JEE Main January 2026

Question Paper With Text Solution

28 January | Shift-2

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN JANUARY 2026 | 28^{TH} JANUARY SHIFT-2

SECTION – A

Question ID : 8606541652

1. Given below are two statements :

Statement I: The function $f : R \rightarrow R$ defined by $f(x) = \dfrac{x}{1+|x|}$ is one-one.

Statement II: The function $f : R \rightarrow R$ defined by $f(x) = \dfrac{x^2+4x-30}{x^2-8x+18}$ is many-one. In the light of the above statements, choose the correct answer from the options given below :

(1) Both Statement I and Statement II are false

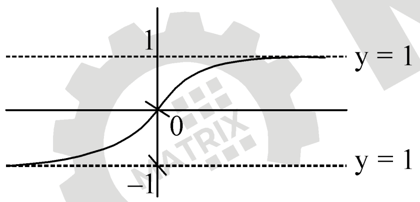
(2) Statement I is false but Statement II is true

(3) Statement I is true but Statement II is false

(4) Both Statement I and Statement II are true

Ans. Official answer NTA (4)

Sol. $f : R \rightarrow R$



$$S-1 : f(x) = \frac{x}{1+|x|}\begin{cases} \dfrac{x}{1+x}; & x \geq 0 \\ \dfrac{x}{1-x}; & x < 0 \end{cases}$$

$\therefore$ f(x) is one-one

$$S-2 : f(x) = \frac{x^2+4x-30}{x^2-8x+18}$$

$A\left(\dfrac{Q \cdot E}{Q \cdot E}\right)$ with no common factor is always many-one.

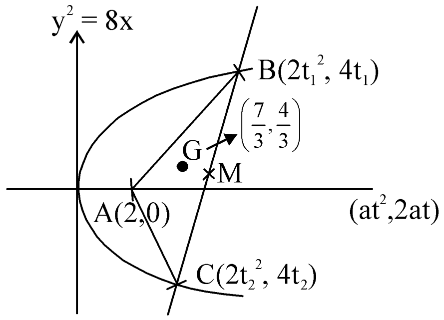
Question ID : 8606541663

2. Let A be the focus of the parabola $y^2 = 8x$. Let the line $y = mx + c$ intersect the parabola at two distinct points B and C. If the centroid of the triangle ABC is $\left(\frac{7}{3}, \frac{4}{3}\right)$, then $(BC)^2$ is equal to :

(1) 41 (2) 32 (3) 89 (4) 80

Ans. Official answer NTA (4)

Sol.



$a = 2$

Let $B\left(2t_1^2, 4t_1\right) \& C\left(2t_2^2, 4t_2\right)$

Now $G\left[\frac{2t_1^2 + 2t_2^2 + 2}{3}, \frac{4t_1 + 4t_2}{3}\right] \equiv \begin{pmatrix} 7 & 4 \\ 3 & 3 \end{pmatrix}$

$\therefore t_1^2 + t_2^2 = \frac{5}{2}; t_1 + t_2 = 1$

Now $(BC)^2 = 4\left(t_1^2 - t_2^2\right)^2 + 16\left(t_1 - t_2\right)^2$

$= 4\left(t_1 + t_2\right)^2 \left(t_1 - t_2\right)^2 + 16\left(t_1 - t_2\right)^2$

$= 4\left(t_1 - t_2\right)^2 \left\{\left(t_1 + t_2\right)^2 + 4\right\}$

Again,

$\left(t_1 + t_2\right)^2 = \frac{5}{2} + 2t_1t_2$

$1 - \frac{5}{2} = 2t_1t_2$

$-\frac{3}{4} = t_1t_2$

And $\left(t_1 - t_2\right)^2 = \left(t_1 + t_2\right)^2 - 4t_1t_2$

$= 1 + 3 = 4$

$\therefore (BC)^2 = 4 \times 4 \times (5)$

$= 4 \times 20 = 80$

Question ID : 8606541653

3. Let $A = \{z \in \mathbb{C} : |z-2| \leqslant 4\}$ and $B = \{z \in \mathbb{C} : |z-2| + |z+2| = 5\}$. Then the max $\{|z_1 - z_2| : z_1 \in A$ and $z_2 \in B\}$ is :

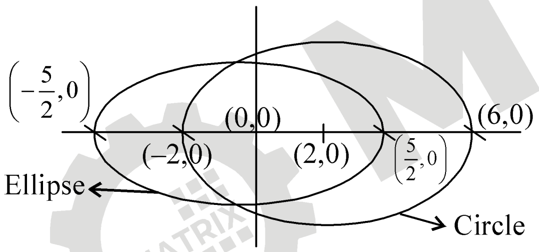
(1) 9 (2) 8 (3) $\frac{15}{2}$ (4) $\frac{17}{2}$

