JEE Main April 2026

Question Paper With Text Solution

02 April | Shift-1

MATHEMATICS

JEE MAIN APRIL 2026 | 2ND APRIL SHIFT-1

SECTION – A

Question ID : 6911211

1. Let $\alpha, \alpha + 2; \alpha \in Z$ be the roots of the quadratic equation $x(x + 2) + (x + 1)(x + 3) + (x + 2)(x + 4) + ... + (x + n - 1)(x + n + 1) = 4n$ for some $n \in N$. Then $n + \alpha$ is equal to :

(1) 0 (2) 1 (3) 2 (4) 3

Ans. (3)

Sol. The general term of the summation on the left-hand side is $T_k = (x + k - 1)(x + k + 1) = (x + k)^2 - 1$.

The equation is :

$$\sum_{k=1}^{n}\left[(x+k)^2-1\right]=4n$$

$$\sum_{k=1}^{n}(x^2+2kx+k^2-1)=4n$$

$$nx^2+2x\frac{n(n+1)}{2}+\frac{n(n+1)(2n+1)}{6}-n=4n$$

Dividing by n (since $n \in \mathbf{N}$):

$$x^2+(n+1)x+\frac{(n+1)(2n+1)}{6}-1=4$$

$$x^2+(n+1)x+\frac{2n^2+3n+1-30}{6}=0$$

$$x^2+(n+1)x+\frac{2n^2+3n-29}{6}=0$$

The roots are α and $\alpha + 2$.

Sum of roots : $\alpha + (\alpha + 2) = -(n + 1) \Rightarrow 2\alpha + 2 = -n - 1$

$\Rightarrow 2\alpha = -n - 3$.

Difference of roots: $(\alpha + 2) - \alpha = 2$.

We know $(\text{Difference})^2 = (\text{Sum})^2 - 4(\text{Product})$:

$$4=(n+1)^2-4\left(\frac{2n^2+3n-29}{6}\right)$$

$$4=n^2+2n+1-\frac{4n^2+6n-58}{3}$$

$12 = 3n^2 + 6n + 3 - 4n^2 - 6n + 58$

$12 = -n^2 + 61 \quad \Rightarrow n^2 = 49 \quad \Rightarrow n = 7$ (as $\in$ N)

Now, substitute n = 7 into the sum relation :

$2\alpha = -7 - 3 = -10 \quad \Rightarrow \alpha = -5$

Therefore, $n + \alpha = 7 + (-5) = 2$

Question ID : 6911212

2. Let x and y be real numbers such that $50\left(\frac{2x}{1+3i} - \frac{y}{1-2i}\right) = 31 + 17i$ where $i = \sqrt{-1}$. Then the value of $10(x - 3y)$ is: :

(1) 20 (2) 31 (3) 35 (4) 75

Ans. (4)

Sol. Rationalize the denominators inside the bracket :

$$\frac{2x}{1+3i} = \frac{2x(1-3i)}{1+9} = \frac{2x(1-3i)}{10} = \frac{x(1-3i)}{5}$$

$$\frac{y}{1-2i} = \frac{y(1+2i)}{1+4} = \frac{y(1+2i)}{5}$$

Substitute these back into the equation:

$$50\left[\frac{x-3xi}{5} - \frac{y+2yi}{5}\right] = 31 + 17i$$

$10 [(x - y) - (3x + 2y)i] = 31 + 17i$

Equating real and imaginary parts:

1. $10(x - y) = 31 \quad \Rightarrow 10x - 10y = 31$

2. $-10(3x + 2y) = 17 \quad \Rightarrow -30x - 20y = 17$

From (1), $30x = 93 + 30y$. Substitute into (2):

$-(93 + 30y) - 20y = 17$

$-93 - 50y = 17 \quad \Rightarrow -50y = 110 \Rightarrow 10y = -22$

Substituting $10y = -22$ into (1):

$10x - (-22) = 31 \Rightarrow 10x + 22 = 31 \Rightarrow 10x = 9$

We need the value of $10(x - 3y) = 10x - 30y$:

$10x - 3(10y) = 9 - 3(-22) = 9 + 66 = 75$

Question ID : 6911213

3. Let $\alpha, \beta \in R$ be such that the system of linear equations

$x + 2y + z = 5,$

$2x + y + \alpha z = 5$

$8x + 4y + \beta z = 18$ has no solution. Then $\frac{\beta}{\alpha}$ is equal to: :

(1) –4 (2) 4 (3) 8 (4) –8

Ans. (2)

Sol. For the system to have no solution, the determinant of the coefficient matrix (Δ) must be zero.

$$\Delta = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & \alpha \\ 8 & 4 & \beta \end{vmatrix} = 0$$

Expanding along the first row:

$1(\beta - 4\alpha) - 2(2\beta - 8\alpha) + 1(8 - 8) = 0$

$\beta - 4\alpha - 4\beta + 16\alpha = 0 \quad \Rightarrow 12\alpha - 3\beta = 0 \quad \Rightarrow 4\alpha = \beta$

$\frac{\beta}{\alpha} = 4$

We must also ensure that the planes are not all coincident (which would give infinite solutions).

Comparing equations (2) and (3):

Eq 2: $2x + y + \alpha z = 5$

Eq 3: $8x + 4y + \beta z = 18 \quad \Rightarrow 2x + y + \frac{\beta}{4}z = \frac{18}{4} = 4.5$

Since $\alpha = \frac{\beta}{4}$, the left-hand sides are identical but the constants (5 vs 4.5) are different. This confirms the system represents parallel planes with no intersection.

