JEE Main April 2026

Question Paper With Text Solution

02 April | Shift-2

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

Office : Piprali Road, Sikar (Raj.) | Ph. 01572-241911
Website : www.matrixedu.in ; Email : smd@matrixacademy.co.in

JEE MAIN APRIL 2026 | 2ND APRIL SHIFT-2

SECTION – A

Question ID : 691121151

1. Let α, β be the roots of the equation $x^2 - 3x + r = 0$ and $\frac{\alpha}{2}, 2\beta$ be the roots of the equation $x^2 + 3x + r = 0$.

If the roots of the equation $x^2 + 6x = m$ are $2\alpha + \beta + 2r$ and $\alpha - 2\beta - \frac{r}{2}$, then m is equal to:

(1) –135 (2) –567 (3) 135 (4) 567

Ans. (4)

Sol. $x^2 - 3x + r = 0 \left\langle \begin{matrix} \alpha \\ \beta \end{matrix} \right.$

$\alpha + \beta = 3$ ....(i)

$\alpha\beta = r$ ....(ii)

$x^2 + 3x + r = 0 \left\langle \begin{matrix} \alpha/2 \\ 2\beta \end{matrix} \right.$

$\frac{\alpha}{2} + 2\beta = -3$ ....(iii)

From (i) and (iii)

$\alpha = 6,\ \beta = -3$

and from (ii)

$r = -18$

Now

$x^2 + 6x - m = 0 \left\langle \begin{matrix} 2\alpha + \beta + 2r \\ \alpha - 2\beta - \frac{r}{2} \end{matrix} \right.$

$2\alpha + \beta + 2r = -27$

and $\alpha - 2\beta - \frac{r}{2} = 21$

so $-m = (-27)(21)$

$\Rightarrow m = 567$

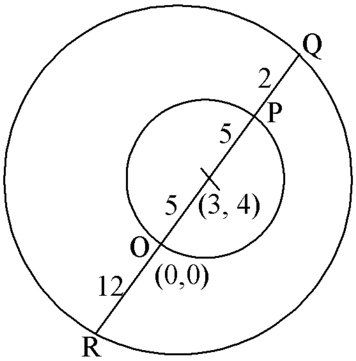
Question ID : 691121152

2. Let the circles $C_1 : |z| = r$ and $C_2 : |z - 3 - 4i| = 5$, $z \in \mathbf{C}$, be such that C_2 lies within C_1. If z_1 moves on C_1, z_2 moves on C_2 and $\min |z_1 - z_2| = 2$, then $\max|z_1 - z_2|$ is equal to :

(1) 12 (2) 17 (3) 22 (4) 24

Ans. (3)

Sol.



$\text{Max } |z_1 - z_2| = OR + OP$

$= 12 + 5 + 5$

$= 22$

Question ID : 691121153

3. If the system of equations

$x + 5y + 6z = 4,$
$2x + 3y + 4z = 7,$
$x + 6y + az = b$

has infinitely many solutions, then the point (a, b) lies on the line :

(1) $y - x = 3$ (2) $x - y = 3$ (3) $x + y = 11$ (4) $x + y = 12$

Ans. (2)

Sol. $x + 5y + 6z = 4$

$2x + 3y + 4z = 7$

$x + 6y + az = b$

$$D = 0 \Rightarrow \begin{vmatrix} 1 & 5 & 6 \\ 2 & 3 & 4 \\ 1 & 6 & a \end{vmatrix} = 0$$

⇒ expanding with R_3

$(20 - 18) -6(4 - 12) + a(3 - 10) = 0$

$$\Rightarrow \quad a = \frac{50}{7}$$

also $D_z = 0$

$$\Rightarrow \begin{vmatrix} 1 & 5 & 4 \\ 2 & 3 & 7 \\ 1 & 6 & b \end{vmatrix} = 0$$