Ans. Official answer NTA (4)

Sol. $|z-2| \le 4$ & $|z-2| + |z+2| = 5$

first curve is region inside circle with centre C(2, 0) and radius = 4 and second is ellipse with centre (0, 0) ellipse focii (–2, 0) and (2, 0)

Circle $\equiv (x-2)^2 + y^2 \le 16$ & $\frac{x^2}{25/4} + \frac{y^2}{9/4} = 1$



$\therefore |z_1 - z_2|_{max}$

$= 6 + \frac{5}{2} = \frac{17}{2}$

Question ID : 8606541658

4. The probability distribution of a random variable X is given below :

x	4k	$\frac{30}{7}k$	$\frac{32}{7}k$	$\frac{34}{7}k$	$\frac{36}{7}k$	$\frac{38}{7}k$	$\frac{40}{7}k$	6k
P(x)	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{1}{5}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{1}{5}$	$\frac{1}{15}$

If $E(X)=\frac{263}{15}$, then $P(X<20)$ is equal to :

(1) $\frac{14}{15}$ (2) $\frac{8}{15}$ (3) $\frac{11}{15}$ (4) $\frac{3}{5}$

Ans. Official answer NTA (3)

Sol. $E(X)=\frac{263}{15}$

E(x) is mean of observations

i.e $\sum P_i x_i = \frac{8k}{15}+\frac{2k}{7}+\frac{64k}{7\times15}+\frac{36k}{7\times15}+\frac{34k}{7\times5}+\frac{76k}{7\times15}+\frac{40k}{7\times5}+\frac{6k}{15}$

$\frac{10k+56k+30k+64k+36k+76k+120k+42k}{15\times7}=\frac{263}{15}$

$\frac{526k}{7}=263$

$k=\frac{7}{2}$

$\therefore P(x<20)$

$$\Rightarrow \begin{array}{c|cccccccc} x & 14 & 15 & 16 & 17 & 18 & 19 & 20 & 21 \\ P(x) & \frac{2}{15} & \frac{1}{15} & \frac{2}{15} & \frac{1}{5} & \frac{1}{15} & \frac{2}{15} & \frac{1}{5} & \frac{1}{15} \end{array}$$

$P(X<20)$

$=1-(P=20+P=21)$

$=1-\left[\frac{1}{5}+\frac{1}{15}\right]=1-\left[\frac{4}{15}\right]=\frac{11}{15}$

Question ID : 8606541667

5. Let $[\cdot]$ denote the greatest integer function. Then $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(\frac{12(3+[x])}{3+[\sin x]+[\cos x]}\right)dx$ is equal to :

(1) $11\pi+2$ (2) $13\pi+1$ (3) $12\pi+5$ (4) $15\pi+4$

Ans. Official answer NTA (1)

Sol. $I=\int_{-\pi/2}^{\pi/2}\frac{12(3+[x])}{3+[\sin x]+[\cos x]}dx$

$$= \int_{-\pi/2}^{-1} \frac{12\cdot 1}{2}dx + \int_{-1}^{0} \frac{12.2}{2}dx + \int_{0}^{1} \frac{12\cdot 3}{3}dx + \int_{1}^{\pi/2} \frac{12.4}{3}dx$$

$$= 6\left(\frac{\pi}{2} - 1\right) + 12(0+1) + 12(1-0) + 16\left(\frac{\pi}{2} - 1\right)$$

$= 3\pi - 6 + 12 + 12 + 8\pi - 16$

$= 11\pi + 2$

Question ID : 8606541654

6. Let the arithmetic mean of $\frac{1}{a}$ and $\frac{1}{b}$ be $\frac{5}{16}$, $a > 2$. If α is such that a, 4, α, b are in A.P., then the equation $\alpha x^2 - ax + 2(\alpha - 2b) = 0$ has :

(1) one root in (0,2) and another in (–4,–2)

(2) complex roots of magnitude less than 2

(3) both roots in the interval (–2, 0)

(4) one root in (1, 4) and another in (–2, 0)

Ans. Official answer NTA (4)

Sol. AM of $\frac{1}{a}$ & $\frac{1}{b} = \frac{5}{16}$

$$\frac{\frac{1}{a} + \frac{1}{b}}{2} = \frac{5}{16}$$

$\frac{1}{a} + \frac{1}{b} = \frac{5}{8}$ ....(1)

a, 4, α, b ________ A.P.