Question ID : 6911214

4. Let $A = \begin{bmatrix} 1 & 2 \\ 1 & \alpha \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 3 \\ \beta & 2 \end{bmatrix}$ and $B^2 - 5B - 6I = O$, then among the two statements:

(S1): $[(B-A)(B+A)]^T = \begin{bmatrix} 13 & 15 \\ 7 & 10 \end{bmatrix}$

(S2): $\det(\text{adj}(A+B)) = -5$,

(1) only (S1) is correct

(2) only (S2) is correct

(3) both (S1) and (S2) are correct

(4) both (S1) and (S2) are wrong

Ans. (2)

Sol. From $A^2 - 4A + I = 0$, using the characteristic equation $\lambda^2 - \text{tr}(A)\lambda + \det(A) = 0$:

$\text{tr}(A) = 1 + \alpha = 4 \quad \Rightarrow \alpha = 3.$

$\det(A) = \alpha - 2 = 3 - 2 = 1$ (consistent). So $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$.

From $B^2 - 5B - 6I = 0$:

$\text{tr}(B) = 3 + 2 = 5$ (consistent).

$\det(B) = 6 - 3\beta = -6 \quad \Rightarrow 3\beta = 12 \quad \Rightarrow \beta = 4$. So $B = \begin{bmatrix} 3 & 3 \\ 4 & 2 \end{bmatrix}$.

Calculate $B - A = \begin{bmatrix} 2 & 1 \\ 3 & -1 \end{bmatrix}$ and $B + A = \begin{bmatrix} 4 & 5 \\ 5 & 5 \end{bmatrix}$.

$$(B-A)(B+A) = \begin{bmatrix} 2(4)+1(5) & 2(5)+1(5) \\ 3(4)-1(5) & 3(5)-1(5) \end{bmatrix} = \begin{bmatrix} 13 & 15 \\ 7 & 10 \end{bmatrix}.$$

Then $[(B-A)(B+A)]^T = \begin{bmatrix} 13 & 7 \\ 15 & 10 \end{bmatrix}$.

Statement (S1) says the transpose is $\begin{bmatrix} 13 & 15 \\ 7 & 10 \end{bmatrix}$, which is false.

For (S2): $\det(\text{adj}(M)) = (\det M)^{n-1}$. For $n = 2$, $\det(\text{adj}(A+B)) = \det(A+B)$.

$\det(A+B) = \det\begin{bmatrix} 4 & 5 \\ 5 & 5 \end{bmatrix} = 20 - 25 = -5$.

Statement (S2) is true.

Question ID : 6911215

5. Let A be the set of first 101 terms of an A.P. whose first term is 1 and common difference is 5 and B be the set of first 71 terms of an A.P. whose first term is 9 and common difference is 7. Then the number of elements in A $\cap$ B which are divisible by 3 is :

(1) 4 (2) 5 (3) 6 (4) 7

Ans. (2)

Sol. Terms of A: 1, 6, 11, 16, 21, ... , 1 + 100(5) = 501.

Terms of B: 9, 16, 23, 30, ... , 9 + 70(7) = 499.

The first common term is 16.

The common difference is lcm(5, 7) = 35.

Common terms C = 16, 51, 86, ...

The general common term is $T_r = 16 + (r - 1)35$.

Max term $\le$ min(501, 499) = 499.

$16 + 35(r - 1) \le 499 \quad \Rightarrow 35(r - 1) \le 483 \quad \Rightarrow r - 1 \le 13.8 \quad \Rightarrow r \le 14.$

There are 14 common terms. We need those divisible by 3:

$16 + 35(r - 1) \equiv 0 \pmod 3$

$1 + 2(r - 1) \equiv 0 \pmod 3$

$2r - 1 \equiv 0 \pmod 3 \quad \Rightarrow 2r \equiv 1 \equiv 4 \pmod 3 \quad \Rightarrow r \equiv 2 \pmod 3.$

Values of r are 2, 5, 8, 11, 14.

The number of such elements is 5.

Question ID : 6911216

6. The number of seven-digit numbers, that can be formed by using the digits 1, 2, 3, 5 and 7 such that each digit is used at least once, is :

(1) 15400 (2) 17800 (3) 16800 (4) 29400

Ans. (3)

Sol. We need to form a 7-digit number using 5 available digits {1, 2, 3, 5, 7} such that each digit appears at least once. This leaves 2 extra positions to be filled by repeating digits. There are two possible cases for the frequency of the digits:

- Case 1: One digit is used three times, and the other four digits are used once each.

- o Number of ways to choose which digit is repeated 3 times: $\binom{5}{1} = 5$ ways.
- o Number of ways to arrange these 7 digits: $\frac{7!}{3!\cdot 1!\cdot 1!\cdot 1!\cdot 1!} = \frac{5040}{6} = 840$ ways.
- o Total for Case 1: $5 \times 840 = 4200$.

• Case 2: Two digits are used twice each, and the other three digits are used once each.

- o Number of ways to choose which 2 digits are repeated: $\binom{5}{2} = 10$ ways.
- o Number of ways to arrange these 7 digits: $\frac{7!}{2!\cdot 2!\cdot 1!\cdot 1!\cdot 1!} = \frac{5040}{4} = 1260$ ways.
- o Total for Case 2: $10 \times 1260 = 12600$.

Summing both cases: $4200 + 12600 = 16800$.

