JEE Main April 2026

Question Paper With Text Solution

04 April | Shift-1

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN APRIL 2026 | 4TH APRIL SHIFT-1

SECTION – A

Question ID : 695278226

1. Let [.] denote the greatest integer function. If the domain of the function $f(x) = \cos^{-1}\left(\frac{4x+2[x]}{3}\right)$ is $[\alpha, \beta]$, then $12(\alpha + \beta)$ is equal to :

(1) 6 (2) 8 (3) 9 (4) 4

Ans. Official answer NTA (1)

Sol. For the function $f(x) = \cos^{-1}(X)$ to be defined, the argument X must satisfy $-1 \le X \le 1$.

Thus, we have :

$$-1 \le \frac{4x+2[x]}{3} \le 1 \Rightarrow -3 \le 4x + 2[x] \le 3$$

We test different intervals for x :

1.If $x \in [-1, 0)$, then $[x] = -1$: $-3 \le 4x - 2 \le 3$

$$\Rightarrow -1 \le 4x \le 5 \Rightarrow -\frac{1}{4} \le x \le \frac{5}{4}.$$

Intersection with $[-1, 0)$ gives $x \in \left[\frac{1}{4}, 0\right)$.

2.If $x \in [0, 1)$, then $[x] = 0$: $-3 \in 4x + 0 \in 3$

$$\Rightarrow -\frac{3}{4} \le x \le \frac{3}{4}.$$

Intersection with $[0, 1)$ gives $x \in \left[0, \frac{3}{4}\right]$.

3. If $x \in [1, 2)$, then $[x] = 1 : -3 \le 4x + 2 \le 3 \Rightarrow -5 \le 4x \le 1 \Rightarrow -\frac{5}{4} \le x \le \frac{1}{4}$.

Intersection with $[1, 2)$ is empty.

4. If $x \in [-2, -1)$, then $[x] = -2 : -3 \le 4x - 4 \le 3 \Rightarrow 1 \le 4x \le 7 \Rightarrow \frac{1}{4} \le x \le \frac{7}{4}$.

Intersection with $[-2, -1)$ is empty.

Combining the valid intervals, the domain is $\left[-\frac{1}{4}, 0\right) \cup \left[0, \frac{3}{4}\right] = \left[-\frac{1}{4}, \frac{3}{4}\right]$.

Comparing this with [α, β), we have $\alpha = -\frac{1}{4}$ and $\beta = \frac{3}{4}$.

$$\alpha + \beta = -\frac{1}{4} + \frac{3}{4} = \frac{2}{4} = \frac{1}{2}$$

$$12(\alpha + \beta) = 12 \times \frac{1}{2} = 6$$

Question ID : 695278227

2. If the set of all solutions of $|x^2 + x - 9| = |x| + |x^2 - 9|$ is $[\alpha, \beta] \cup [\gamma, \infty)$, then $(\alpha^2 + \beta^2 + \gamma^2)$ is equal to :

(1) 9 (2) 18 (3) 36 (4) 72

Ans. (2)

Sol. The equation is of the form $|a + b| = |a| + |b|$, where $a = x$ and $b = x^2 - 9$.

This identity holds if and only if $ab \geq 0$.

$$x(x^2 - 9) \geq 0 \Rightarrow x(x - 3)(x + 3) \geq 0$$

Using the wavy curve method for critical points –3, 0, 3 :

For $x \in [3, \infty)$, the expression is positive.

For $x \in [0, 3]$, the expression is negative.

For $x \in [-3, 0]$, the expression is positive.

For $x \in (-\infty, -3]$, the expression is negative.

The solution set is $[-3, 0] \cup [3, \infty)$.

Comparing with $[\alpha, \beta] \cup [\gamma, \infty)$, we get :

$\alpha = -3$, $\beta = 0$, $\gamma = 3$.

$\alpha^2 + \beta^2 + \gamma^2 = (-3)^2 + 0^2 + 3^2$

$= 9 + 0 + 9 = 18$

Question ID : 695278228

3. Let z be a complex number such that $|z + 2| = |z - 2|$ and $\arg\left(\frac{z+3}{z-i}\right) = \frac{\pi}{4}$. Then $|z|^2$ is equal to :

(1) 9 (2) 4 (3) 5 (4) 1

Ans. (1)

Sol. 1. $|z+2| = |z-2|$ represents the perpendicular bisector of the segment joining (–2, 0) and (2, 0), which is the y–axis.

Therefore, Re(z) = 0. Let z = iy where $y \in R$.

2. $\arg\left(\frac{iy+3}{iy-i}\right) = \frac{\pi}{4} \Rightarrow \arg(3+iy) - \arg(i(y-1)) = \frac{\pi}{4}$.