⇒ expanding with R_3

$(35 - 12) - 6(7 - 8) + b(3 - 10) = 0$

$$\Rightarrow b = \frac{29}{7}$$

so $a - b = 3$

Question ID : 691121154

4. Let $a_1, a_2, a_3, \ldots\ldots$ be an A.P. and $g_1 = a_1, g_2, g_3, \ldots\ldots$ be an increasing G.P. If $a_1 = a_2 + g_2 = 1$ and $a_3 + g_3 = 4$, then $a_{10} + g_5$ is equal to :

(1) 81 (2) 76 (3) 62 (4) 55

Ans. (4)

Sol. Let $a_1 = g_1, a = 1$

$a_2 = 1 + d,$ $a_3 = 1 + 2d$

$g_2 = r,$ $g_3 = r^2$

Now, $a_2 + g_2 = 1$

$\Rightarrow 1 + d + r = 1$

$\Rightarrow d + r = 0$ $\Rightarrow d = -r$

and $a_3 + g_3 = 4$

$\Rightarrow 1 + 2d + r^2 = 4$

put $d = -r$

$\Rightarrow (r - 1)^2 = 4$

$\Rightarrow r - 1 = \pm 2$

$\Rightarrow r = 3, -1$

$r = 3$ for increassing GP

$d = -3$

Now $a_{10} + g_5$

$= 1 + 9d + r^4$

$= 1 - 27 + 81$

$= 55$

Question ID : 691121155

5. The sum $\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+\ldots$ up to 8 terms, is :

(1) 70 (2) 71 (3) 72 (4) 73

Ans. (2)

Sol. $\text{Tr}=\frac{1^3+2^3+\cdots+r^3}{1+3+5+\cdots+(2r-1)}$

$$=\frac{\left(\frac{r(r+1)}{2}\right)^2}{r^2}=\frac{(r+1)^2}{4}$$

$$\text{So sum}=\frac{1}{4}\sum_{r=1}^{8}(r+1)^2$$

$$=\frac{1}{4}\left[1^2+2^2+\cdots+9^2-1^2\right]$$

$$=\frac{1}{4}\left(\frac{9\cdot10\cdot19}{6}-1\right)=71$$

Question ID : 691121156

6. If for $3 \le r \le 30$, $\left({}^{30}C_{30-r}\right)+3\left({}^{30}C_{31-r}\right)+3\left({}^{30}C_{32-r}\right)+\left({}^{30}C_{33-r}\right)={}^{m}C_{r}$ then m equals :

(1) 31 (2) 32 (3) 33 (4) 34

Ans. (3)

Sol. ${}^{3}C_{3}\,{}^{30}C_{30-r} + {}^{3}C_{2} \cdot {}^{30}C_{31-r} + {}^{3}C_{1} \cdot {}^{30}C_{32-r} + {}^{3}C_{8} \cdot {}^{30}C_{33-r}$

$= {}^{33}C_{33-r} = {}^{33}C_{r}$

Question ID : 691121157

7. Let P_n denote the total number of triangles formed by joining the vertices of an n–side regular polygon. If $P_{n+1} - P_n = 66$ then the sum of all distinct prime divisors of n is :

(1) 7 (2) 8 (3) 5 (4) 6

Ans. (3)

Sol. $P_n = {}^{n}C_3$

$\Rightarrow P_{n+1} - P_n = 66$

$\Rightarrow {}^{n+1}C_3 - {}^{n}C_3 = 66$

$$\Rightarrow \frac{(n+1)\cdot n\cdot(n-1)}{6} - \frac{n(n-1)(n-2)}{6} = 66$$

$\Rightarrow n(n-1).3 = 6\ .\ 66$

$\Rightarrow n(n-1) = 132$

$\Rightarrow n^2 - n - 132 = 0$

$\Rightarrow (n-12)(n+11) = 0$

$\Rightarrow n = 12,$ $n = -11$ (Rejected)