Question ID : 6911217

7. The number of elements in the set $\left\{ S = \{(r,k) : k \in \mathbb{Z} \text{ and } {}^{36}C_{r+1} = \frac{6\binom{35}{r}}{k^2 - 3}\right\}$, is :

(1) 2 (2) 4 (3) 8 (4) 16

Ans. (2)

Sol. Using the property of combinations ${}^{n}C_{r} = \frac{n}{r} \cdot {}^{n-1}C_{r-1}$, we can rewrite the equation :

$${}^{36}C_{r+1} = \frac{36}{r+1} \cdot {}^{35}C_{r}$$

Substitute this into the given equation:

$$\frac{36}{r+1} \cdot {}^{35}C_{r} = \frac{6 \cdot {}^{35}C_{r}}{k^2 - 3}$$

$$\frac{6}{r+1} = \frac{1}{k^2 - 3}$$

$$k^2 - 3 = \frac{r+1}{6} \qquad \Rightarrow k^2 = 3 + \frac{r+1}{6}$$

Since r is from the term ${}^{35}C_{r}$, we know $0 \le r \le 35$.

Thus, the range for $\frac{r+1}{6}$ is:

$\frac{1}{6} \le \frac{r+1}{6} \le \frac{36}{6}$ $\Rightarrow 0.166 \le \frac{r+1}{6} \le 6$

Substituting this into the expression for k^2:

$3 + 0.166 \le k^2 \le 3 + 6$ $\Rightarrow 3.166 \le k^2 \le 9$

Since $k \in Z$, the possible values for k^2 in this range are 4 and 9.

• If $k^2 = 4$: $k = \pm 2$.

Then $3+\frac{r+1}{6} = 4 \Rightarrow \frac{r+1}{6} = 1 \Rightarrow r = 5$. (2 pairs : (5, 2), (5, –2))

• If $k^2 = 9$: $k = \pm 3$.

Then $3+\frac{r+1}{6} = 9 \Rightarrow \frac{r+1}{6} = 6 \Rightarrow r = 35$. (2 pairs: (35, 3), (35, –3))

Total elements in S are 4.

Question ID : 6911218

8. If the mean of the data

Class	5 – 10	10 – 15	15 – 20	20 – 25	25 – 30	30 – 35
Frequency	2	k	28	54	k + 1	5

is 21, then k is one of the roots of the equation :

(1) $2x^2 - 23x - 10 = 0$ (2) $4x^2 - 35x + 24 = 0$ (3) $2x^2 - 19x - 10 = 0$ (4) $2x^2 - 35x + 98 = 0$

Ans. (3)

Sol. First, find the midpoints (x_i) and total frequency $\left(\sum f_i\right)$:

- Midpoints: 7.5, 12.5, 17.5, 22.5, 27.5, 32.5.
- $\sum f_i = 2 + k + 28 + 54 + (k+1) + 5 = 90 + 2k$.
- $\sum f_i x_i = 2(7.5) + k(12.5) + 28(17.5) + 54(22.5) + (k+1)(27.5) + 5(32.5)$
- $\sum f_i x_i = 15 + 12.5k + 490 + 1215 + 27.5k + 27.5 + 162.5 = 1910 + 40k$.

Mean $(\bar{x})$ is given as 21:

$$\frac{1910+40k}{90+2k}=21$$

$1910 + 40k = 21(90 + 2k) = 1890 + 42k$

$20 = 2k \qquad \Rightarrow k = 10$

Now check which equation has x=10 as a root:

For (C): $2(10)^2 - 19(10) - 10 = 200 - 190 - 10 = 0$.

Question ID : 6911219

9. Let the mid points of the sides of a triangle ABC be $\left(\frac{5}{2},7\right),\left(\frac{5}{2},3\right)$ and (4, 5). If its incentre is (h, k), then $3h+k$ is equal to: :

(1) 11 (2) 12 (3) 13 (4) 14

Ans. (3)

Sol. Let the vertices be A, B, C. The midpoints are $M_1\left(\frac{5}{2},7\right), M_2\left(\frac{5}{2},3\right), M_3(4,5)$.

The vertices can be found using the property that the vertex is the sum of the adjacent midpoints minus the opposite midpoint:

$$A = M_1 + M_3 - M_2 = \left(\frac{5}{2}+4-\frac{5}{2}, 7+5-3\right) = (4,9)$$

$$B = M_1 + M_2 - M_3 = \left(\frac{5}{2}+\frac{5}{2}-4, 7+3-5\right) = (1,5)$$

$$C = M_2 + M_3 - M_1 = \left(\frac{5}{2}+4-\frac{5}{2}, 3+5-7\right) = (4,1)$$

Calculate side lengths :

$$c = AB = \sqrt{(4-1)^2+(9-5)^2} = \sqrt{3^2+4^2} = 5$$

$$a = BC = \sqrt{(4-1)^2+(1-5)^2} = \sqrt{3^2+(-4)^2} = 5$$

$$b = AC = \sqrt{(4-4)^2+(9-1)^2} = \sqrt{0^2+8^2} = 8$$

Incentre $I(h,k) = \left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right)$:

$$h = \frac{5(4)+8(1)+5(4)}{5+8+5} = \frac{20+8+20}{18} = \frac{48}{18} = \frac{8}{3}$$

$$k = \frac{5(9)+8(5)+5(1)}{5+8+5} = \frac{45+40+5}{18} = \frac{90}{18} = 5$$

Value of $3h + k = 3\left(\frac{8}{3}\right) + 5 = 8 + 5 = 13$.

Question ID : 69112110

10. Let an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a < b$, pass through the point (4, 3) and have eccentricity $\frac{\sqrt{5}}{3}$. Then the length of its latus rectum is :

(1) $\frac{4\sqrt{5}}{3}$ (2) $2\sqrt{5}$ (3) $\frac{7\sqrt{5}}{3}$ (4) $\frac{8\sqrt{5}}{3}$

Ans. (4)

Sol. For the ellipse with b > a, eccentricity $e = \sqrt{1 - \frac{a^2}{b^2}}$. $\frac{5}{9} = 1 - \frac{a^2}{b^2} \Rightarrow \frac{a^2}{b^2} = \frac{4}{9} \Rightarrow a^2 = \frac{4}{9}b^2$.