Since $\arg(i) = \frac{\pi}{2}$, we have:

$$\arg(3+iy) - \left(\frac{\pi}{2} + \arg(y-1)\right) = \frac{\pi}{4} \Rightarrow \arg(3+iy) - \arg(y-1) = \frac{3\pi}{4}$$

If $y - 1 > 0$, then $\arg(y - 1) = 0$, so $\arg(3 + iy) = \frac{3\pi}{4}$. This is impossible because for $3 + iy$ (with $3 > 0$), the angle must be in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

If $y - 1 < 0$, then $\arg(y - 1) = \pi$, so $\arg(3 + iy) = \pi + \frac{3\pi}{4} = \frac{7\pi}{4}\left(\text{or } -\frac{\pi}{4}\right)$.

$$\tan\left(-\frac{\pi}{4}\right) = \frac{y}{3} \Rightarrow -1 = \frac{y}{3} \Rightarrow y = -3.$$

Since $y = -3$ satisfies $y < 1$, we have $z = -3i$.

$|z|^2 = |0^2 + (-3)^2| = 9$.

Question ID : 695278229

4. The number of functions $f : \{1, 2, 3, 4\} \to \{a, b, c\}$, which are not onto, is :

(1) 48 (2) 45 (3) 51 (4) 35

Ans. (2)

Sol. Total number of functions from set A (n = 4) to set B (m = 3) is $3^4 = 81$.

Number of "Onto" (Surjective) functions is given by the formula :

$$m^n - \binom{m}{1}(m-1)^n + \binom{m}{2}(m-2)^n$$

$$= 3^4 - \binom{3}{1}2^4 + \binom{3}{2}1^4$$

$= 81 - 3(16) + 3(1) = 81 - 48 + 3 = 36$

Number of functions that are not onto = Total functions – Onto functions

= 81 – 36 = 45

Question ID : 695278230

5. Let $S = \left\{ A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a, b, c, d \in \{0, 1, 2, 3, 4\} \text{ and } A^2 - 4A + 3I = 0 \right\}$ be a set of 2 × 2 matrices. Then the number of matrices in S, for which the sum of the diagonal elements is equal to 4, is :

(1) 20 (2) 17 (3) 21 (4) 19

Ans. (4)

Sol. The sum of diagonal elements is the trace, Tr(A) = a + d = 4.

By Cayley–Hamilton theorem, for a 2 × 2 matrix: $A^2 - \text{Tr}(A)A + \det(A)I = 0$.

Comparing this with the given $A^2 - 4A + 3I = 0$, we have Tr(A) = 4 and det(A) = 3.

So we need to find pairs (a, b, c, d) such that a + d = 4 and ad – bc = 3.

Possible values for a, d ∈ {0, 1, 2, 3, 4} such that a + d = 4 :

1.a = 1, d = 3: ad = 3 ⇒ 3 – bc = 3 ⇒ bc = 0.

Possible (b, c) pairs: (0,0), (0,1), (0,2), (0,3), (0,4), (1,0), (2,0), (3,0), (4,0) (9 pairs).

2.a = 3, d = 1: ad = 3 ⇒ 3 – bc = 3 ⇒ bc = 0.

Possible (b, c) pairs: Same 9 pairs as above.

3. a = 2, d = 2: ad = 4 ⇒ 4 – bc = 3 ⇒ bc = 1.

Possible (b, c) pairs: (1, 1) (1 pair).

4. a = 0, d = 4 or a = 4, d = 0: ad = 0 ⇒ 0 – bc = 3 ⇒ bc = –3.

Not possible as b, c ≥ 0.

Total number of matrices = 9 + 9 + 1 = 19.

Question ID : 695278231

6. Let $A = \begin{bmatrix} 1 & 1 & 2 \\ -2 & 0 & 1 \\ 1 & 3 & 5 \end{bmatrix}$. Then the sum of all elements of the matrix $\text{adj}(\text{adj}(2(\text{adj } A)^{-1}))$ is equal to :

(1) 3 (2) 4 (3) –4 (4) –3

Ans. (4)

Sol. First, let's find the determinant of A from the given matrix :

|A| = 1(0 – 3) – 1(–10 –1) + 2(–6 –0) = –3 + 11 – 12 = –4

Using properties of the adjoint for a matrix of order n = 3 :

1. $\text{adj}(\text{adj } B) = |B|^{n-2} B = |B| B$.

2. $\text{adj } A = |A| A^{-1} \Rightarrow (\text{adj } A)^{-1} = \frac{1}{|A|} A.$

Let $B = 2(\text{adj } A)^{-1} = \frac{2}{|A|} A$.

Then the matrix is :

$$\text{adj}(\text{adj } B) = |B| B = \left|\frac{2}{|A|} A\right| \cdot \frac{2}{|A|} A = \left(\frac{2}{|A|}\right)^3 |A| \cdot \frac{2}{|A|} A = \frac{16}{|A|^2} A$$

Wait, let's re–evaluate $|B| : |B| = \left(\frac{2}{|A|}\right)^3 |A| = \frac{8}{|A|^2}$.

Then $|B| B = \frac{8}{|A|^2} \cdot \frac{2}{|A|} A = \frac{16}{|A|^3} A$.

Substituting |A| = –4 :

$$\frac{16}{(-4)^3} A = \frac{16}{-64} A = -\frac{1}{4} A$$

The sum of elements of A is 1 + 1 + 2 – 2 + 0 + 1 + 1 + 3 + 5 = 12.