Now

sum of prime diviser of 12

$= 2 + 3 = 5$

Question ID : 691121158

8. A man throws a fair coin repeatedly. He gets 10 points for each head he throws and 5 points for each tail he throws. If the probability that he gets exactly 30 points is $\frac{m}{n}$ where gcd (m, n) = 1, then m + n is equal to :

(1) 53 (2) 55 (3) 107 (4) 105

Ans. (3)

Sol. HHH + HHTT + HTTTT + TTTTTT

$$\frac{1}{8}+{}^{4}C_{2}\cdot\frac{1}{16}+{}^{5}C_{1}\cdot\frac{1}{32}+\frac{1}{64}$$

$$=\frac{1}{8}+\frac{6}{16}+\frac{5}{32}+\frac{1}{64}$$

$$=\frac{8+24+10+1}{64}=\frac{43}{64}$$

so m + n = 43 + 64

= 107

Question ID : 691121159

9. The mean and variance of n observations are 8 and 16, respectively. If the sum of the first (n – 1) observations is 48 and the sum of squares of the first (n – 1) observations is 496, then the value of n is :

(1) 21 (2) 16 (3) 13 (4) 7

Ans. (4)

Sol. $$\frac{\left(x_1+x_2+\cdots+x_{n-1}\right)+x_n}{n}=8$$

$\Rightarrow 48 + x_n = 8n$

$\Rightarrow x_n = 8(n-6)$ ....(i)

and $$\frac{x_1^{\ 2}+x_2^{\ 2}+\cdots+x_{n-1}^{\ \ 2}+x_n^{\ 2}}{n}-64=16$$

$\Rightarrow 496 + x_n^{\ 2} = 80n$

$\Rightarrow x_n^{\ 2} = 80n - 496$ ....(ii)

from (1) and (ii)

$64(n-6)^2 = 16(5n-31)$

$\Rightarrow 4(n^2 - 12n + 36) = 5n - 31$

$\Rightarrow 4n^2 - 53n + 175 = 0$

$\Rightarrow (4n-25)(n-7) = 0$

$$\Rightarrow n = 7,\ n=\frac{25}{4}\notin N$$

Question ID : 691121160

10. Let a circle pass through the origin and its centre be the point of intersection of two mutually perpendicular lines $x + (k-1)y + 3 = 0$ and $2x + k^2y - 4 = 0$. If the line $x - y + 2 = 0$ intersects the circle at the points A and B, then $(AB)^2$ is equal to :

(1) 10 (2) 27 (3) 18 (4) 34

Ans. (3)

Sol. $x + (k-1)y + 3 = 0$

and $2x + k^2 y - 4 = 0$

lines are perpendicular so

$$\left(\frac{-1}{k-1}\right)\left(\frac{-2}{k^2}\right) = -1$$

$\Rightarrow k^3 - k^2 = -2$

$\Rightarrow k = -1$

so lines are

$x - 2y + 3 = 0$

and $2x + y - 4 = 0$

POI is (1, 2) which is centre

Circe passes through origin so $r = \sqrt{5}$

Equation of circle

$x^2 + y^2 - 2x - 4y = 0$

solving it with line

$y = x + 2$

gives

$x^2 + (x+2)^2 - 2x - 4(x+2) = 0$

$\Rightarrow x^2 + x^2 + 4x + 4 - 2x - 4x - 8 = 0$

$\Rightarrow 2x^2 - 2x - 4 = 0$

$\Rightarrow x^2 - x - 2 = 0$

$\Rightarrow (x-2)(x+1) = 0$

$\Rightarrow x = -1, 2$

so points A(–1, 1) and B(2, 4)

$$AB = \sqrt{18} \quad \Rightarrow \quad AD^2 = 18$$

Question ID : 691121161

11. Let O be the origin, and P and Q be two points on the rectangular hyperbola xy =12 such that the mid point of the line segment PQ is $\left(\frac{1}{2}, -\frac{1}{2}\right)$ Then the area of the triangle OPQ equals :