The ellipse passes through (4, 3) :

$$\frac{16}{a^2} + \frac{9}{b^2} = 1$$

Substitute a^2 :

$$\frac{16}{\frac{4}{9}b^2} + \frac{9}{b^2} = 1 \Rightarrow \frac{36}{b^2} + \frac{9}{b^2} = 1 \Rightarrow \frac{45}{b^2} = 1 \Rightarrow b^2 = 45 \Rightarrow b = 3\sqrt{5}$$

Then $a^2 = \frac{4}{9}(45) = 20$. For a vertical ellipse (b > a), the length of the latus rectum is $\frac{2a^2}{b}$:

$$LR = \frac{2(20)}{3\sqrt{5}} = \frac{40}{3\sqrt{5}} = \frac{40\sqrt{5}}{15} = \frac{8\sqrt{5}}{3}$$

Question ID : 69112111

11. If $\sin\left(\frac{\pi}{18}\right)\sin\left(\frac{5\pi}{18}\right)\sin\left(\frac{7\pi}{18}\right) = K$, then the value of $\sin\left(\frac{10K\pi}{3}\right)$ is :

(1) $\frac{\sqrt{3}+1}{2\sqrt{2}}$ (2) $\frac{\sqrt{3}-1}{\sqrt{2}}$ (3) $\frac{\sqrt{3}}{2}$ (4) $\frac{1}{2}$

Ans. (1)

Sol. First, we simplify the expression for K:

$K = \sin(10°)\sin(50°)\sin(70°)$ (converting to degrees)

Using $\sin\theta = \cos(90^{\circ} - \theta)$:

$K = \cos(80°)\cos(40°)\cos(20°)$

Rearranging :

$K = \cos(20°)\cos(40°)\cos(80°)$

Using the product formula $\prod_{r=0}^{n-1}\cos(2^r\theta) = \frac{\sin(2^n\theta)}{2^n\sin\theta}$:

$$K = \frac{\sin(2^3 \cdot 20°)}{2^3\sin(20°)} = \frac{\sin(160°)}{8\sin(20°)}$$

Since $\sin(160°) = \sin(180° - 20°) = \sin(20°)$:

$$K = \frac{\sin(20°)}{8\sin(20°)} = \frac{1}{8}$$

Now, find the value of the required trigonometric expression:

$$\sin\left(\frac{10K\pi}{3}\right) = \sin\left(\frac{10\cdot\frac{1}{8}\pi}{3}\right) = \sin\left(\frac{5\pi}{12}\right)$$

$$\frac{5\pi}{12} = 75°$$

$$\sin(75°) = \sin(45° + 30°) = \sin 45°\cos 30° + \cos 45°\sin 30°$$

$$= \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}\cdot\frac{1}{2} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

Question ID : 69112112

12. Let $S = \{x \in [-\pi, \pi] : \sin x(\sin x + \cos x) = a, a \in \mathbb{Z}\}$. Then n(S) is equal to :

(1) 3 (2) 6 (3) 7 (4) 9

Ans. (4)

Sol. Let $f(x) = \sin^2 x + \sin x \cos x$. Using double angle identities:

$$f(x) = \frac{1-\cos 2x}{2} + \frac{\sin 2x}{2} = \frac{1}{2} + \frac{1}{2}(\sin 2x - \cos 2x)$$

$$f(x) = \frac{1}{2} + \frac{1}{\sqrt{2}}\sin\left(2x - \frac{\pi}{4}\right)$$

Since $-1 \le \sin\left(2x - \frac{\pi}{4}\right) \le 1$:

$$\frac{1}{2} - \frac{1}{\sqrt{2}} \le f(x) \le \frac{1}{2} + \frac{1}{\sqrt{2}}$$

$\frac{1}{\sqrt{2}} \approx 0.707$, so $0.5 - 0.707 \le f(x) \le 0.5 + 0.707$

$\Rightarrow -0.207 \le f(x) \le 1.207$.

As $a \in Z$, the possible values for a are 0 and 1.

Case 1: $a = 0$

$\sin x (\sin x + \cos x) = 0 \qquad \Rightarrow \sin x = 0$ or $\tan x = -1$.

- From $\sin x = 0$ in $[-\pi, \pi]$: $x = \pi, 0, \pi$. (3 solutions)
- From $\tan x = -1$ in $[-\pi, \pi]$: $x = -\frac{\pi}{4}, \frac{3\pi}{4}$. (2 solutions)

Total solutions for $a = 0$ is 5.

Case 2: $a = 1$

$$\frac{1}{2} + \frac{1}{\sqrt{2}}\sin\left(2x - \frac{\pi}{4}\right) = 1 \Rightarrow \sin\left(2x - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}.$$

For $x \in [-\pi, \pi]$, $2x - \frac{\pi}{4} \in \left[-\frac{9\pi}{4}, \frac{7\pi}{4}\right]$.

The values are $\frac{\pi}{4}, \frac{3\pi}{4}, -\frac{5\pi}{4}, -\frac{7\pi}{4}$.

- $2x - \frac{\pi}{4} = \frac{\pi}{4} \qquad \Rightarrow x = \frac{\pi}{4}$
- $2x - \frac{\pi}{4} = \frac{3\pi}{4} \qquad \Rightarrow x = \frac{\pi}{2}$
- $2x - \frac{\pi}{4} = -\frac{5\pi}{4} \qquad \Rightarrow x = -\frac{\pi}{2}$

- $2x - \frac{\pi}{4} = -\frac{7\pi}{4} \Rightarrow x = -\frac{3\pi}{4}$

Total solutions for a = 1 is 4.