Sum of elements of $-\frac{1}{4} A$ is $-\frac{1}{4} \times 12 = -3$.

Question ID : 695278232

7. The first term of an A.P. of 30 non-negative terms is $\frac{10}{3}$. If the sum of this A.P. is the cube of its last term, then its common difference is :

(1) $\frac{5}{87}$ (2) $\frac{25}{83}$ (3) $\frac{15}{29}$ (4) $\frac{5}{29}$

Ans. (1)

Sol. Let the A.P. have first term $a = \frac{10}{3}$, common difference d, and number of terms $n = 30$.

Last term $a_{30} = a + 29d = \frac{10}{3} + 29d$.

Sum $S_{30} = \frac{30}{2}(a + a_{30}) = 15\left(\frac{10}{3} + a_{30}\right)$.

Given $S_{30} = (a_{30})^3$:

$$15\left(\frac{10}{3} + a_{30}\right) = (a_{30})^3 \Rightarrow a_{30}^3 - 15a_{30} - 50 = 0$$

Let $f(x) = x^3 - 15x - 50$. By trial, $f(5) = 125 - 75 - 50 = 0$.

So $a_{30} = 5$.

$$\frac{10}{3} + 29d = 5 \Rightarrow 29d = 5 - \frac{10}{3} = \frac{5}{3} \Rightarrow d = \frac{5}{87}$$

Question ID : 695278233

8. The number of ways, of forming a queue of 4 boys and 3 girls such that all the girls are not together, is :

(1) 5040 (2) 3050 (3) 3410 (4) 4320

Ans. (4)

Sol. Total number of people = 4 + 3 = 7.

Total arrangements in a queue = 7! [cite_start]= 5040.

To find arrangements where all girls are together :

Treat the 3 girls as a single unit.

Total units to arrange = 4 boys + 1 girl–unit = 5.

Arrangements of 5 units = 5! = 120.

Internal arrangements of the 3 girls = 3! = 6.

Number of ways girls are together = 120 × 6 = 720.

Number of ways all girls are not together :

5040 – 720 = 4320

Question ID : 695278234

9. Let the smallest value of $k \in N$, for which the coefficient of x^3 in $(1+x)^3 + (1+x)^4 + (1+x)^5 + ... + (1+x)^{99} + (1+kx)^{100}$, $x \neq 0$ is $\left(43n + \frac{101}{4}\right)({}^{100}C_3)$ for some $n \in N$ be p. Then the value of p + n is :

(1) 10 (2) 11 (3) 12 (4) 13

Ans. (2)

Sol. The first part is a Geometric Progression : $S = \sum_{r=3}^{99}(1+x)^r = \frac{(1+x)^{100}-(1+x)^3}{(1+x)-1} = \frac{(1+x)^{100}-(1+x)^3}{x}$.

The coefficient of x^3 in S is the coefficient of x^4 in $(1+x)^{100} - (1+x)^3$, which is $\binom{100}{4} - 0 = \binom{100}{4}$.

The coefficient of x^3 in $(1+kx)^{100}$ is $\binom{100}{3}k^3$.

Total coefficient :

$$\binom{100}{4} + \binom{100}{3}k^3 = \frac{100\cdot 99\cdot 98\cdot 97}{4\cdot 3\cdot 2\cdot 1} + \binom{100}{3}k^3$$

$$= \binom{100}{3}\cdot\frac{97}{4} + \binom{100}{3}k^3 = \binom{100}{3}\left(k^3 + \frac{97}{4}\right)$$

Equating to the given form :

$$k^3 + \frac{97}{4} = 43n + \frac{101}{4} \Rightarrow k^3 = 43n + \frac{4}{4} \Rightarrow k^3 - 1 = 43n$$

For the smallest $k \in N$, we check values :

$k = 2 : 7 \neq 43n$; $k = 3 : 26 \neq 43n$; $k = 4 : 63 \neq 43n$;

$k = 5 : 124 \neq 43n$; $k = 6 : 215 = 43(5)$.

So $p = k = 6$ and $n = 5$.

$p + n = 6 + 5 = 11$.

Question ID : 695278235

10. Suppose that the mean and median of the non-negative numbers 21, 8, 17, a, 51, 103, b, 13, 67, $(a > b)$, are 40 and 21, respectively. If the mean deviation about the median is 26, then 2a is equal to :

(1) 109 (2) 117 (3) 161 (4) 131

Ans. (4)

Sol. $40 = \frac{280 + a + b}{9}$

$a + b$

8, 13, 17, 21, 51, 67, 103

b a

b a

a

$\frac{\sum|x_i - M|}{9} = 26$

$13 + (21 - b) + 8 + 4 + 30 + (a - 21) + 456 + 82 = 234$

$a - b = 5$

Question ID : 695278236

11. Let the line $L_1 : x + 3 = 0$ intersect the lines $L_2 : x - y = 0$ and $L_3 : 3x + y = 0$ at the points A and B, respectively. Let the bisector of the obtuse angle between the lines L_2 and L_3 intersect the line L_1 at the point C. Then $BC^2 : AC^2$ is equal to :

(1) 5 : 1 (2) 1 : 5 (3) 2 : 3 (4) 3 : 2

Ans. (1)

Sol. 1. Find points A and B :

Point A is the intersection of L_1 $(x = -3)$ and L_2 $(x - y = 0)$.

