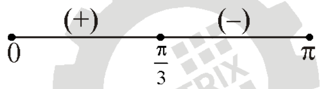
$k = 4\sin x(1 + \cos x)$

$k = 4\sin x + 2\sin 2x$

$\frac{dk}{dx} = 4\cos x + 4\cos 2x$

$\frac{dk}{dx} = 4\left(\cos x + 2\cos^2 x - 1\right)$

$\frac{dk}{dx} = 4(2\cos x - 1)(\cos x + 1)$



for $k_{max} \Rightarrow x = \frac{\pi}{3}$

$k_{max} = 4\sin\frac{\pi}{3} + 2\sin\frac{2\pi}{3}$

$= 3\sqrt{3}$

Question ID : 695278388

13. The shortest distance between the lines $\vec{r} = \left(\frac{1}{3}\hat{i} + 2\hat{j} + \frac{8}{3}\hat{k}\right) + \lambda(2\hat{i} - 5\hat{j} + 6\hat{k})$ and $\vec{r} = (-\frac{2}{3}\hat{i} - \frac{1}{3}\hat{k}) + \mu(\hat{j} - \hat{k})$, $\lambda, \mu \in N$, i is :

(1) $\sqrt{5}$ (2) 3 (3) $2\sqrt{3}$ (4) $\sqrt{15}$

Ans. (2)

Sol. $L_1 : A = \left(\frac{1}{3}, 2, \frac{8}{3}\right), \vec{P} = 2i - 5j + 6k$

$L_2 : \; B = \left(-\frac{2}{3}, 0, \frac{-1}{3}\right), \vec{q} = j - k$

$\overrightarrow{BA} = i + 2j + 3k$

$$\vec{p} \times \vec{q} = \begin{vmatrix} i & j & k \\ 2 & -5 & 6 \\ 0 & 1 & -1 \end{vmatrix} = -i + 2j + 2k$$

$$SD = \left| \frac{\overrightarrow{BA} \cdot (\vec{p} \times \vec{q})}{|\vec{p} \times \vec{q}|} \right|$$

$$SD = \frac{9}{3} = 3$$

Question ID : 695278389

14. If $\left(2\alpha + 1, \alpha^2 - 3\alpha, \frac{\alpha - 1}{2}\right)$ is the image of $(\alpha, 2\alpha, 1)$ in the line $\frac{x-2}{3} = \frac{y-1}{2} = \frac{z}{1}$, then the possible value(s) of α is (are) :

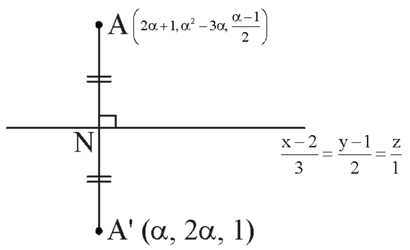
(1) Only 3 (2) Only 3 and -1 (3) Only 3, $\frac{1}{4}$ and -1 (4) Only 3 and $\frac{1}{4}$

Ans. (1)

Sol. $\overrightarrow{A'A} = (\alpha + 1)i + (\alpha^2 - 5\alpha)j + \left(\frac{\alpha - 3}{2}\right)k$

$\vec{P} = 3i + 2j + k$

$\overrightarrow{A'A} \cdot \vec{P} = 0$



$$3(\alpha + 1) + 2(\alpha^2 - 5\alpha) + \left(\frac{\alpha - 3}{2}\right) = 0$$

$4\alpha^2 - 13\alpha + 3 = 0$

$(4\alpha - 1)(\alpha - 3) = 0$

$\alpha = 3$ or $\frac{-1}{4}$

$$N=\left(\frac{3\alpha+1}{2},\frac{\alpha^2-\alpha}{2},\frac{\alpha+1}{4}\right)$$

for $\alpha = 3 \Rightarrow N = (5, 3, 1)$ lies on the line

$\alpha = -1/4 \Rightarrow N = \left(\frac{1}{8},\frac{3}{16},\frac{3}{16}\right)$ Does not lie on the line

So $\alpha = 3$ only

Question ID : 695278390

15. Let $\hat{u}$ and $\hat{v}$ be unit vectors inclined at an acute angle such that $|\hat{u}\times\hat{v}| = \frac{\sqrt{3}}{2}$. If $\vec{A} = \lambda\hat{u} + \hat{v} + (\hat{u}\times\hat{v})$ then λ is equal to :