Total elements n(S) = 5 + 4 = 9.

Question ID : 69112113

13. If the point of intersection of the lines $\frac{x+1}{3} = \frac{y+a}{5} = \frac{z+b+1}{7}$ and $\frac{x-2}{1} = \frac{y-b}{4} = \frac{z-2a}{7}$ lies on the xy-plane, then the value of a + b is :

(1) 2 (2) 5 (3) 7 (4) 9

Ans. (3)

Sol. A point on the xy-plane has z = 0. For the first line:

$\frac{x+1}{3} = \frac{y+a}{5} = \frac{0+b+1}{7} \Rightarrow \frac{x+1}{3} = \frac{b+1}{7}$ and $\frac{y+a}{5} = \frac{b+1}{7}$.

From $\frac{x+1}{3} = \frac{b+1}{7} \Rightarrow x = \frac{3b+3-7}{7} = \frac{3b-4}{7}$. From the second line: $\frac{x-2}{1} = \frac{y-b}{4} = \frac{0-2a}{7} \Rightarrow \frac{x-2}{1} = -\frac{2a}{7}$ and $\frac{y-b}{4} = -\frac{2a}{7}$. From $x-2 = -\frac{2a}{7} \Rightarrow x = 2 - \frac{2a}{7} = \frac{14-2a}{7}$.

Equating the x values:

$\frac{3b-4}{7} = \frac{14-2a}{7} \Rightarrow 2a + 3b = 18 \quad \ldots(1)$

Now equate y values.

From first line: $y = \frac{5b+5}{7} - a = \frac{5b+5-7a}{7}$.

From second line: $y = -\frac{8a}{7} + b = \frac{7b-8a}{7}$.

Equating:

$\frac{5b+5-7a}{7} = \frac{7b-8a}{7} \Rightarrow 5b+5-7a = 7b-8a \Rightarrow a-2b = -5 \Rightarrow a = 2b-5$

Substitute a into (1):

$2(2b-5) + 3b = 18 \Rightarrow 4b - 10 + 3b = 18 \Rightarrow 7b = 28 \Rightarrow b = 4$

Then a = 2(4) – 5 = 3.

a+b = 3+4 = 7.

Question ID : 69112114

14. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}| = 2$ and $|\vec{b}| = 3$, then the maximum value of $3\left|\left(3\vec{a}+2\vec{b}\right)\right| + 4\left|\left(3\vec{a}-2\vec{b}\right)\right|$ is :

(1) 30 (2) 36 (3) 60 (4) 72

Ans. (3)

Sol. Let θ be the angle between $\vec{a}$ and $\vec{b}$.

$|3\vec{a}+2\vec{b}|^2 = 9|\vec{a}|^2 + 4|\vec{b}|^2 + 12\vec{a}\cdot\vec{b} = 9(4)+4(9)+12(2)(3)\cos\theta = 72+72\cos\theta$.

Using $1+\cos\theta = 2\cos^2\left(\frac{\theta}{2}\right)$: $|3\vec{a}+2\vec{b}|^2 = 144\cos^2\left(\frac{\theta}{2}\right)$

$\Rightarrow |3\vec{a}+2\vec{b}| = 12\left|\cos\left(\frac{\theta}{2}\right)\right|$. Similarly, $|3\vec{a}-2\vec{b}|^2 = 72-72\cos\theta = 144\sin^2\left(\frac{\theta}{2}\right)$

$\Rightarrow |3\vec{a}-2\vec{b}| = 12\left|\sin\left(\frac{\theta}{2}\right)\right|$. The expression to maximize is:

$$f(\theta) = 3\left(12\cos\frac{\theta}{2}\right) + 4\left(12\sin\frac{\theta}{2}\right) = 36\cos\frac{\theta}{2} + 48\sin\frac{\theta}{2}$$

The maximum value of $A\cos\alpha + B\sin\alpha$ is $\sqrt{A^2+B^2}$.

Max Value = $\sqrt{36^2+48^2} = \sqrt{(12\cdot3)^2+(12\cdot4)^2} = 12\sqrt{3^2+4^2} = 12\cdot5 = 60$

Question ID : 69112115

15. Let a line L passing through the point (1, 1, 1) be perpendicular to both the vectors $2\hat{i}+2\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}+2\hat{k}$. If P(a, b, c) is the foot of perpendicular from the origin on the line L, then the value of 34(a +b + c) is :

(1) 50 (2) 80 (3) 100 (4) 120

Ans. (3)

Sol. The direction vector $\vec{d}$ of line L is perpendicular to both given vectors:

$$\vec{d} = (2\hat{i}+2\hat{j}+\hat{k})\times(\hat{i}+2\hat{j}+2\hat{k}) = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 2 & 1\\ 1 & 2 & 2\end{vmatrix}$$

$= \hat{i}(4-2) - \hat{j}(4-1) + \hat{k}(4-2) = 2\hat{i} - 3\hat{j} + 2\hat{k}$

The equation of line L passing through A(1, 1, 1) is :

$\vec{r} = (1,1,1) + t(2,-3,2) = (1+2t, 1-3t, 1+2t)$

The foot of the perpendicular P(a, b, c) from the origin (0, 0, 0) satisfies $\overrightarrow{OP} \perp \vec{d}$:

(1 + 2t, 1 – 3t, 1 + 2t) .... (2, –3, 2) = 0

2(1 + 2t) – 3(1 – 3t) + 2(1 + 2t) = 0

$\Rightarrow 2 + 4t - 3 + 9t + 2 + 4t = 0$

$\Rightarrow 17t + 1 = 0 \qquad \Rightarrow t = -\frac{1}{17}$

Substitute t back into the coordinates of P:

$a = 1 + 2\left(-\frac{1}{17}\right) = \frac{15}{17}$

$b = 1 - 3\left(-\frac{1}{17}\right) = \frac{20}{17}$

$c = 1 + 2\left(-\frac{1}{17}\right) = \frac{15}{17}$

Sum $a + b + c = \frac{15+20+15}{17} = \frac{50}{17}$. Value of $34(a+b+c) = 34 \cdot \frac{50}{17} = 100$.