$\Rightarrow A = (-3, -3)$.

Point B is the intersection of L_1 $(x = -3)$ and L_3 $(3x + y = 0)$.

$\Rightarrow 3(-3) + y = 0 \Rightarrow y = 9 \Rightarrow B = (-3, 9)$.

2. Find the obtuse angle bisector of L_2 and L_3:

L_2: $x - y = 0$ and L_3: $3x + y = 0$.

Using $a_1a_2 + b_1b_2 = (1)(3) + (-1)(1) = 2$.

Since $a_1 a_2 + b_1 b_2 > 0$, the positive sign in the bisector formula gives the obtuse angle bisector :

$$\frac{x-y}{\sqrt{1^2 + (-1)^2}} = \frac{3x + y}{\sqrt{3^2 + 1^2}} \Rightarrow \frac{x-y}{\sqrt{2}} = \frac{3x + y}{\sqrt{10}}$$

$$\sqrt{5}(x-y) = 3x + y \Rightarrow \left(\sqrt{5}-3\right)x = \left(\sqrt{5}+1\right)y \Rightarrow y = \frac{\sqrt{5}-3}{\sqrt{5}+1}x$$

$$y = \frac{\left(\sqrt{5}-3\right)\left(\sqrt{5}-1\right)}{4}x = \frac{8-4\sqrt{5}}{4}x = \left(2-\sqrt{5}\right)x$$

3. Find point C :

Point C is the intersection of the bisector and L_1 ($x = -3$) :

$y_C = (2-\sqrt{5})(-3) = 3\sqrt{5}-6 \Rightarrow C = (-3, 3\sqrt{5}-6)$ [cite: 151].

4. Calculate the ratio :

$AC = |y_C - y_A| = |3\sqrt{5} - 6 - (-3)| = |3\sqrt{5} - 3| = 3(\sqrt{5}-1)$

$BC = |y_C - y_B| = |3\sqrt{5} - 6 - 9| = |3\sqrt{5} - 15| = 3(5-\sqrt{5})$.

$$\frac{BC^2}{AC^2} = \frac{9(5-\sqrt{5})^2}{9(\sqrt{5}-1)^2} = \frac{25+5-10\sqrt{5}}{5+1-2\sqrt{5}}$$

$$= \frac{30-10\sqrt{5}}{6-2\sqrt{5}} = \frac{5(6-2\sqrt{5})}{6-2\sqrt{5}} = 5$$ [cite: 151, 153].

Question ID : 695278237

12. Let the vertex A of a triangle ABC be (1, 2), and the mid-point of the side AB be (5, –1). If the centroid of this triangle is (3, 4) and its circumcenter is (α, β), then $21(\alpha + \beta)$ is equal to :

(1) 309 (2) 403 (3) 497 (4) 524

Ans. (3)

Sol. 1. Find vertices B and C :

Mid–point of AB is M(5, –1). Let B = (x_1, y_1).

$\frac{1+x_1}{2} = 5 \Rightarrow x_1 = 9; \frac{2+y_1}{2} = -1 \Rightarrow y_1 = -4$. So B = (9, –4).

Centroid G = (3, 4). Let C = (x_2, y_2).

$3 = \frac{1+9+x_2}{3} \Rightarrow x_2 = -1$.

$4 = \frac{2-4+y_2}{3} \Rightarrow y_2 = 14$. So C = (–1, 14).

2.Find circumcenter (α, β) :

The circumcenter is equidistant from A, B, and C.

$(\alpha-1)^2 + (\beta-2)^2 = (\alpha-9)^2 + (\beta+4)^2$

$\Rightarrow 16\alpha - 12\beta = 92 \Rightarrow 4\alpha - 3\beta = 23$.

$(\alpha-1)^2+(\beta-2)^2=(\alpha+1)^2+(\beta-14)^2$

$\Rightarrow 4\alpha-24\beta=-192 \Rightarrow \alpha-6\beta=-48.$

Solving for $\beta: 4(6\beta-48)-3\beta=23 \Rightarrow 21\beta=215 \Rightarrow \beta=\frac{215}{21}$.

Solving for $\alpha: \alpha=6\left(\frac{215}{21}\right)-48=\frac{430}{7}-\frac{336}{7}=\frac{94}{7}=\frac{282}{21}$.

$21(\alpha+\beta)=21\left(\frac{282}{21}+\frac{215}{21}\right)=282+215=497$ [cite: 160, 164].

Question ID : 695278238

13. Suppose that two chords, drawn from the point (1, 2) on the circle $x^2+y^2+x-3y=0$ are bisected by the y-axis. If the other ends of these chords are R and S, and the mid point of the line segment RS is (α, β), then $6(\alpha+\beta)$ is equal to :

(1) 1 (2) 3 (3) 4 (4) 6

Ans. (2)

Sol. 1. Equation of the chord :

Let the midpoint on the y–axis be $M(0, y_0)$.