(1) $\frac{3}{2}$ (2) $\frac{5}{2}$ (3) $\frac{7}{2}$ (4) $\frac{9}{2}$

Ans. (3)

Sol. $\left(t, \frac{12}{t}\right) \qquad \left(\frac{1}{2}, \frac{-1}{2}\right) \qquad \left(P, \frac{12}{P}\right)$

$$\frac{t+p}{2} = \frac{1}{2} \text{ and } \frac{12}{t} + \frac{12}{p} = -1$$

$\Rightarrow t + P = 1$ ....(i)

$$\Rightarrow \frac{t+p}{tp} = -\frac{1}{12}$$

$\Rightarrow tP = -12$ ....(ii)

From (i) and (ii)

t = 4, P = –3

so P(4, 3) and Q(–3, –4)

$$\text{Area of } \Delta OPQ = \left|\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 4 & 3 & 1 \\ -3 & -4 & 1 \end{vmatrix}\right|$$

$$= \left|\frac{1}{2}(-16+9)\right| = \frac{7}{2}$$

Question ID : 691121162

12. Let the parabola $y = x^2 + px + q$ passing through the point (1, –1) be such that the distance between its vertex and the x-axis is minimum. Then the value of $p^2 + q^2$ is :

(1) 2 (2) 4 (3) 5 (4) 8

Ans. (2)

Sol. $y = x^2 + px + q$

(1, –1) satisfies it

$-1 = 1 + p + q \Rightarrow p + q = -2$

Co-ordinate of vertex $\left(-\frac{p}{2}, -\frac{(p^2-4q)}{4}\right)$

$$= \left(-\frac{p}{2}, -\frac{(p^2+4p+4)}{4}\right)$$

distance of vertex from x-axis

$$= \left|\frac{4+p^2+4p}{4}\right| = \left|\frac{(p+2)^2}{4}\right|$$

minimum distance = 0 when p = –2

so $p = -2$ $\Rightarrow q = 0$

$p^2 + q^2 = 4$

Question ID : 691121163

13. Let $P = \{\theta \in [0, 4\pi] : \tan^2\theta \neq 1\}$ and $S = \{a \in \mathbf{Z} : 2(\cos^8\theta - \sin^8\theta) \sec 2\theta = a^2, \theta \in P\}$. Then n(S) is :

(1) 0 (2) 1 (3) 2 (4) 3

Ans. (1)

Sol. $2\left(\cos^8\theta - \sin^8\theta\right)\sec(2\theta) = a^2$

$\Rightarrow 2\left(\cos^4\theta + \sin^4\theta\right)\left(\cos^2\theta - \sin^2\theta\right)\sec 2\theta = a^2$

$\Rightarrow 2\left(1 - \frac{1}{2}\sin^2 2\theta\right) = a^2$

$\Rightarrow 2 - \sin^2 2\theta = a^2$

$\Rightarrow \sin^2 2\theta = 2 - a^2$

$\because a \in z$, only possibility $a^2 = 1$

$\Rightarrow \sin^2 2\theta = 1$

$\Rightarrow 2\theta = (2n+1)\frac{\pi}{2}$

$\Rightarrow \theta = (2n+1)\frac{\pi}{4}$

but at $\theta = (2n+1)\frac{\pi}{4}, \tan^2\theta = 1$

so no possible value of θ

Question ID : 691121164

14. Let the vectors $\vec{a} = -\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} + \hat{k}$. For some λ, $\mu \in$ **R**, let $\vec{c} = \lambda\vec{a} + \mu\vec{b}$. If $\vec{c}\cdot\left(3\hat{i} - 6\hat{j} + 2\hat{k}\right) = 10$ and $\vec{c}\cdot\left(\hat{i} + \hat{j} + \hat{k}\right) = -2$, then $|\vec{c}|^2$ is equal to :

(1) 8 (2) 12 (3) 14 (4) 15

Ans. (2)