Question ID : 69112116

16. If $\lim_{x \to 2} \frac{\sin\left(x^3 - 5x^2 + ax + b\right)}{\left(\sqrt{x-1} - 1\right)\log_e(x-1)} = m$, then a + b + m is equal to :

(1) 5 (2) 6 (3) 8 (4) 10

Ans. (2)

Sol. For the limit to exist and be finite, the numerator must approach zero as $x \to 2$ because the denominator becomes zero. Let $f(x) = x^3 - 5x^2 + ax + b$. Since $\lim_{x \to 2} f(x) = 0$:

$8 - 20 + 2a + b = 0 \qquad \Rightarrow 2a + b = 12$ __________(1)

Furthermore, for the limit to be non-zero and finite, f(x) must contain $(x-2)^2$ as a factor to match the order of the denominator. Let x – 1 = u. As $x \to 2$, $u \to 1$.

The denominator is $\left(\sqrt{u} - 1\right)\ell n\, u$. Using u = 1 + t where $t \to 0$:

$\left(\sqrt{1+t}-1\right)\ell n(1+t) \approx \left(\frac{1}{2}t\right)(t) = \frac{1}{2}t^2 = \frac{1}{2}(x-2)^2$. Thus, f(x) must be of the form $k(x-2)^2(x-c)$.

Comparing x^3 coefficients, k = 1. $x^3 - 5x^2 + ax + b = (x-2)^2(x-c) = (x^2-4x+4)(x-c) = x^3-(c+4)x^2 + (4c+4)x - 4c$. Comparing coefficients: $c + 4 = 5 \Rightarrow c = 1$. $a = 4(1)+4 = 8$.

$b = -4(1) = -4$. Check Eq (1): $2(8) - 4 = 12$ (Satisfied). Now calculate m :

$$m = \lim_{x\to 2}\frac{(x-2)^2(x-1)}{\frac{1}{2}(x-2)^2} = 2(2-1) = 2.$$

$a + b + m = 8 - 4 + 2 = 6.$

Question ID : 69112117

17. If the curve y = f(x) passes through the point (1, e) and satisfies the differential equation $dy = y(2+\log_e x)dx$, x > 0, then f(e) is equal to :

(1) e^e (2) e^{e^2} (3) e^{2e} (4) e^{2^e}

Ans. (3)

Sol. The given differential equation is:

$$\frac{dy}{y} = (2+\ln x)dx$$

Integrating both sides:

$$\int \frac{1}{y}dy = \int (2+\ln x)dx$$

$$\ln y = 2x + (x\ln x - x) + C = x\ln x + x + C$$

The curve passes through (1, e):

$$\ln e = 1\ln 1 + 1 + C \Rightarrow 1 = 1 + C \Rightarrow C = 0$$

So, $\ln y = x\ln x + x$. To find f(e), substitute x = e:

$\ln y = e\ln e + e = e + e = 2e$

$y = e^{2e}$

Question ID : 69112118

18. The number of critical points of the function $f(x)=\begin{cases}\left|\dfrac{\sin x}{x}\right|, & x\neq 0\\ 1, & x=0\end{cases}$ in the interval $(-2\pi, 2\pi)$ is equal to :

(1) 1 (2) 3 (3) 5 (4) 7

Ans. (3)

Sol. Critical points occur where $f'(x)=0$ or $f'(x)$ is undefined

1. At $x=0$: $\lim_{x\to 0}\dfrac{\sin x}{x}=1$, so f(x) is continuous. $f'(0)=\lim_{h\to 0}\dfrac{\frac{\sin h}{h}-1}{h}=0$

Thus, $x=0$ is a stationary point

2. At points where |sinx| changes sign : In $(-2\pi, 2\pi)$, these are $x=\pi$ and $x=-\pi$. At these points, the function has sharp corners, so f'(x) is undefined. (2 points)

3. Where $f'(x)=0$:

For $x\in(0,\pi)$, $f(x)=\dfrac{\sin x}{x}$, $f'(x)=\dfrac{x\cos x-\sin x}{x^2}=0\Rightarrow \tan x=x$

This has no solution in $(0, \pi)$

For $x\in(\pi, 2\pi)$, $f(x)=-\dfrac{\sin x}{x}$

$f'(x)=0\Rightarrow \tan x=x$. There is one intersection between $y=x$ and $y=\tan x\in(\pi, 2\pi)$. (1 point)

Due to symmetry (f(x) is even), there is one solution in $(-2\pi, -\pi)$. (1 point)

Total critical points = 1 (at 0) + 2(at $\pm\pi$) + 2 (where $\tan x = x$) = 5

Question ID : 69112119

19. Let $[\cdot]$ denote the greatest integer function. Then the value of $\int_0^3\left(\dfrac{e^x+e^{-x}}{[x]!}\right)dx$ is :