det $a = 4 - d$, $\alpha = 4 + d$, $b = 4 + 2d$

$\alpha x^2 - ax + 2(\alpha - 2b) = 0$

$(4 + d)x^2 - (4 - d)x + 2(4 + d - 2(4 + 2d)) = 0$

$(4 + d)x^2 - (4 - d)x + 2(-4 - 3d) = 0$ ....(2)

and from equation (1)

$$\frac{1}{4-d} + \frac{1}{4+2d} = \frac{5}{8} \Rightarrow d = 2$$

$\therefore$ eq. (2) becomes

$6x^2 - 2x - 20 = 0$

$3x^2 - x - 10 = 0$

$\Rightarrow x = 2, -\frac{5}{3}$

Question ID : 8606541661

7. Considering the principal values of inverse trigonometric functions, the value of the expression $\tan\left(2\sin^{-1}\left(\frac{2}{\sqrt{13}}\right) - 2\cos^{-1}\left(\frac{3}{\sqrt{10}}\right)\right)$ is equal to :

(1) $\frac{16}{63}$ (2) $\frac{33}{56}$ (3) $-\frac{33}{56}$ (4) $-\frac{16}{63}$

Ans. Official answer NTA (2)

Sol. $\tan\left(2\sin^{-1}\left(\frac{2}{\sqrt{13}}\right) - 2\cos^{-1}\left(\frac{3}{\sqrt{10}}\right)\right)$

$\tan\left(2\tan^{-1}\frac{2}{3} - 2\tan^{-1}\frac{1}{3}\right)$

$\tan\left(2\left(\tan^{-1}\frac{2}{3} - \tan^{-1}\frac{1}{3}\right)\right)$

$\tan\left(2\tan\left(\frac{\frac{2}{3}-\frac{1}{3}}{1+\frac{2}{9}}\right)\right)$

$\tan\left(2\tan^{-1}\left(\frac{1}{3}\times\frac{9}{11}\right)\right)$

$\tan\left(2\tan^{-1}\left(\frac{3}{11}\right)\right)$

$\frac{2\cdot\frac{3}{11}}{1-\frac{y}{121}} \Rightarrow \frac{6}{11}\times\frac{121}{112} \Rightarrow \frac{33}{56}$

Question ID : 8606541656

8. The sum of the coefficients of x^{499} and x^{500} in $(1+x)^{1000}+x(1+x)^{999}+x^2(1+x)^{998}+\ldots+x^{1000}$ is :

(1) ${}^{1000}C_{501}$ (2) ${}^{1002}C_{501}$ (3) ${}^{1002}C_{500}$ (4) ${}^{1001}C_{501}$

Ans. Official answer NTA (3)

Sol. $(1+x)^{1000}+x(1+x)^{999}+x^2(1+x)^{998}+\ldots+x^{1000}$

G.P. with $a = (1+x)^{1000}$

and $r = \dfrac{x}{1+x}$

$$\frac{(1+x)^{1000}\left\{1-\left(\frac{x}{1+x}\right)^{1001}\right\}}{1-\frac{x}{1+x}}$$

$= (1+x)^{1001} - x^{1001}$

Required sum $= {}^{1001}C_{499} + {}^{1001}C_{500} = {}^{1002}C_{500}$

Question ID : 8606541651

9. The sum of all the elements in the range of

$f(x) = \text{Sgn}(\sin x)+\text{Sgn}(\cos x)+\text{Sgn}(\tan x)+\text{Sgn}(\cot x),\ x \neq \dfrac{n\pi}{2}, n \in Z$, where $\text{Sgn}(t) = \begin{cases} 1, & \text{if } t > 0 \\ -1, & \text{if } t < 0 \end{cases}$

, is :

(1) –2 (2) 2 (3) 0 (4) 4

Ans. Official answer NTA (2)

Sol. $f(x) = \text{sgn}(\sin x)+\text{sgn}(\cos x)+\text{sgn}(\tan x)+\text{sgn}(\cot x);\ x \neq \dfrac{n\pi}{2}$