Sol. $\vec{c} = \lambda\vec{a} + \mu\vec{b}$

$= \lambda(-1,1,3) + \mu(1,3,1)$

$= (-\lambda+\mu, \lambda+3\mu, 3\lambda+\mu)$

Now $\vec{c}\cdot(3,-6,2) = 10$

$\Rightarrow 3(-\lambda+\mu) - 6(\lambda+3\mu) + 2(3\lambda+\mu) = 10$

$\Rightarrow -3\lambda - 13\mu = 10$

$\Rightarrow 3\lambda + 13\mu = 10$ ....(i)

and $\vec{c}\cdot(1,1,1) = -2$

$\Rightarrow -\lambda + \mu + \lambda + 3\mu + 3\lambda + \mu = -2$

$\Rightarrow 3\lambda + 5\mu = -2$ ....(ii)

from (i) and (ii)

$\lambda = 1, \mu = -1$

so $\vec{c} = \vec{a} - \vec{b} = (-2,-2,2)$

$\Rightarrow |\vec{c}|^2 = 12$

Question ID : 691121165

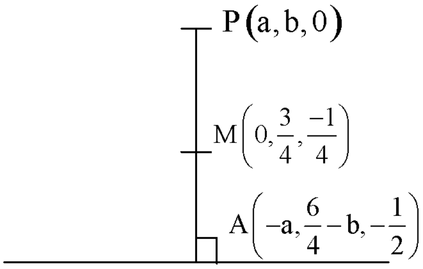
15. Let the point A be the foot of perpendicular drawn from the point P(a, b, 0) on the line $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-\alpha}{3}$.

If the midpoint of the line segment PA is $\left(0, \frac{3}{4}, \frac{-1}{4}\right)$ then the value of $a^2 + b^2 + \alpha^2$ is equal to :

(1) 1 (2) 2 (3) 6 (4) 9

Ans. (1)

Sol.



$$\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-\alpha}{3}=\lambda$$

putting point A in line

$$\frac{-a-1}{2}=\frac{6}{4}-b-2=\frac{\frac{-1}{2}-\alpha}{3}$$

by solving we get

$a = 0,\ b = 0,\ \alpha = 1$

so $a^2 + b^2 + \alpha^2 = 0 + 0 + 1 = 1$

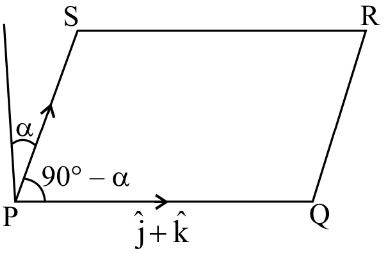
Question ID : 691121166

16. Two adjacent sides of a parallelogram PQRS are given by $\overrightarrow{PQ}=\hat{j}+\hat{k}$ and $\overrightarrow{PS}=\hat{i}-\hat{j}$. If the side PS is rotated about the point P by an acute angle α in the plane of the parallelogram so that it becomes perpendicular to the side PQ, then $\sin^2\left(\frac{5\alpha}{2}\right)-\sin^2\left(\frac{\alpha}{2}\right)$ is equal to :

(1) $\frac{1}{2}$ (2) $\frac{\sqrt{3}}{2}$ (3) $\frac{\sqrt{3}}{4}$ (4) $\frac{2\sqrt{3}}{5}$

Ans. (2)

Sol.



$$\cos(90^\circ-\alpha)=\left|\frac{(1,-1,0)\cdot(0,1,1)}{\sqrt{2}\sqrt{2}}\right|$$

$$= \frac{1}{2}$$

$$\Rightarrow \quad \sin\alpha = \frac{1}{2} \quad \Rightarrow \quad \alpha = \frac{\pi}{6}$$

Now $\sin^2\frac{5\alpha}{2} - \sin^2\frac{\alpha}{2}$

$$= \sin 3\alpha \cdot \sin 2\alpha$$

$$= \sin\frac{3\pi}{6} \cdot \sin\frac{2\pi}{6} = \frac{\sqrt{3}}{2}$$

Question ID : 691121167

17. The value of $\int_0^{20\pi} \left(\sin^4 x + \cos^4 x\right) dx$ is equal to :

(1) $\frac{15\pi}{2}$ (2) 25π (3) 15π (4) $\frac{25\pi}{2}$

Ans. (3)