(1) $e^2+e^3-\dfrac{1}{e^2}-\dfrac{1}{e^3}$

(2) $\dfrac{1}{2}\left(e^2+e^3-\dfrac{1}{e^2}-\dfrac{1}{e^3}\right)$

(3) $e^2+e^3-\dfrac{1}{2e^2}-\dfrac{1}{2e^3}$

(4) $\dfrac{1}{2}\left(e^2+e^3\right)-\dfrac{1}{e^2}-\dfrac{1}{e^3}$

Ans. (2)

Sol. Break the integral based on the steps of [x]:

$$I=\int_0^1 \frac{e^x+e^{-x}}{0!}dx+\int_1^2 \frac{e^x+e^{-x}}{1!}dx+\int_2^3 \frac{e^x+e^{-x}}{2!}dx$$

Since 0! = 1 and 1! = 1

$$I=[e^x-e^{-x}]_0^1+[e^x-e^{-x}]_1^2+\frac{1}{2}[e^x-e^{-x}]_2^3$$

$$I=(e-e^{-1}-0)+(e^2-e^{-2}-e+e^{-1})+\frac{1}{2}(e^3-e^{-3}-e^2+e^{-2})$$

$$I=e^2-e^{-2}+\frac{1}{2}e^3-\frac{1}{2}e^{-3}-\frac{1}{2}e^2+\frac{1}{2}e^{-2}$$

$$I=\frac{1}{2}e^2+\frac{1}{2}e^3-\frac{1}{2}e^{-2}-\frac{1}{2}e^{-3}$$

$$=\frac{1}{2}e^2+e^3-\frac{1}{e^2}-\frac{1}{e^3}$$

Question ID : 69112120

20. Let $y=y(x)$ be the solution curve of the differential equation $(1+\sin x)\frac{dy}{dx}+(y+1)\cos x=0$, $y(0)=0$. If the curve passes through the point $\left(\alpha,-\frac{1}{2}\right)$, then a value of α is :

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{2}$

Ans. (4)

Sol. Rearranging the differential equation:

$$(1+\sin x)dy=-(y+1)\cos x dx \Rightarrow \frac{dy}{y+1}=-\frac{\cos x}{1+\sin x}dx$$

Integrating both sides

$$\ln|y+1|=-\ln|1+\sin x|+C$$

Given $y(0)=0$

$$\ln|0+1|=-\ln|1+\sin 0|+C \Rightarrow 0=0+C \Rightarrow C=0$$

$$\text{So, } \ln|y+1|=\ln\left|\frac{1}{1+\sin x}\right| \Rightarrow y+1=\frac{1}{1+\sin x}$$

Substituting the point $\left(\alpha,-\frac{1}{2}\right)$ into the equation

$-\frac{1}{2}+1=\frac{1}{1+\sin\alpha} \Rightarrow \frac{1}{2}=\frac{1}{1+\sin\alpha}$

$1 + \sin\alpha = 2 \Rightarrow \sin\alpha = 1$

In the standard principal range, $\alpha=\frac{\pi}{2}$

SECTION – B

Question ID : 69112121

21. If the domain of the function $f(x)=\sqrt{\log_{(0.6)}\left(\left|\frac{2x-5}{x^2-4}\right|\right)}$ is $(-\infty, a]\cup\{b\}\cup[c,d)\cup(e,\infty)$, then the value of $a+b+c+d+e$ is_______.

Ans. (4)

Sol. For the function f(x) to be defined, the expression inside the square root must be non-negative:

$\log_{0.6}\left(\left|\frac{2x-5}{x^2-4}\right|\right)\geq 0$

Since the base of the logarithm is 0.6 (where $0<0.6<1$), the inequality sign reverses when we remove the log:

$\left|\frac{2x-5}{x^2-4}\right|\leq(0.6)^0 \Rightarrow \left|\frac{2x-5}{x^2-4}\right|\leq 1$

Additionally, the argument of the logarithm must be positive, and denominators cannot be zero, implying

$x\neq\pm 2$ and $x\neq 2.5$

$\frac{|2x-5|}{|x^2-4|}\leq 1 \Rightarrow |x^2-4|\geq|2x-5|$

Squaring both sides

$(x^2-4)^2\geq(2x-5)^2 \Rightarrow (x^2-4)^2-(2x-5)^2\geq 0$

Using $A^2-B^2=(A-B)(A+B)$

$(x^2-4-2x+5)(x^2-4+2x-5)\geq 0$

$(x^2-2x+1)(x^2+2x-9)\geq 0 \Rightarrow (x-1)^2(x^2+2x-9)\geq 0$

Since $(x-1)^2\geq 0$, the inequality is satisfied if

1. $(x-1)^2=0 \Rightarrow x=1$ (Isolated point)

2. $x^2+2x-9\geq 0$ Roots of $x^2+2x-9=0$ are $x=\frac{-2\pm\sqrt{4+36}}{2}=-1\pm\sqrt{10}$

So, $x \in (-\infty, -1-\sqrt{10}] \cup [-1+\sqrt{10}, \infty)$. Combining this with the exclusion of x = 2, –2, 2.5, we note that –2 and 2 are not in the intervals [– 4.16] and [2.16], but 2.5 is. Domain:

$(-\infty, -1-\sqrt{10}] \cup \{1\} \cup [-1+\sqrt{10}, 2.5) \cup (2.5, \infty)$

Comparing with $(-\infty, a] \cup \{b\} \cup [c, d) \cup (e, \infty)$:

$a = -1-\sqrt{10}$, $b = 1$, $c = -1+\sqrt{10}$, $d = 2.5$, $e = 2.5$

$a+b+c+d+e = (-1-\sqrt{10}) + 1 + (-1+\sqrt{10}) + 2.5 + 2.5 = 4$

Question ID : 69112122

22. If $\sum_{k=1}^{n} a_k = 6n^3$, then $\sum_{k=1}^{6}\left(\frac{a_{k+1}-a_k}{36}\right)^2$ is equal to: ______.

Ans. (91)