$C-1:\ 0 < x < \dfrac{\pi}{2}$

$y = 1 + 1 + 1 + 1 = 4$

$C-2:\ \dfrac{\pi}{2} < x < \pi$

$y = 1 - 1 - 1 - 1 = -2$

$C-3:\ \pi < x < \dfrac{3\pi}{2}$

$y = -1 - 1 + 1 + 1 = 0$

$C-4: \frac{3\pi}{2} < x < 2\pi$

$y = -1 + 1 - 1 - 1 = -2$

Range : $\{-2, 0, 4\}$

Sum $= -2 + 0 + 4 = 2$

Question ID : 8606541655

10. $\frac{6}{3^{26}} + \frac{10\cdot 1}{3^{25}} + \frac{10\cdot 2}{3^{24}} + \frac{10\cdot 2^2}{3^{23}} + \ldots + \frac{10\cdot 2^{24}}{3}$ is equal to :

(1) 2^{26} (2) 3^{26} (3) 2^{25} (4) 3^{25}

Ans. Official answer NTA (1)

Sol. $\frac{6}{3^{26}} + \frac{10.1}{3^{25}} + \frac{10.2}{3^{24}} + \ldots + \frac{10.2^{24}}{3}$

$$S = \frac{6}{3^{26}} + \frac{10}{3^{25}}\{1 + 2\cdot 3 + 2^2\cdot 3^2 + \cdots + 2^{24}3^{24}\}$$

$$= \frac{6}{3^{26}} + \frac{10}{3^{25}}\left\{\frac{6^{25}-1}{6-1}\right\}$$

$$= \frac{2}{3^{25}}\left(6^{25}-1\right) + \frac{6}{3^{26}}$$

$$= \frac{2\times 3^{25}\times 2^{25}}{3^{25}} - \frac{2}{3^{25}} + \frac{6}{3^{26}}$$

$$= 2^{26} - \frac{2}{3^{25}} + \frac{3\cdot 2}{3^{26}}$$

$$= 2^{26}$$

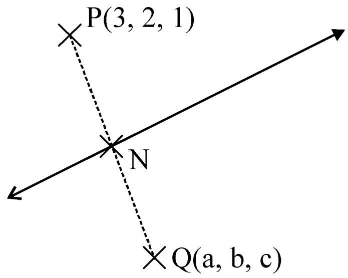
Question ID : 8606541665

11. Let Q(a, b, c) be the image of the point P(3,2,1) in the line $\frac{x-1}{1} = \frac{y}{2} = \frac{z-1}{1}$. Then the distance of Q from the line $\frac{x-9}{3} = \frac{y-9}{2} = \frac{z-5}{-2}$ is :

(1) 7 (2) 5 (3) 6 (4) 8

Ans. Official answer NTA (1)

Sol.



$L \equiv \frac{x-1}{1} = \frac{y}{2} = \frac{z-1}{1} = \lambda$

General pt of line $\{\lambda + 1, 2\lambda, \lambda + 1\}$

DR of PN = $\{\lambda - 2, 2\lambda - 2, \lambda\}$

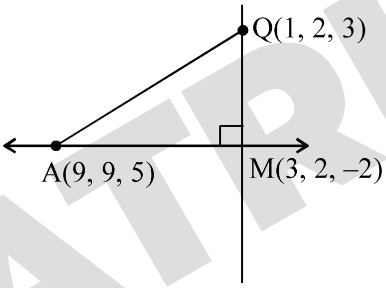
Again, $\lambda - 2 + 2(2\lambda - 2) + \lambda = 0$

$6\lambda = 6$

$\lambda = 1$

$\therefore$ N = (2, 2, 2)

$\Rightarrow Q \equiv (1, 2, 3)$



$AQ = \sqrt{64+49+4}$

$= \sqrt{117}$

$AM = \left|\frac{24+14-4}{\sqrt{9+4+4}}\right| = \frac{34}{\sqrt{17}}$

$QM = \sqrt{117-68} = \sqrt{49} = 7$

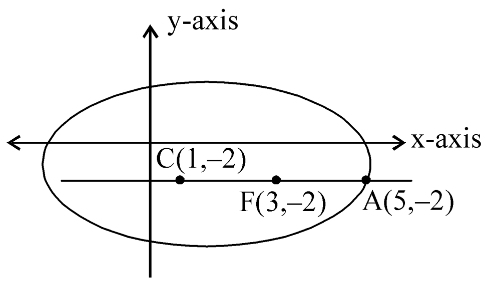
Question ID : 8606541659

12. An ellipse has its center at (1, –2), one focus at (3, –2) and one vertex at (5, –2). Then the length of its latus rectum is :

(1) $\frac{16}{\sqrt{3}}$ (2) $4\sqrt{3}$ (3) 6 (4) $6\sqrt{3}$

Ans. Official answer NTA (3)

Sol.