Sol. Let $I = \int_0^{20\pi} \left(\sin^4 x + \cos^4 x\right) dx$

$$= 20\int_0^{\pi} \left(\sin^4 x + \cos^4 x\right) dx$$

$$= 40\int_0^{\pi/2} \left(\sin^4 x + \cos^4 x\right) dx$$

$$\because \quad \int_0^{\pi/2} \sin^4 x\, dx = \int_0^{\pi/2} \cos^4 x\, dx = \frac{3\pi}{16}$$

$$\Rightarrow I = 40 \cdot \frac{6\pi}{16} = 15\pi$$

Question ID : 691121168

18. Let f(x) be a polynomial of degree 5, and have extrema at x = 1 and x = –1. If $\lim_{x\to 0}\left(\frac{f(x)}{x^3}\right) = -5$, then f(2) – f(–2) is equal to :

(1) 0 (2) 50 (3) 92 (4) 112

Ans. (4)

Sol. $\lim\limits_{x\to 0}\dfrac{f(x)}{x^3}=-5$

so let

$f(x)=ax^5+bx^4-5x^3$

$\Rightarrow f'(x)=5ax^4+4bx^3-15x^2$ ....(i)

Now, $f'(1)=0$

$\Rightarrow 5a+4b-15=0$ ....(ii)

and $f'(-1)=0$

$\Rightarrow 5a-4b-15=0$ ....(iii)

from (i) and (ii)

$\Rightarrow f(x)=3x^5-5x^3$

Now $f(2)-f(-2)$

$=3.32-5.8-(3(-32)-5(-8))$

$=192-80=112$

Question ID : 691121169

19. Let $f(x)=\int\left(\dfrac{16x+24}{x^2+2x-15}\right)dx$. If $f(4)=14\log_e 3$ and $f(7)=\log_e(2^{\alpha}\cdot 3^{\beta})$, $\alpha, \beta \in \mathbf{N}$, then $\alpha+\beta$ is equal to:

(1) 31 (2) 37 (3) 39 (4) 41

Ans. (3)

Sol. $f(x)=\int\left(\dfrac{16x+24}{x^2+2x-15}\right)dx$

$=\int\dfrac{8(2x+2)+8}{x^2+2x-15}dx$

$=8\int\dfrac{2x+2}{x^2+2x-15}dx+\int\dfrac{(x+5)-(x-3)}{(x+5)(x-3)}dx$

$\Rightarrow f(x)=8\ln\left|x^2+2x-15\right|+\ln\left|\dfrac{x-3}{x+5}\right|+c$

Given $f(4)=14\ln 3$

$\Rightarrow 14\ln 3=8\ln 9+\ln\left|\dfrac{1}{9}\right|+c$

$\Rightarrow c=0$

Now f(7) = 8 ln 48 – ln3

$$= \ln\left(\frac{48^8}{3}\right) = \ln\left(2^{32}\cdot 3^7\right)$$

so $\alpha = 32$, $\beta = 7$

Question ID : 691121170

20. Let $x = x(y)$ be the solution of the differential equation $2y^2\frac{dx}{dy} - 2xy + x^2 = 0$, $y > 1$, $x(e) = e$. Then $x(e^2)$ is equal to :

(1) $\frac{3}{2}e^2$ (2) $\frac{2}{3}e^2$ (3) e^2 (4) $2e^2$

Ans. (2)