(1) $\frac{4}{3}(\vec{A}\cdot\hat{u}) - \frac{2}{3}(\vec{A}\cdot\hat{v})$

(2) $\frac{2}{3}(\vec{A}\cdot\hat{u}) - \frac{1}{3}(\vec{A}\cdot\hat{v})$

(3) $\frac{4}{3}(\vec{A}\cdot\hat{u}) + \frac{2}{3}(\vec{A}\cdot\hat{v})$

(4) $(\vec{A}\cdot\hat{u}) - \frac{1}{2}(\vec{A}\cdot\hat{v})$

Ans. (1)

Sol. 1. Find the angle θ between $\hat{u}$ and $\hat{v}$:

$$|\hat{u}\times\hat{v}| = |\hat{u}||\hat{v}|\sin\theta = \sin\theta = \frac{\sqrt{3}}{2}$$

Since θ is acute, $\theta = 60°$. Thus $\hat{u}\cdot\hat{v} = \cos 60^\circ = \frac{1}{2}$

2. Take dot products of $\vec{A}$ with $\hat{u}$ and $\hat{v}$:

$$\vec{A}\cdot\hat{u} = \lambda(\hat{u}\cdot\hat{u}) + (\hat{v}\cdot\hat{u}) + (\hat{u}\times\hat{v})\cdot\hat{u} = \lambda(1) + \frac{1}{2} + 0 = \lambda + \frac{1}{2}$$

$$\vec{A}\cdot\hat{v} = \lambda(\hat{u}\cdot\hat{v}) + (\hat{v}\cdot\hat{v}) + (\hat{u}\times\hat{v})\cdot\hat{v} = \lambda(\frac{1}{2}) + 1 + 0 = \frac{\lambda}{2} + 1$$

3. Solve for λ:

From the first equation: $\lambda = (\vec{A}\cdot\hat{u}) - \frac{1}{2}$.

From the second equation: $\frac{\lambda}{2} = (\vec{A}\cdot\hat{v}) - 1 \Rightarrow \lambda = 2(\vec{A}\cdot\hat{v}) - 2$

oTo eliminate the constants, multiply the first by $\frac{4}{3}$ and the second by $\frac{2}{3}$:

$$\frac{4}{3}(\vec{A}\cdot\hat{u})-\frac{2}{3}(\vec{A}\cdot\hat{v})=\frac{4}{3}(\lambda+\frac{1}{2})-\frac{2}{3}(\frac{\lambda}{2}+1)$$

$$=\frac{4\lambda}{3}+\frac{2}{3}-\frac{\lambda}{3}-\frac{2}{3}=\frac{3\lambda}{3}=\lambda$$

Question ID : 695278391

16. Let for some $\alpha \in R$, $f : R \to R$ be a function satisfying $f(x + y) = f(x) + 2y^2 + y + axy$ for all $x, y \in R$. If $f(0) = -1$ and $f(1) = 2$, then the value of $\sum_{n=1}^{5}(a+f(n))$ is :

(1) 110 (2) 140 (3) 150 (4) 170

Ans. (2)

Sol. To find the functional form, set $x = 0$ in the given equation:

$$f(0+y)=f(0)+2y^2+y+a(0)y$$

$$f(y)=-1+2y^2+y \Rightarrow f(x)=2x^2+x-1$$

Verify this with $f(1)=2 : f(1)=2(1)^2+1-1=2$. This is consistent. Now, substitute f(x) back into the original functional equation to find a:

$$f(x+y)=2(x+y)^2+(x+y)-1=2x^2+4xy+2y^2+x+y-1$$

$$(2x^2+x-1)+2y^2+y+4xy=f(x)+2y^2+y+4xy$$

Comparing with $f(x) + 2y^2 + y + axy$, we get $a = 4$. The summation is:

$$\sum_{n=1}^{5}(4+f(n))=\sum_{n=1}^{5}(4+2n^2+n-1)=\sum_{n=1}^{5}(2n^2+n+3)$$

$$=2\sum_{n=1}^{5}n^2+\sum_{n=1}^{5}n+\sum_{n=1}^{5}3$$

$$=2\left(\frac{5\cdot6\cdot11}{6}\right)+\frac{5\cdot6}{2}+5(3)$$

$= 110 + 15 + 15 = 140$

Question ID : 695278392

17. Let $A=\{(a, b, c) : a, b, c \text{ are non-negative integers and } a+b+2c=22\}$. Then n(A) is equal to :

(1) 121 (2) 124 (3) 144 (4) 169

Ans. (3)