Length of LR $= \frac{2b^2}{a}$

CA = 4 = a

a.e = 2

$e = \frac{2}{4} = \frac{1}{2}$

$b^2 = a^2 (1 - e^2)$

$= 16\left(1 - \frac{1}{4}\right) = 16 \cdot \frac{3}{4}$

$b^2 = 12$

$\therefore$ length of LR $= \frac{2.12}{4} = 6$

Question ID : 8606541668

13. Let $f(x) = \int \frac{dx}{x^{\left(\frac{2}{3}\right)} + 2x^{\left(\frac{1}{2}\right)}}$ be such that $f(0) = -26 + 24\log_e(2)$. If $f(1) = a + b\log_e(3)$, where $a, b \in Z$, then $a + b$ is equal to :

(1) –26 (2) –11 (3) –5 (4) –18

Ans. Official answer NTA (2)

Sol. $f(x) = \int \frac{dx}{x^{2/3} + 2x^{1/2}}$

$f(0) = -26 + 24\ell n 2$

Put $x = t^6$

$dx = 6t^5\, dt$

$\int \frac{6t^5 dt}{t^4 + 2t^3}$

$6\int \frac{t^2 - 4 + 4}{t + 2} dt$

$6\left\{\int (t-2)dt + 4\left\{\frac{1}{t+2}dt\right\}\right\}$

$6\left\{\frac{t^2}{2} - 2t + 4\ell n(t+2)\right\} + C$

$f(x) = 3x^{1/3} - 12x^{1/6} + 24\ell n\left(x^{1/6} + 2\right) + C$

$f(0) = 24\ell n 2 + C = -26 + 24\ell n 2$

$\Rightarrow C = -26$

$f(1) = -35 + 24\, ln3$

$a = -35 \quad b = 24$

$a + b = -11$

Question ID : 8606541660

14. Let the circle $x^2 + y^2 = 4$ intersect x-axis at the points A(a, 0), a > 0 and B(b, 0). Let $P(2\cos\alpha, 2\sin\alpha),\ 0 < \alpha < \frac{\pi}{2}$ and $Q(2\cos\beta, 2\sin\beta)$ be two points such that $(\alpha - \beta) = \frac{\pi}{2}$. Then the point of intersection of AQ and BP lies :

(1) $x^2 + y^2 - 4x - 4 = 0$

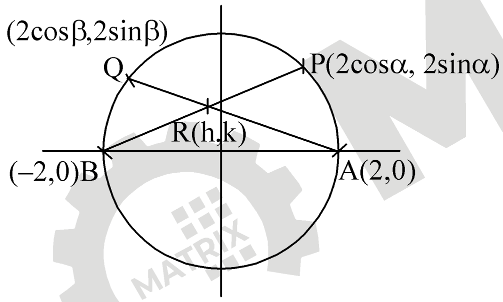
(2) $x^2 + y^2 - 4x - 4y = 0$

(3) $x^2 + y^2 - 4y - 4 = 0$

(4) $x^2 + y^2 - 4x - 4y - 4 = 0$

Ans. Official answer NTA (3)

Sol.



det Point of intersection by P(h, k)

$$\frac{k}{h+2} = \frac{2\sin\alpha}{2\cos\alpha + 2}$$

$$\Rightarrow \frac{k}{h+2} = \tan\alpha/2$$

$$\frac{k}{h-2} = \frac{2\sin\beta}{2\cos\beta - 2}$$

$$\frac{k}{h-2} = -\cot\frac{\beta}{2}$$

Now $\frac{\alpha}{2} - \frac{\beta}{2} = \frac{\pi}{4}$

$$\frac{\tan\alpha/2-\tan\beta/2}{1+\tan\alpha/2\cdot\tan\beta/2}=1$$

$$\frac{\frac{k}{h+2}+\frac{h-2}{k}}{1+\frac{k}{h+2}\cdot\frac{2-h}{k}}=1$$

$\Rightarrow x^2+y^2-4y-4=0$

Question ID : 8606541657

15. Given below are two statements :

Statement I : $25^{13}+20^{13}+8^{13}+3^{13}$ is divisible by 7 .

Statement II: The integral part of $(7+4\sqrt{3})^{25}$ is an odd number.