Sol. $2y^2\frac{dx}{dy} - 2xy + x^2 = 0$

$$\Rightarrow \quad 2y\frac{(ydx - xdy)}{x^2} + dy = 0$$

$$\Rightarrow \quad -2\cdot d\left(\frac{y}{x}\right) + \frac{dy}{y} = 0$$

$$\Rightarrow \quad \frac{-2y}{x} + \ln y = c$$

$\because x(e) = e$

$-2 + 1 = c \Rightarrow c = -1$

so $-\frac{2y}{x} + \ln y = -1$

to find $x(e^2)$ put $y = e^2$

$$-\frac{2e^2}{x} + 2 = -1$$

$$\Rightarrow \frac{2e^2}{x} = 3$$

$$\Rightarrow x = \frac{2e^2}{3}$$

SECTION – B

Question ID : 691121171

21. Let A= {2, 3, 4, 5, 6}. Let R be a relation on the set A × A given by (x, y)R(z, w) if and only if x divides z and $y \le w$ Then the number of elements in R is ________.

Ans. (120)

Sol. A= {2, 3, 4, 5, 6}

(x, y) R(z, w) if x divides z and $y \le w$

for x = 2, z = 2, 4, 6

x = 3, z = 3, 6

x = 4, z = 4

x = 5, z = 5

x = 6, z = 6

and for $y \le w$

total cases = 5 + 4 + 3 + 2 + 1

= 15

number of elements is R = 8 × 15 = 120

Question ID : 691121172

22. Consider the matrices $A = \begin{bmatrix} 2 & -2 \\ 4 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 9 \\ 1 & 3 \end{bmatrix}$. If matrices P and Q are such that PA = B and AQ=B then the absolute value of the sum of the diagonal elements of 2(P + Q) is ________.

Ans. (34)

Sol. $A = \begin{bmatrix} 2 & -2 \\ 4 & -2 \end{bmatrix}, B = \begin{bmatrix} 3 & 9 \\ 1 & 3 \end{bmatrix}$

PA = B

$\Rightarrow P = BA^{-1}$

$$\Rightarrow P = \begin{bmatrix} 3 & 9 \\ 1 & 3 \end{bmatrix} \frac{1}{4} \begin{bmatrix} -2 & 2 \\ -4 & 2 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} -21 & 12 \\ -7 & 4 \end{bmatrix}$$

AQ = B

$\Rightarrow Q = A^{-1}. b$

$$=\frac{1}{2}\begin{bmatrix}-1 & 1\\-2 & 1\end{bmatrix}\begin{bmatrix}3 & 9\\1 & 3\end{bmatrix}$$

$$=\frac{1}{2}\begin{bmatrix}-2 & -6\\-5 & -15\end{bmatrix}$$

sum of diagonal elements of 2(P + Q)

$$=2\left[-\frac{21}{2}+\frac{4}{2}-\frac{2}{2}-\frac{15}{2}\right]$$

$= -17 -17$

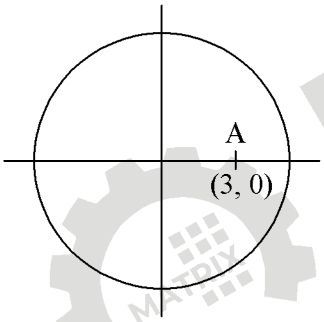
$= -34$

Question ID : 691121173

23. Let A be the point (3, 0) and circles with variable diameter AB touch the circle $x^2 + y^2 = 36$ internally. Let the curve C be the locus of the point B. If the eccentricity of C is e, then $72e^2$ is equal to __________.

Ans. (18)

Sol.



Let B(h, k)

Circle with AB as diameter

$(x - h)(x - 3) + y(y - k) = 0$

$\Rightarrow x^2 + y^2 - (h + 3)x - ky + 3h = 0$

Centre $\left(\frac{h+3}{2}, \frac{k}{2}\right)$

Circle touch internelly so

$c_1c_2 = |r_1 - r_2|$

$$\Rightarrow \sqrt{\left(\frac{h+3}{2}\right)^2+\left(\frac{k}{2}\right)^2}=6-\frac{1}{2}\sqrt{(h-3)^2+k^2}$$