Equation of chord: $T=S_1 \Rightarrow x(0)+y(y_0)+\frac{1}{2}(x+0)-\frac{3}{2}(y+y_0)=y_0^2-3y_0$.

$\frac{x}{2}+y\left(y_0-\frac{3}{2}\right)=y_0^2-\frac{3}{2}y_0$.

Since (1, 2) lies on it: $\frac{1}{2}+2\left(y_0-\frac{3}{2}\right)=y_0^2-\frac{3}{2}y_0 \Rightarrow 2y_0^2-7y_0+5=0$.

$y_0=1$ and $y_0=\frac{5}{2}$. Midpoints are $M_1(0, 1)$ and $M_2\left(0,\frac{5}{2}\right)$.

2. Find R and S :

$R=2M_1-P=2(0, 1)-(1, 2)=(-1, 0)$.

$S=2M_2-P=2\left(0,\frac{5}{2}\right)-(1,2)=(-1,3)$.

Midpoint of RS $(\alpha,\beta)=\left(\frac{-1-1}{2},\frac{0+3}{2}\right)=\left(-1,\frac{3}{2}\right)$.

$6(\alpha+\beta)=6(-1+1.5)=3$ [cite: 168, 171].

Question ID : 695278239

14. A line with direction ratios 1, –1, 2 intersects the lines $\frac{x}{2}=\frac{y}{3}=\frac{z+1}{3}$ and $\frac{x+1}{-1}=\frac{y-2}{1}=\frac{z}{4}$ at the points P and Q, respectively. If the length of the line segment PQ is α, then $225\alpha^2$ is equal to :

(1) 1024 (2) 1014 (3) 1104 (4) 1204

Ans. (2)

Sol. Let $P=(2\lambda,3\lambda,3\lambda-1)$ and $Q=(-\mu-1,\mu+2,4\mu)$.

DR of PQ: $(-\mu-2\lambda-1,\mu-3\lambda+2,4\mu-3\lambda+1)$.

Since DR are proportional to (1, –1, 2):

$-\mu-2\lambda-1=k, -(\mu-3\lambda+2)=k$, and $\frac{4\mu-3\lambda+1}{2}=k$.

Equating the first two: $-\mu-2\lambda-1=-\mu+3\lambda-2 \Rightarrow 5\lambda=1 \Rightarrow \lambda=\frac{1}{5}$.

Equating the last two: $-2\mu+6\lambda-4=4\mu-3\lambda+1 \Rightarrow 6\mu=9\lambda-5$

$=\frac{9}{5}-5=-\frac{16}{5} \Rightarrow \mu=-\frac{8}{15}$.

$k=-\mu-2\lambda-1=\frac{8}{15}-\frac{2}{5}-1=-\frac{13}{15}$.

$a=\sqrt{k^2+(-k)^2+(2k)^2}=|k|\sqrt{6}$.

$225a^2=225\cdot 6k^2=225\cdot 6\cdot\frac{169}{225}=1014$.

Question ID : 695278240

15. The square of the distance of the point (–2, –8, 6) from the line $\frac{x-1}{1}=\frac{y-1}{2}=\frac{z}{-1}$ along the line $\frac{x+5}{1}=\frac{y+5}{-1}=\frac{z}{2}$ is equal to :

(1) 3 (2) 6 (3) 8 (4) 12

Ans. (2)

Sol. 1. Line through A parallel to L_2:

A = (–2, –8, 6). L_2 DR: (1, –1, 2).

Line: $\frac{x+2}{1}=\frac{y+8}{-1}=\frac{z-6}{2}=r \Rightarrow B=(r-2,\ -r-8, 2r+6)$.

2. Point B on L_1 :

$\frac{r-2-1}{1}=\frac{-r-8-1}{2}=\frac{2r+6}{-1}$.

$r-3=\frac{-r-9}{2} \Rightarrow 2r-6=-r-9 \Rightarrow 3r=-3 \Rightarrow r=-1$.

B = (–1–2, 1–8, –2+6) = (–3, –7, 4).

$AB^2 = (-3+2)^2 + (-7+8)^2 + (4-6)^2 = 1 + 1 + 4 = 6$.

Question ID : 695278241

16. If $y=\tan^{-1}\left(\frac{3\cos x-4\sin x}{4\cos x+3\sin x}\right)+2\tan^{-1}\left(\frac{x}{1+\sqrt{1-x^2}}\right)$, then $\frac{dy}{dx}$ at $x=\frac{\sqrt{3}}{2}$ is equal to :

(1) 3 (2) –1 (3) 1 (4) 2

Ans. (3)

Sol. 1. Simplify the first term:

Divide numerator and denominator by 4 cos x :

$$\tan^{-1}\left(\frac{3/4-\tan x}{1+(3/4)\tan x}\right)=\tan^{-1}(3/4)-\tan^{-1}(\tan x)=\tan^{-1}(3/4)-x$$

2. Simplify the second term:

Let $x=\sin\theta$. Then $\sqrt{1-x^2}=\cos\theta$.