Sol. Let $S_n = \sum_{k=1}^{n} a_k = 6n^3$

The n^{th} term is given by $a_n = S_n - S_{n-1}$:

$a_n = 6n^3 - 6(n-1)^3 = 6[n^3 - (n^3 - 3n^2 + 3n - 1)] = 6(3n^2 - 3n + 1) = 18n^2 - 18n + 6$

Now find $a_{k+1} - a_k$:

$a_{k+1} = 18(k+1)^2 - 18(k+1) + 6 = 18(k^2 + 2k + 1) - 18k - 18 + 6 = 18k^2 + 18k + 6$

$a_{k+1} - a_k = (18k^2 + 18k + 6) - (18k^2 - 18k + 6) = 36k$

Substituting this into the required summation

$$\sum_{k=1}^{6}\left(\frac{36k}{36}\right)^2 = \sum_{k=1}^{6} k^2 = \frac{6(6+1)(2\cdot 6+1)}{6} = 7\cdot 13 = 91$$

Question ID : 69112123

23. Let a, b, c ∈ {1, 2, 3, 4}. If the probability, that $ax^2 + 2\sqrt{2}bx + c > 0$ for all x ∈ R, is $\frac{m}{n}$, where $\gcd(m, n) = 1$, then m + n is equal to________.

Ans. (81)

Sol. For the quadratic $ax^2 + 2\sqrt{2}bx + c > 0$ to hold for all x ∈ R, we require the coefficient of $x^2 > 0$ (which is a ∈ {1, 2, 3, 4}, so always true) and the Discriminant D < 0

$D = (2\sqrt{2}b)^2 - 4ac = 8b^2 - 4ac < 0 \Rightarrow 2b^2 < ac$

Total outcomes = 4 × 4 × 4 = 64. Favorable cases for $2b^2 < ac$

If b = 1 : ac > 2. Out of 16 pairs of (a, c), the cases ac ≤ 2 are (1, 1), (1, 2), (2, 1). Favorable = 16 – 3 = 13

If b = 2 : ac > 8. Favorable pairs (a, c) are (3, 3), (3, 4), (4, 3), (4, 4). Favorable = 4

If b = 3 : ac > 18. Maximum ac = 4 × 4 = 16. Favorable = 0

If b = 4 : ac > 32. Favorable = 0. Total favorable outcomes = 13 + 4 = 17. Probability $P = \frac{17}{64} = \frac{m}{n}$. Since 17 is prime and does not divide 64, gcd(17, 64) = 1. m + n = 17 + 64 = 81

Question ID : 69112124

24. Let a circle C have its centre in the first quadrant, intersect the coordinate axes at exactly three points and cut off equal intercepts from the coordinate axes. If the length of the chord of C on the line x + y = 1 is $\sqrt{14}$, then the square of the radius of C is________.

Ans. (8)

Sol. A circle in the first quadrant intersecting the axes at exactly three points with equal intercepts must pass through the origin (0, 0). Let the center be (h, k). Since it passes through the origin, $r^2 = h^2 + k^2$. The x-intercept is 2h and the y-intercept is 2k. Given they are equal, h = k. Thus, center is (h, h) and $r^2 = 2h^2$. The circle equation is: $(x-h)^2 + (y-h)^2 = 2h^2 \Rightarrow x^2 + y^2 - 2hx - 2hy = 0$. The distance d from center (h, h) to the line x + y – 1= 0 is

$$d = \frac{|h+h-1|}{\sqrt{1^2+1^2}} = \frac{|2h-1|}{\sqrt{2}}$$

Chord length $L = 2\sqrt{r^2 - d^2} = \sqrt{14}$

$$14 = 4\left(2h^2 - \frac{(2h-1)^2}{2}\right) = 8h^2 - 2(4h^2 - 4h + 1) = 8h^2 - 8h^2 + 8h - 2$$

$$14 = 8h - 2 \Rightarrow 8h = 16 \Rightarrow h = 2$$

The square of the radius is $r^2 = 2h^2 = 2(2^2) = 8$

Question ID : 69112125

25. If $\alpha = \int_0^{2\sqrt{3}} \log_2(x^2+4)dx + \int_2^4 \sqrt{2^x - 4}\,dx$, then α^2 is equal to________.

Ans. (192)

Sol. Let $f(x) = \log_2(x^2 + 4)$. To find its inverse $f^{-1}(x)$

$x = \log_2(y^2+4) \Rightarrow 2^x = y^2 + 4 \Rightarrow y = \sqrt{2^x - 4}$

So $f^{-1}(x) = \sqrt{2^x - 4}$. Check the limits of integration

For f(x), the limits are $[0, 2\sqrt{3}]$

At x = 0, $f(0) = \log_2(4) = 2$

At $x = 2\sqrt{3}, f(2\sqrt{3}) = \log_2(12+4) = \log_2(16) = 4$. The second integral has limits [2, 4], which matches $[f(0), f(2\sqrt{3})]$. Using the property $\int_a^b f(x)dx + \int_{f(a)}^{f(b)} f^{-1}(x)dx = b \cdot f(b) - a \cdot f(a)$:

$\alpha = [2\sqrt{3} \cdot f(2\sqrt{3})] - [0 \cdot f(0)] = 2\sqrt{3} \cdot 4 - 0 = 8\sqrt{3}$

Therefore, $\alpha^2 = (8\sqrt{3})^2 = 64 \cdot 3 = 192$