$$\Rightarrow \sqrt{(x+3)^2+y^2}+\sqrt{(x-3)^2+y^2}=12$$

$\Rightarrow 2a = 12$ $\Rightarrow a = 6$ ....(i)

focii are (–3, 0) and (3, 0)
2ae = 6 ....(ii)
from (i) and (ii)

$$e=\frac{1}{2}\Rightarrow\quad 72e^{2}=72\cdot\frac{1}{4}=18$$

Question ID : 691121174

24. If the area of the region bounded by $16x^2 - 9y^2 = 144$ and $8x - 3y = 24$ is A, then $3(A+6\log_e(3))$ is equal to __________.

Ans. (24)

Sol. $16x^2 - 9y^2 = 144$

solving it with $8x - 3y = 24$

$16x^2 - (8x - 24)^2 = 144$

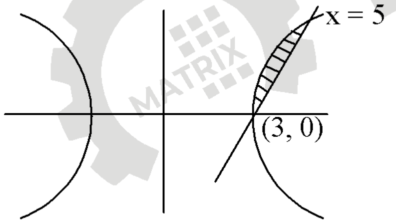
$\Rightarrow 16x^2 - 64(x^2 - 6x + 9) = 144$

$\Rightarrow x^2 - 4(x^2 - 6x + 9) = 9$

$\Rightarrow 3x^2 - 24x + 45 = 0$

$\Rightarrow x^2 - 8x + 15 = 0$

$\Rightarrow x = 3, 5$



$$\text{Area }=\int_{3}^{5}\left(\sqrt{\frac{16x^{2}-144}{9}}-\left(\frac{8x-24}{3}\right)\right)dx$$

$$=\frac{4}{3}\int_{3}^{5}\left(\sqrt{x^{2}-9}-\frac{2x}{3}+18\right)dx$$

$$=\frac{4}{3}\left(\frac{x}{2}\sqrt{x^{2}-9}-\frac{9}{2}\ln\left|x+\sqrt{x^{2}-9}\right|\right)_{3}^{5}-\frac{8}{3}\frac{\left(x^{2}\right)^{5}}{2}+8(x)_{3}^{5}$$

$$=\frac{4}{3}\left[\frac{5}{2}\cdot4-\frac{9}{2}\ln9+\frac{9}{2}\ln3\right]-\frac{8}{3}\frac{16}{2}+16$$

$$=\frac{40}{3}-\frac{9}{2}\cdot\frac{4}{3}\ln 3-\frac{64}{3}+16$$

$\Rightarrow A = 8 - 6\ln 3$

so $3(A + 6\ln 3)$

$= 3(8 - 6\ln 3 + 6\ln 3) = 24$

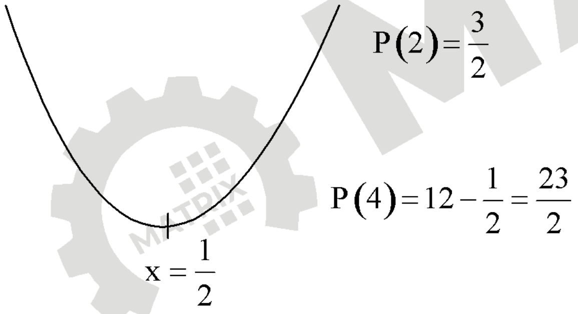
Question ID : 691121175

25. The number of points in the interval [2, 4], at which the function $f(x)=\left[x^2-x-\frac{1}{2}\right]$, where $[\cdot]$ denotes the greatest integer function, is discontinuous, is ________.

Ans. (10)

Sol. $f(x)=\left[x^2-x-\frac{1}{2}\right]$

let $P(x)=x^2-x-\frac{1}{2}$



so range of P(x) $\leftarrow$ [1.5, 11.5]
Total 10 linegers in the range.
so P(x) has 1 x such that

$P(x) = n,\ n \in \{2, 3, ..., 11\}$

$\Rightarrow$ Total 10 discontinuities.