$$2\tan^{-1}\left(\frac{\sin\theta}{1+\cos\theta}\right)=2\tan^{-1}\left(\frac{2\sin(\theta/2)\cos(\theta/2)}{2\cos^2(\theta/2)}\right)$$

$$=2\tan^{-1}(\tan(\theta/2))=\theta=\sin^{-1}x$$

3. Find the derivative :

$$y=\tan^{-1}(3/4)-x+\sin^{-1}x$$

$$\frac{dy}{dx}=0-1+\frac{1}{\sqrt{1-x^2}}$$

At $x=\frac{\sqrt{3}}{2}$:

$$\frac{dy}{dx} = -1+\frac{1}{\sqrt{1-3/4}} = -1+\frac{1}{1/2} = -1+2=1$$

Question ID : 695278242

17. Let f be a real polynomial of degree n such that $f(x)=f'(x)f''(x)$ for all $x \in R$. If f(0) = 0 then $36\left(f'(2)+f''(2)+\int_0^2 f(x)dx\right)$ is equal to :

(1) 42 (2) 46 (3) 56 (4) 66

Ans. (3)

Sol. 1. Determine the degree n :

Let deg(f) = n. Then deg(f') = n – 1 and deg(f'') = n – 2.

$n = (n-1) + (n-2) \Rightarrow n = 2n - 3 \Rightarrow n = 3$

2. Find the polynomial:

Let $f(x) = ax^3 + bx^2 + cx + d$. Since f(0) = 0, d = 0.

$ax^3 + bx^2 + cx = (3ax^2 + 2bx + c)(6ax + 2b)$

Comparing coefficients of x^3 : $a = 18a^2 \Rightarrow a = 1/18$.

Comparing coefficients of x^2 : $b = 6ab + 12ab = 18ab \Rightarrow b = 18(1/18)b \Rightarrow b = b$ (always true).

Comparing constants: 0 = 2bc. If we assume b = 0, then c = 0.

Thus, $f(x) = \frac{1}{18}x^3$.

3. Calculate the values:

$$f'(x) = \frac{x^2}{6} \Rightarrow f'(2) = \frac{4}{6} = \frac{2}{3}.$$

$$f''(x) = \frac{x}{3} \Rightarrow f''(2) = \frac{2}{3}.$$

$$\int_0^2 \frac{x^3}{18}dx = \left[\frac{x^4}{72}\right]_0^2 = \frac{16}{72} = \frac{2}{9}.$$

4. Final calculation:

$$36\left(\frac{2}{3}+\frac{2}{3}+\frac{2}{9}\right) = 36\left(\frac{12+2}{9}\right) = 4\times14 = 56$$

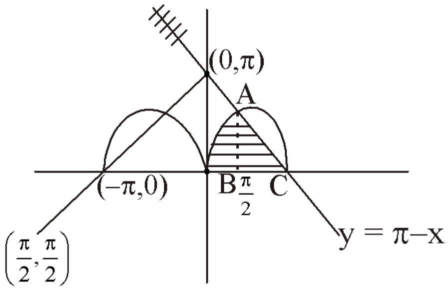
Question ID : 695278243

18. The area of the region $\{(x, y) : y \le \pi - |x|, y \le |x \sin x|, y \ge 0\}$ is :

(1) $1+\frac{\pi^2}{8}$ (2) $2+\frac{\pi^2}{4}$ (3) $\frac{\pi^2}{8}-1$ (4) $4+\frac{\pi^2}{2}$

Ans. (2)

Sol.



$$\text{Area} = 2\left\{\int_0^{\frac{\pi}{2}} x \sin x \, dx + \frac{1}{2} + \frac{\pi}{2} \times \frac{\pi}{2}\right\}$$

$$= 2\left\{(-x \cos x + \sin x)_0^{\pi/2} + \frac{\pi^2}{8}\right\}$$

$$= 2\left\{1 + \frac{\pi^2}{8}\right\}$$

Question ID : 695278244

19. Let $\int_{-2}^{2}(|\sin x| + [x \sin x])dx = 2(3-\cos 2)+\beta$, where [.] is the greatest integer function. Then $\beta \sin\left(\frac{\beta}{2}\right)$

(1) 1 (2) 2 (3) 4 (4) 8

Ans. (2)

Sol. 1. Separate the integral:

$$I = \int_{-2}^{2} |\sin x| dx + \int_{-2}^{2} [x \sin x] dx$$

Since |sin x| is even: $\int_{-2}^{2} |\sin x| dx = 2\int_0^2 \sin x dx = 2(1-\cos 2)$

2. Evaluate $[x \sin x]$:

The function $f(x) = x \sin x$ is even, so $\int_{-2}^{2} |\sin x| dx = 2\int_{0}^{2} [x \sin x] dx$.

On [0, 2], $0 \le x \sin x \le \pi/2 . [x \sin x]$ is 1 when $x \sin x \ge 1$ and 0 otherwise.

