JEE Main January 2026

Question Paper With Text Solution

28 January | Shift-1

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN JANUARY 2026 | 28TH JANUARY SHIFT-1

SECTION – A

Question ID : 444792691

1. Let f be a polynomial function such that $f\left(x^2+1\right)=x^4+5x^2+2$, for all $x \in \mathbb{R}$.

Then $\int_0^3 f(x)dx$ is equal to

(1) $\frac{27}{2}$ (2) $\frac{33}{2}$ (3) $\frac{5}{3}$ (4) $\frac{41}{3}$

Ans. Official answer NTA (2)

Sol. $f\left(x^2+1\right)=x^4+5x^2+2$

$f\left(x^2+1\right)=\left(x^2+1\right)^2+3\left(x^2+1\right)-2$

$f(x)=x^2+3x-2$

$\int_0^3 f(x)dx=\int_0^3\left(x^2+3x-2\right)dx$

$$=\left[\frac{x^3}{3}+\frac{3x^2}{2}-2x\right]_0^3$$

$\Rightarrow\left[9+\frac{27}{2}-6-0\right]=\frac{33}{2}$

Question ID : 444792684

2. Let $S=\{1,2,3,4,5,6,7,8,9\}$. Let x be the number of 9-digit numbers formed using the digits of the set S such that only one digit is repeated and it is repeated exactly twice. Let y be the number of 9–digit numbers formed using the digits of the set S such that only two digits are repeated and each of these is repeated exactly twice. Then :

(1) $56x=9y$ (2) $29x=5y$ (3) $21x=4y$ (4) $45x=7y$

Ans. Official answer NTA (3)

Sol. $S=\{1,2,3,4,5,6,7,8,9\}$

$x={}^9C_8\cdot{}^8C_1\frac{9!}{2}=9\times4\times9!$

$y={}^9C_7\;{}^7C_2\times\frac{9!}{2!2!}=\frac{9\times8\times7\times6\times9!}{2\times2\times2\times2}$

$21x = 4y$

Question ID : 444792694

3. Let $y = y(x)$ be the solution of the differential equation $x\frac{dy}{dx} - \sin 2y = x^3\left(2 - x^3\right)\cos^2 y, x \neq 0$. If y(2)=0, then tan (y(1)) is equal to :

(1) $-\frac{3}{4}$ (2) $\frac{7}{4}$ (3) $-\frac{7}{4}$ (4) $\frac{3}{4}$

Ans. Official answer NTA (2)

Sol. $x\frac{dy}{dx} - \sin 2y = x^3\left(2 - x^3\right)\cos^2 y$

$$\frac{1}{\cos^2 y}\frac{dy}{dx} - \frac{1}{x}\frac{2\sin y\cos y}{\cos^2 y} = x^2\left(2 - x^3\right)$$

$$\sec^2 y\frac{dy}{dx} - \frac{1}{x}(2\tan y) = x^2\left(2 - x^3\right)$$

Let $\tan y = z$

$$\sec^2 y\frac{dy}{dx} = \frac{dz}{dx}$$

$$\frac{dz}{dx} - \frac{2}{x}z = x^2\left(2 - x^3\right)$$

$$\text{I.F.} = e^{-\int\frac{2}{x}dx} \Rightarrow e^{-2\ell n|x|} = \frac{1}{x^2}$$

$$\Rightarrow Z\left(\frac{1}{x^2}\right) = \int x^2\left(2 - x^3\right)\times\frac{1}{x^2}dx$$

$$\frac{Z}{x^2} = 2x - \frac{x^4}{4} + C$$

$$\frac{\tan y}{x^2} = 2x - \frac{x^4}{4} + C$$

$y(2) = 0$

$$0 = 4 - \frac{16}{4} + C$$

$C = 0$

$$\frac{\tan y}{x^2} = 2x - \frac{x^4}{4}$$

$\tan y = 2x^3 - \frac{x^6}{4}$

$y(1) = 3$

$x = 1$

$\tan(y(1)) = 2 - \frac{1}{4} = \frac{7}{4}$

Question ID : 444792680

4. Let A, B and C be three 2 × 2 matrices with real entries such that $B = (I + A)^{-1}$ and A + C = I. If $BC = \begin{bmatrix} 1 & -5 \\ -1 & 2 \end{bmatrix}$ and $CB\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 12 \\ -6 \end{bmatrix}$, then $x_1 + x_2$ is :