Sol. We need to find the number of non-negative integer solutions to $a + b + 2c = 22$. The variable c can take values from 0 to 11 because $2c \le 22$. For a fixed value of c, the equation becomes:

$a + b = 22 - 2c$

The number of non-negative integer solutions for $a + b = k$ is given by $k + 1$. Here, for each c, the number of solutions is $(22 - 2c) + 1 = 23 - 2c$. Total number of solutions n(A) is the sum of solutions for all possible c:

$$n(A) = \sum_{c=0}^{11}(23 - 2c)$$

This is an Arithmetic Progression:

$c = 0 \Rightarrow 23$

$c=1 \Rightarrow 21$

$c = 11 \Rightarrow 1$

The sum is $\frac{\text{number of terms}}{2}(\text{first term} + \text{last term}) = \frac{12}{2}(23+1) = 6 \times 24 = 144$

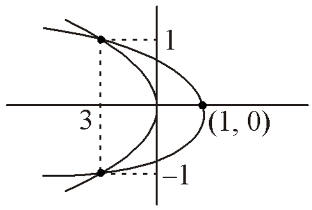
Question ID : 695278393

18. The area of the region bounded by the curves $x + 3y^2 = 0$ and $x + 4y^2=1$ is equal to :

(1) $\frac{1}{3}$ (2) $\frac{2}{3}$ (3) $\frac{4}{3}$ (4) $\frac{5}{3}$

Ans. (3)

Sol. $P_1 : y^2 = -\frac{x}{3}$



$P_1 : y^2 = -\frac{x}{3}$

$y^2 = -\frac{x}{3} = -\frac{1}{4}(x-1)$

$-4x = -3x + 3$

$x = -3 \Rightarrow y^2 = 1 \Rightarrow y = \pm 1$

$A = \int_{-1}^{1}\{(1-4y^2)-(-3y^2)\}dy$

$A = 2\int_0^1(1-y^2)dy$

$A = \frac{4}{3}$

Question ID : 695278394

19. Let $y = y(x)$ be the solution of the differential equation:

$$\frac{dy}{dx} + \left(\frac{6x^2 + (3x^2 + 2x^3 + 4)e^{-2x}}{(x^3 + 2)(2 + e^{-2x})}\right)y = 2 + e^{-2x}.$$

$x \in (-1, 2).$, satisfying $y(0) = \frac{3}{2}$. If $y(1) = \alpha(2 + e^{-2})$. then a is equal to :

(1) $\frac{13}{8}$ (2) $\frac{6}{13}$ (3) $\frac{12}{13}$ (4) $\frac{13}{12}$

Ans. (4)

Sol. $IF = e^{\int \frac{6x^2 + (3x^2 + 2x^3 + 4)e^{e^{-2x}}}{(x^3+2)(2+e^{-2x})}dx} = e^{\int \frac{6x^2 + (3x^2 - 2x^3 - 4)e^{-2x} + (4x^3 + 8)e^{-2x}}{(x^3+2)(2+e^{-2x})}dx}$

$IF = e^{\ln\left(\frac{x^3+2}{2+e^{-2x}}\right)} = \frac{x^3 + 2}{2 + e^{-2x}}$

$y \cdot \left(\frac{x^3 + 2}{2 + e^{-2x}}\right) = \int \left(2 + e^{-2x}\right)\left(\frac{x^3 + 2}{2 + e^{-2x}}\right)dx$

$y\left(\frac{x^3 + 2}{2 + e^{-2x}}\right) = \frac{x^4}{4} + 2x + c$

$y(0) = \frac{3}{2} \Rightarrow \frac{3}{2} \cdot \frac{2}{3} = c \Rightarrow c = 1$

$y\left(\frac{x^3 + 2}{2 + e^{-2x}}\right) = \frac{x^4}{4} + 2x + 1$

$x = 1$

$y(1) \cdot \left(\frac{3}{2 + e^{-2}}\right) = \frac{1}{4} + 2 + 1 = \frac{13}{4}$