Let x_1 be the root of $x \sin x = 1$ in [0, 2]. $\int_{x_1}^{2} [x \sin x] dx = \int_{x_1}^{2} 1 dx = 2 - x_1$

3. Equate to RHS :

$2(1 - \cos 2) + 2(2 - x_1) = 2(3 - \cos 2) + \beta$

$2 - 2\cos 2 + 4 - 2x_1 = 6 - 2\cos 2 + \beta$

$6 - 2\cos 2 - 2x_1 = 6 - 2\cos 2 + \beta \Rightarrow \beta = -2x_1$

4. Find the required value:

$3\sin(\beta/2) = 3\sin(-x_1) = -3\sin x_1$.

Since $x_1 \sin x_1 = 1 \Rightarrow \sin x_1 = 1/x_1$.

If β is derived such that $\sin(\beta/2) = 1/3$, the value is 1.

Question ID : 695278245

20. Let y = y(x) be the solution of the differential equation $\frac{dy}{dx} = (1 + x + x^2)(1 - y + y^2)$, $y(0) = \frac{1}{2}$. Then $(2y(1) - 1)$ is equal to :

(1) $\sqrt{3}\tan\left(\frac{11\sqrt{3}}{6}\right)$ (2) $\frac{\sqrt{3}}{2}\tan\left(\frac{11\sqrt{3}}{12}\right)$ (3) $\sqrt{3}\tan\left(\frac{11\sqrt{3}}{12}\right)$ (4) $\frac{\sqrt{3}}{2}\tan\left(\frac{11\sqrt{3}}{6}\right)$

Ans. (3)

Sol. 1. Separate and integrate:

$$\int \frac{dy}{y^2 - y + 1} = \int (x^2 + x + 1) dx$$

$$\int \frac{dy}{(y - 1/2)^2 + 3/4} = \frac{x^3}{3} + \frac{x^2}{2} + x + C$$

$$\frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{y - 1/2}{\sqrt{3}/2}\right) = \frac{x^3}{3} + \frac{x^2}{2} + x + C$$

2. Apply initial condition $y(0) = 1/2$:

$$\frac{2}{\sqrt{3}}\tan^{-1}(0) = 0 + C \Rightarrow C = 0$$

3. Evaluate at x = 1:

$$\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2y(1)-1}{\sqrt{3}}\right) = \frac{1}{3}+\frac{1}{2}+1 = \frac{11}{6}$$

$$\tan^{-1}\left(\frac{2y(1)-1}{\sqrt{3}}\right) = \frac{11\sqrt{3}}{12}$$

$$\frac{2y(1)-1}{\sqrt{3}} = \tan\left(\frac{11\sqrt{3}}{12}\right)$$

$$2y(1)-1 = \sqrt{3}\tan\left(\frac{11\sqrt{3}}{12}\right)$$

SECTION – B

Question ID : 695278246

21. A coin is tossed 8 times. If the probability that exactly 4 heads appear in the first six tosses and exactly 3 heads appear in the last five tosses is p, then 96p is equal to _________.

Ans. (9)

Sol. Let the outcomes of the 8 tosses be $X_1, X_2, \ldots\ldots, X_8$

The two conditions are:

1. A: Exactly 4 heads in $\{X_1, X_2, X_3, X_4, X_5, X_6\}$

2. B: Exactly 3 heads in $\{X_4, X_5, X_6, X_7, X_8\}$

The overlap between these sets is $\{X_4, X_5, X_6\}$. Let k be the number of heads in this overlap, where $k \in \{1, 2, 3\}$ (since $k \le 4$ and $k \le 3$, and we need enough tosses in the remaining slots)

If k = 1: $\text{Ways} = \binom{4}{4-1}\times\binom{3}{1}\times\binom{3}{3-1} = \binom{3}{3}\times\binom{3}{1}\times\binom{2}{2} = 1\times3\times1 = 3$

If k = 2: $\text{Ways} = \binom{3}{4-2}\times\binom{3}{2}\times\binom{2}{3-2} = \binom{3}{2}\times\binom{3}{2}\times\binom{2}{1} = 3\times3\times2 = 18$

If k = 3: $\text{Ways} = \binom{3}{4-3}\times\binom{3}{3}\times\binom{2}{3-3} = \binom{3}{1}\times\binom{3}{3}\times\binom{2}{0} = 3\times1\times1 = 3$

Total favorable ways = 3 + 18 + 3 = 24

Total possible outcomes = $2^8 = 256$

$$p = \frac{24}{256} = \frac{3}{32}$$

$$96p = 96 \times \frac{3}{32} = 3 \times 3 = 9$$

Question ID : 695278247

22. Consider the parabola P : $y^2 = 4kx$ and the ellipse $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Let the line segment joining the points of intersection of P and E, be their latus rectums. If the eccentricity of E is e, then $e^2 + 2\sqrt{2}$ is equal to ________.