In the light of the above statements, choose the correct answer from the options given below :

(1) Statement I is false but Statement II is true

(2) Statement I is true but Statement II is false

(3) Both Statement I and Statement II are false

(4) Both Statement I and Statement II are true

Ans. Official answer NTA (4)

Sol. Statement I

$25^{13}+3^{13}+20^{13}+8^{13}$

$25+3 \qquad 20+8$

$=28 \qquad =28$

Divisible by 7

Statement II

$\left(7+4\sqrt{3}\right)^{25}=I+f$

$\left(7-4\sqrt{3}\right)^{25}=f'$

$I+f+f'=2\left\{{}^{25}C_0 7^{25}+{}^{25}C_2 7^{23}+......\right\}$

= Ever integer

$I=\text{odd}\ ;\ f+f'=1$

Question ID : 8606541666

16. Let $f(x)=\lim_{\theta\to 0}\left(\dfrac{\cos\pi x - x^{\left(\frac{2}{\theta}\right)}\sin(x-1)}{1+x^{\left(\frac{2}{\theta}\right)}(x-1)}\right), x\in R$. Consider the following two statements :

(I) f(x) is discontinous at x = 1.

(II) f(x) is continous at x = –1.

Then,

(1) Neither (I) nor (II) is True

(2) Only (I) is True

(3) Only (II) is True

(4) Both (I) and (II) are True

Ans. Official answer NTA (1)

Sol. $f(x)=\underset{\theta\to 0}{\text{dt}}\dfrac{\cos\pi x - x^{\frac{2}{\theta}}\sin(x-1)}{1+x^{2/\theta}(x-1)}$

$$f(x)=\begin{cases}\cos\pi x & ;\ x<1\\ \dfrac{-\sin(x-1)}{x-1} & ;\ x>1^{+}\end{cases}$$

LHS : $\dfrac{\text{dt}}{x\to 1^{-}}\cos\pi x=-1$

RHL : $\dfrac{\text{dt}}{x\to 1^{+}}-\dfrac{\sin(x-1)}{x-1}=-1$

Continous at x = 1

Again : $f(x)=\begin{cases}-\dfrac{\sin(x-1)}{-(x-1)} ; & x<-1\\ \cos\pi x ; & x>-1\end{cases}$

RHL $=\dfrac{\text{dt}}{x\to -1}\cos\pi x=-1$

LHL $=\dfrac{\text{dt}}{x\to -1}-\dfrac{\sin(x-1)}{x-1}=\dfrac{\sin 2}{-2}$

Discontinous at x = –1.

Question ID : 8606541669

17. Let y = y(x) be the solution of the differential equation $x\frac{dy}{dx}-y=x^2\cot x, x\in(0,\pi)$. If $y\left(\frac{\pi}{2}\right)=\frac{\pi}{2}$ then $6y\left(\frac{\pi}{6}\right)-8y\left(\frac{\pi}{4}\right)$ is equal to :

(1) 3π (2) $-\pi$ (3) π (4) -3π

Ans. Official answer NTA (2)

Sol. $x\frac{dy}{dx}-y=x^2\cot x$

$\frac{dy}{dx}-\frac{1}{x}\cdot y=x\cot x$

$P=-\frac{1}{x};\quad Q=x\cot x;\quad I\cdot F=e^{\int -\frac{1}{x}dx}$

$=e^{-\ell n x}=\frac{1}{x}$

$y\cdot\frac{1}{x}=\int x\cot x\cdot\frac{1}{x}dx$

$\frac{y}{x}=\ell n\sin x+C$

$y=x\ell n\sin x+Cx$

Given : $x=\frac{\pi}{2}; y=\frac{\pi}{2}$

$\frac{\pi}{2}=\frac{\pi}{2}\ell n1+C\cdot\frac{\pi}{2}$

$C=1$

$\therefore y=x\ell n\sin x+x$

Now $6y\left(\frac{\pi}{6}\right)-8y\left(\frac{\pi}{4}\right)$

At $x=\frac{\pi}{6}, y=\frac{\pi}{6}\ell n\left(\frac{1}{2}\right)+\frac{\pi}{6}$ $\Rightarrow 6y=-\pi\ell n2+1\cdot\pi$

At $x=\frac{\pi}{4}\quad y=\frac{\pi}{4}\ell n\left(\frac{1}{\sqrt{2}}\right)+\frac{\pi}{4}$