$y(1) = \frac{13}{12}\left(2 + e^{-2}\right)$

$\alpha = \frac{13}{12}$

Question ID : 695278395

20. The integral $\int_0^1 \cot^{-1}(1+x+x^2)dx$ is equal to :

(1) $2\tan^{-1}2+\frac{1}{2}\log_e\left(\frac{5}{4}\right)+\frac{\pi}{2}$

(2) $2\tan^{-1}2+\frac{1}{2}\log_e\left(\frac{5}{4}\right)-\frac{\pi}{2}$

(3) $2\tan^{-1}2-\frac{1}{2}\log_e\left(\frac{5}{4}\right)+\frac{\pi}{2}$

(4) $2\tan^{-1}2-\frac{1}{2}\log_e\left(\frac{5}{4}\right)-\frac{\pi}{2}$

Ans. (4)

Sol. $I=\int_0^1 \cot^{-1}\left(1+x+x^2\right)dx=\int_0^1 \tan^{-1}\left(\frac{1}{1+x+x^2}\right)dx$

$I=\int_0^1 \tan^{-1}\left(\frac{(x+1)-x}{1+(x+1)x}\right)dx$

$I=\int_0^1 \tan^{-1}(x+1)dx-\int_0^1 \tan^{-1}x dx$

$x + 1 = t$

$dx = dt$

$I=\int_1^2 \tan^{-1}+dt-\int_0^1 \tan^{-1}x\,dx$

Using integration by parts

$I=\left[t\tan^{-1}t-\frac{1}{2}\ell n\left(1+t^2\right)\right]_1^2-\left[n\tan^{-1}n-\frac{1}{2}\ell n\left(1+n^2\right)_0^1\right]$

$=2\tan^{-1}2-\frac{1}{2}\ell n\left(\frac{5}{4}\right)$

SECTION – B

Question ID : 695278396

21. From a month of 31 days, 3 different dates are selected at random. If the probability that these dates are in an increasing A.P. is equal to $\frac{a}{b}$ where a, b $\in$ N and gcd(a, b) =1, then a + b is equal to _________.

Ans. (944)

Sol. Let selected dates are D_1, D_2, D_3

$2D_2 = D_1 + D_3 = \text{Even}$

$p=\frac{\text{Both Even } + \text{ Both odd}}{\text{Total}}$

$$p = \frac{^{15}C_2 + {}^{16}C_2}{^{31}C_3} = \frac{45}{899} = \frac{a}{b}$$

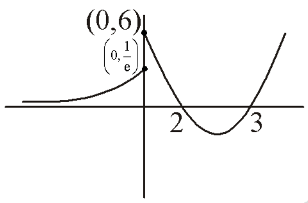
$a + b = 944$

Question ID : 695278397

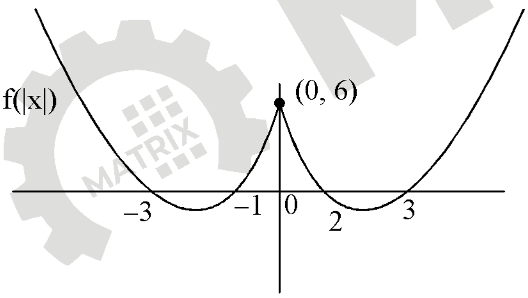
22. Let $f(x) = \begin{cases} e^{x-1} & , x < 0 \\ x^2 - 5x + 6 & , x \geq 0 \end{cases}$ and $g(x) = f(|x|) + |f(x)|$. If the number of points where g is not continuous and is not differentiable are α and β respectively, then $\alpha + \beta$ is equal to_________.

Ans. (4)

Sol. $f(x) = \begin{cases} e^{x-1} & , x < 0 \\ x^2 - 5x + 6 & , x \geq 0 \end{cases}$

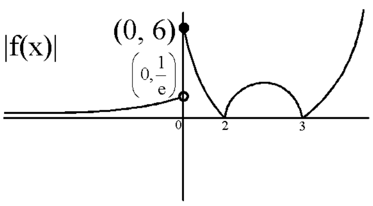


y = f(|x|)



always continuous
but Not Differentiable
at x = 0

y = |f(x)|



Discontinuous at x = 0
and Not differentiable at x = 0, 2, 3
For g(x) = f(|x|) + |f(x)|
Discontinuous at x = 0