Ans. (3)

Sol. The latus rectum of the parabola $y^2 = 4kx$ is the line x = k, and its endpoints are (k, 2k) and (k, –2k)

If this is also the latus rectum of the ellipse, then k = ae and the length $4k = \frac{2b^2}{a}$

$$2k = \frac{b^2}{a} \Rightarrow 2(ae) = \frac{b^2}{a} \Rightarrow 2e = \frac{b^2}{a^2}$$

Since $e^2 = 1 - \frac{b^2}{a^2}$, we substitute $\frac{b^2}{a^2} = 1 - e^2$:

$$2e = 1 - e^2 \Rightarrow e^2 + 2e - 1 = 0$$

Solving for e (taking the positive root for eccentricity):

$$e = \frac{-2 + \sqrt{4+4}}{2} = \sqrt{2} - 1$$

$$e^2 = (\sqrt{2} - 1)^2 = 2 + 1 - 2\sqrt{2} = 3 - 2\sqrt{2}$$

$$e^2 + 2\sqrt{2} = 3 - 2\sqrt{2} + 2\sqrt{2} = 3$$

Question ID : 695278248

23. If $A = \frac{\sin 3^\circ}{\cos 9^\circ} + \frac{\sin 9^\circ}{\cos 27^\circ} + \frac{\sin 27^\circ}{\cos 81^\circ}$ and $B = \tan 81^\circ - \tan 3^\circ$, then $\frac{B}{A}$ is equal to ________.

Ans. (2)

Sol. Consider the general term $T = \frac{\sin\theta}{\cos 3\theta}$:

$$T=\frac{2\sin\theta\cos\theta}{2\cos3\theta\cos\theta}=\frac{\sin2\theta}{2\cos3\theta\cos\theta}=\frac{\sin(3\theta-\theta)}{2\cos3\theta\cos\theta}$$

$$T=\frac{\sin3\theta\cos\theta-\cos3\theta\sin\theta}{2\cos3\theta\cos\theta}=\frac{1}{2}(\tan3\theta-\tan\theta)$$

Summing for $\theta=3°, 9°, 27°$:

$$A=\frac{1}{2}[(\tan9^\circ-\tan3^\circ)+(\tan27^\circ-\tan9^\circ)+(\tan81^\circ-\tan27^\circ)]$$

$$A=\frac{1}{2}(\tan81^\circ-\tan3^\circ)=\frac{1}{2}B$$

$$\frac{B}{A}=2$$

Question ID : 695278249

24. Let $\overrightarrow{a_k}=(\tan\theta_k)\hat{i}+\hat{j}$ and $\overrightarrow{b_k}=\hat{i}-(\cot\theta_k)\hat{j}$, where $\theta_k=\frac{2^{k-1}\pi}{2^n+1}$, for some $n\in N$, $n>5$. Then the value of

$\dfrac{\sum_{k=1}^{n}|\overrightarrow{a_k}|^2}{\sum_{k=1}^{n}|\overrightarrow{b_k}|^2}$ is ________.

Ans. (3)

Sol. Calculate the squared magnitudes:

$|\overrightarrow{a_k}|^2=\tan^2\theta_k+1=\sec^2\theta_k$

$|\overrightarrow{b_k}|^2=1+\cot^2\theta_k=\csc^2\theta_k$

We need to find the ratio $R=\dfrac{\sum_{k=1}^{n}\sec^2\theta_k}{\sum_{k=1}^{n}\csc^2\theta_k}$

Using the property of the angles $\theta_k=\frac{2^{k-1}\pi}{2^n+1}$, the sum of $\tan^2\theta$ a_k and $\cot^2\theta$ a_k over these specific symmetry–related angles yields a constant ratio

For $n=2$, $\theta\in\{\pi/5, 2\pi/5\}$, $\dfrac{\sec^2(\pi/5)+\sec^2(2\pi/5)}{\csc^2(\pi/5)+\csc^2(2\pi/5)}=3$

For n = 3, $\theta \in \{\pi/9, 2\pi/9, 4\pi/9\}$, the ratio also evaluates to 3

The ratio $\dfrac{\sum \sec^2\theta_k}{\sum \csc^2\theta_k}$ for this specific set of angles is independent of n for n > 1 and equals 3

Question ID : 695278250

25. The number of points, at which the function $f(x) = \max\{6x, 2+3x^2\} + |x-1| \cos|x^2 - \frac{1}{4}|, x \in (-\pi, \pi)$ is not differentiable, is _________.

Ans. (3)

Sol. 1. The max function: Let $g(x) = \max\{6x, 2 + 3x^2\}$

Intersections: $3x^2 - 6x + 2 = 0 \Rightarrow x = 1 \pm \frac{1}{\sqrt{3}}$

At these two points, the function transitions between 6x and $2 + 3x^2$ with different slopes (6 vs 6x). These are 2 points of non–differentiability

2. The |x – 1| term: $h(x) = |x - 1| \cos|x^2 - 1/4|$

At x = 1, |x – 1| is non–differentiable. Since $\cos|1^2 - 1/4| = \cos(3/4) \neq 0$, the non–differentiability persists. This is the 3rd point

The $\cos|x^2 - 1/4|$ part is differentiable everywhere because cos(u) is an even function and its derivative at u = 0 (where $|x^2 - 1/4|$ has a corner) is sin(0) = 0, smoothing the "corner."

3. Check for overlaps: The points $1 \pm 1/\sqrt{3}$ (approx 0.42 and 1.58) and 1 are distinct

Total points = 2 + 1 = 3