$=-\frac{\pi}{8}\ell n2+\frac{\pi}{4}$

$-8y = \pi \ell n 2 = 2\pi$

$\therefore\ 6y\left(\frac{\pi}{6}\right) - 8y\left(\frac{\pi}{4}\right) = -\pi$

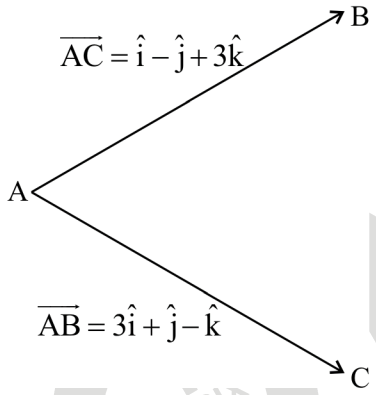
Question ID : 8606541664

18. Let P be a point in the plane of the vectors $\overrightarrow{AB} = 3\hat{i} + \hat{j} - \hat{k}$ and $AC = \hat{i} - \hat{j} + 3\hat{k}$ such that P is equidistant from the lines AB and AC. If $\left|\overrightarrow{AP}\right| = \frac{\sqrt{5}}{2}$, then the area of the triangle ABP is :

(1) $\frac{3}{2}$ (2) $\frac{\sqrt{30}}{4}$ (3) $\frac{\sqrt{26}}{4}$ (4) 2

Ans. Official answer NTA (2)

Sol.



$\cos 2\theta = \frac{3-1-3}{\sqrt{11}\sqrt{11}} = -\frac{1}{11}$

$1 - 2\sin^2\theta = -\frac{1}{2}$

$2\sin^2\theta = \frac{12}{11}$

$\sin\theta = \sqrt{\frac{6}{11}}$

$\therefore$ Area of $\Delta APB = \frac{1}{2}\sqrt{11}\cdot\frac{\sqrt{5}}{2}\cdot\sqrt{\frac{6}{11}} = \frac{\sqrt{30}}{4}$

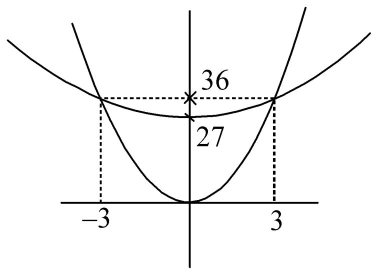
Question ID : 8606541670

19. Let $P_1 : y = 4x^2$ and $P_2 : y = x^2 + 27$ be two parabolas. If the area of the bounded region enclosed between P_1 and P_2 is six times the area of the bounded region enclosed between the line $y = \alpha x, \alpha > 0$ and P_1, then α is equal to :

(1) 8 (2) 6 (3) 15 (4) 12

Ans. Official answer NTA (4)

Sol. $P_1 : y = 4x^2$; $P_2 : y = x^2 + 27$



Area bounded between P_1 and P_2 is

$\int_{-3}^{3}\left(x^2 + 27 - 4x^2\right)dx$

$\Rightarrow 2\int_{0}^{3}\left(27 - 3x^2\right)dx$

$\Rightarrow 2[27x - x^3]^3$

$= 2[81 \times 27]$

$= 108$

$\therefore$ Area bounded betwen P_1 and P_2 is 18 sq. unit $x^2 = 4ay$ and $x = my$ is $\frac{8a^2}{3m^3}$

$\therefore$ Area between $x^2 = \frac{y}{4}$ and $x^2 = \frac{y}{4}$ is

$$\frac{8\left(\frac{1}{16}\right)^2}{3\left(\frac{1}{\alpha}\right)^3} = 18$$

$\Rightarrow \alpha^3 = 2^6 . 3^3$

$\Rightarrow \alpha = 12$

Question ID : 8606541662

20. Let the ellipse $E:\frac{x^2}{144}+\frac{y^2}{169}=1$ and the hyperbola $H:\frac{x^2}{16}-\frac{y^2}{\lambda^2}=-1$ have the same foci. If e and L respectively denote the eccentricity and the length of the latus rectum of H, then the value of $24(e+L)$ is :

(1) 148 (2) 67 (3) 296 (4) 126

Ans. Official answer NTA (3)

Sol. $E:\frac{x^2}{144}\times\frac{y^2}{169}=1$

$H:\frac{x^2}{16}-\frac{y^2}{\lambda^2}=-1$

$e'=\sqrt{1-\frac{144}{169}}=\frac{5}{13}$

four $\Rightarrow (0, 5)$

$\Rightarrow \lambda\left(\sqrt{1+\frac{16}{\lambda^2}}\right)=5\Rightarrow \lambda=3$

eccentricity of hyperbola $=\sqrt{1+\frac{16}{\lambda^2}}=\frac{5}{3}$

length of LR of Hyperbola $=\frac{2\times16}{3}=\frac{32}{3}$

$24(e+\ell)=24\left(\frac{5}{3}+\frac{32}{3}\right)=296$

SECTION – B

Question ID : 8606541674

21. If the distance of the point $P(43,\alpha,\beta),\beta<0$, from the line $\vec{r}=4\hat{i}-\hat{k}+\mu(2\hat{i}+3\hat{k}),\mu\in R$ along a line with direction ratios 3, –1, 0 is $13\sqrt{10}$ then $\alpha^2+\beta^2$ is equal to ________.