Not differentiable at x = 0, 2, 3

$\alpha = 1$

$\beta = 3$

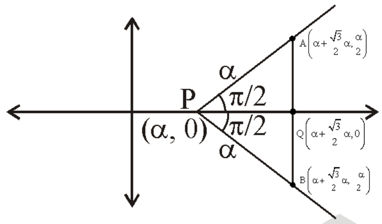
$\alpha + \beta = 4$

Question ID : 695278398

23. Let A, B be points on the two half-lines $x - \sqrt{3}\,|y| = a, \alpha > 0$ at a distance of a from their point of intersection P. The line segment AB meets the angle bisector of the given half-lines at the point Q. If $PQ = \frac{9}{2}$ and R is the radius of the circumcircle of Δ PAB, then $\frac{\alpha^2}{R}$ is equal to ________.

Ans. (9)

Sol.



$$PQ = \frac{\sqrt{3}}{2}\alpha = \frac{9}{2} \Rightarrow \alpha = 3\sqrt{3}$$

Since Δ^{le} is equilaterep Δ^{le}

$$R = \frac{abc}{4\Delta} = \frac{a^3}{4\cdot\frac{\sqrt{3}}{4}a^2} = \frac{a}{\sqrt{3}} = \frac{\alpha}{\sqrt{3}}$$

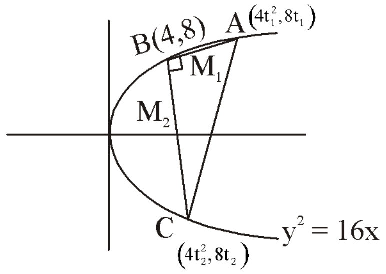
R = 3

Question ID : 695278399

24. Let A, B and C be the vertices of a variable right angled triangle inscribed in the parabola $y^2=16x$. Let the vertex B containing the right angle be (4, 8) and the locus of the centroid of Δ ABC be a conic C_0 Then three times the length of latus rectum of C_0 is ________.

Ans. (16)

Sol. $\frac{\alpha^2}{R} = \frac{27}{3} = 9$



$m_1 \cdot m_2 = -1$

$$\frac{8(t_1-1)}{4(t_1^2-1)} \cdot \frac{8(t_2^2-1)}{4(t_2^2-1)} = -1$$

$$\frac{2}{t_1+1} \cdot \frac{2}{t_2+1} = -1$$

$(t_1+1)(t_2+1) = -4$

$\Rightarrow t_1^2 + t_2^2 + t_1 t_2 + 5 = 0$

Let Centroid be (h, k)

$$h = \frac{4(t_1^2 + t_2^2 + 1)}{3} = \frac{4(-t_1 t_2 - 4)}{3}$$

$$k = \frac{8(t_1 + t_2 + 1)}{3}$$

$(3k-8)^2 = 64(t_1^2 + t_2^2 + 2t_1 t_2)$

$(3k-8)^2 = 64(t_1 t_2 - 5)$

$$(3k-8)^2 = 64\left(\frac{16-3h}{4} - 5\right)$$

$$\left(y - \frac{8}{3}\right)^2 = -\frac{16}{3}\left(x + \frac{4}{3}\right)$$

$3\, l(LR) = 16$

Question ID : 695278400

25. Let f be a twice differentiable function such that $f(x)=\int_0^x \tan(t-x)dt-\int_0^x f(t)\tan t\,dt, x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$. Then

$f''\left(\frac{\pi}{6}\right)+12f'\left(-\frac{\pi}{6}\right)+f\left(\frac{\pi}{6}\right)$ is equal to ________.

Ans. (5)

Sol. $f(x)=\int_0^x \tan(t-x)dt-\int_0^x f(t)\tan t\,dt$

$f(x)=\int_0^x \tan(-t)dt-\int_0^x f(t)\tan t\,dt$

$f'(x)=-\tan x-f(x)\tan x$

$\int\frac{f'(x)dx}{1+f(x)}=-\int \tan x dx$

$\ln(1+f(x))=-\ln\sec x+\ln c$

$1+f(x)=\frac{c}{\sec x}=c\cos x$

$f(x)=c\cos x-1$

$f(0)=0\Rightarrow c=1$

$f(x)=\cos x-1$

$f'(x)=-\sin x$

$f''(x)=-\cos x$

$f''\left(\frac{\pi}{6}\right)+12f'\left(-\frac{\pi}{6}\right)+f\left(\frac{\pi}{6}\right)=5$