Ans. Official answer NTA (170)

Sol. $\frac{x-43}{3}-\frac{y-\alpha}{-1}=\frac{z-\beta}{0}=\lambda$

$P_1(43+3\lambda,\alpha-\lambda,\beta)$

$\frac{x-4}{2}=\frac{y}{0}=\frac{z+1}{3}=\mu$

$P_2(2\mu+4, 0, 3\mu-1)$

$\therefore \mu = \dfrac{3\lambda+39}{2}, \alpha = \lambda, \beta = \dfrac{9\lambda-115}{2}$

$P_1(43, \alpha, \beta), P_2(43+3\alpha, 0, \beta)$

$(P_1P_2)^2 = 1690 = 10\alpha^2$

$\alpha = 13, \beta = 1$

$\Rightarrow \alpha^2 + \beta^2 = 170$

Question ID : 8606541675

22. Let f be a differentiable function satisfying $f(x) = 1 - 2x + \int_0^x e^{(x-t)} f(t)dt, x \in R$ and let $g(x) = \int_0^x (f(t)+2)^{15}(t-4)^6(t+12)^{17}dt, x \in R$. If p and q are respectively the points of local minima and local maxima of g, then the value of $|p+q|$ is equal to ________.

Ans. Official answer NTA (9)

Sol. $f(x) = 1 - 2x + e^x \int_0^x e^{-t} \cdot f(t)dt$

$e^{-x}f(x) = (1-2x)e^{-x} + \int_0^x e^{-t}f(t)dt$

$e^{-x}f'(x) - e^{-x}f(x) = -2e^{-x} + (1-2x)e^{-x}(-1) + e^{-x}f(x)$

$f'(x) - 2f(x) = 2x - 3$

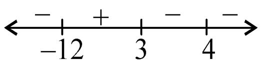
$\dfrac{dy}{dx} - 2y = 2x - 3$

$ye^{-2x} = \int e^{-2x}(2x-3)dx$

Simplify $f(x) = 1 - x$

$g(x) = \int_0^x (3-t)^{15}(t-4)^6(t+12)^{17}dt$

$g'(x) = (3-x)^{15}(x-4)^6(x+12)^{17}$



Min $\equiv q = 3$ $\quad |p+q| = 9$

Max $\equiv p = -12$

Question ID : 8606541672

23. If $\sum_{r=1}^{25}\left(\frac{r}{r^4+r^2+1}\right)=\frac{p}{q}$, where p and q are positive integers such that $\gcd(p,q)=1$, then $p+q$ is equal to ________.

Ans. Official answer NTA (976)

Sol. $\sum_{r=1}^{25}\frac{r}{\left(r^2+r+1\right)\left(r^2-r+1\right)}$

$\frac{1}{2}\left\{\sum_{r=1}^{25}\frac{1}{r^2-r+1}-\frac{1}{r^2+r+1}\right\}$

$\frac{1}{2}\left\{\left(\frac{1}{1}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{7}\right)+\ldots+\left(\frac{1}{601}-\frac{1}{651}\right)\right\}$

$\frac{1}{2}\times\frac{650}{651}=\frac{325}{651}\Rightarrow p+q=976$

Question ID : 8606541671

24. Let $A=\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}$ and B be two matrices such that $A^{100}=100B+I$. Then the sum of all the elements of B^{100} is ________.

Ans. Official answer NTA (0)

Sol. $A=I+\begin{bmatrix}2 & -4\\ 1 & -2\end{bmatrix}$ Let $M=\begin{bmatrix}2 & -4\\ 1 & -2\end{bmatrix}$

$M^2=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}=M^3=M^4=.......=M^{100}$

$A^{100}=(I+M)^{100}=\sum_{r=0}^{100}{}^{100}C_r M^r I$

$= I + 100\ M$

$M = B$

$\Rightarrow M^{100}=B^{100}=0$

Question ID : 8606541673

25. Three persons enter in a lift at the ground floor. The lift will go upto 10^{th} floor. The number of ways, in which the three persons can exit the lift at three different floors, if the lift does not stop at first, second and third floors, is equal to _________.

Ans. Official answer NTA (210)

Sol. ${}^{7}C_{3} \times 3!$

$= 210$