(1) –2 (2) 2 (3) 4 (4) 0

Ans. Official answer NTA (4)

Sol. $B = (I + A)^{-1}$

pre multiply by (I + A)

$(I + A)B = (I + A)(I + A)^{-1}$

$B + AB = I$ ....(1)

post multiply by (I + A)

$B(I + A) = (I + A)^{-1}(I + A)$

$B + BA = I$ ....(2)

from (1) and (2)

$AB = BA$

Now $A + C = I$

pre multiply by B

$BA + BC = B$ ....(3)

post multiply by B

$AB + CB = B$ ....(4)

from (3) and (4)

$BC = CB$

Now $BC = CB = \begin{bmatrix} 1 & -5 \\ -1 & 2 \end{bmatrix}$

$$CB\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 12 \\ -6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -5 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 12 \\ -6 \end{bmatrix}$$

$$\begin{bmatrix} x_1 - 5x_2 \\ -x_1 + 2x_2 \end{bmatrix} = \begin{bmatrix} 12 \\ -6 \end{bmatrix}$$

$x_1 - 5x_2 = 12$

$-x_1 + 2x_2 = -6$

$x_1 = -2,\ x_2 = 2$

Question ID : 444792690

5. The value of $\sum_{k=1}^{\infty}(-1)^{k+1}\left(\frac{k(k+1)}{k!}\right)$ is :

(1) 1/e (2) 2/e (3) $\sqrt{e}$ (4) e/2

Ans. Official answer NTA (1)

Sol. $$\sum_{k=1}^{\infty}(-1)^{k+1}\frac{k(k+1)}{k!}$$

$$\sum_{k=1}^{\infty}(-1)^{k+1}\frac{k+1}{(k-1)!}$$

$$\sum_{k=1}^{\infty}(-1)^{k+1}\frac{k-1}{(k-1)!}+\frac{2}{(k-1)!}$$

$$\sum_{k=2}^{\infty}(-1)^{k+1}\frac{1}{(k-2)!}+2\sum_{k=1}^{\infty}(-1)^{k+1}\frac{1}{(k-1)!}$$

$$\left(-\frac{1}{0!}+\frac{1}{1!}-\frac{1}{2!}+\frac{1}{3!}.....\infty\right)+2\left(\frac{1}{0!}-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}.....\right)$$

$$\Rightarrow\left(\frac{1}{0!}-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}.....\infty\right)=e^{-1}=\frac{1}{e}$$

Question ID : 444792682

6. A bag contains 10 balls out of which k are red and (10 –k) are black, where $0 \le k \le 10$. If three balls are drawn at random without replacement and all of them are found to be black, then the probability that the bag contains 1 red and 9 black balls is :

(1) $\frac{14}{55}$ (2) $\frac{7}{55}$ (3) $\frac{7}{110}$ (4) $\frac{7}{11}$

Ans. Official answer NTA (1)

Sol. $10 \begin{cases} \nearrow k \text{ Red} \\ \searrow 10-k \text{ Black} \end{cases}$

$$= \frac{\dfrac{{}^{9}C_3}{{}^{10}C_3}}{\dfrac{{}^{3}C_3 + {}^{4}C_3 + {}^{5}C_3 + \cdots {}^{9}C_3 + {}^{10}C_3}{{}^{10}C_3}}$$

$$\Rightarrow \frac{{}^{9}C_3}{{}^{3}C_3 + {}^{4}C_3 + {}^{5}C_3 \ldots\ldots {}^{10}C_3}$$

$$\Rightarrow \frac{{}^{9}C_3}{{}^{11}C_4} = \frac{\dfrac{9!}{3!6!}}{\dfrac{11!}{4!7!}} = \frac{14}{55}$$

Question ID : 444792679

7. Let z be a complex number such that $|z-6| = 5$ and $|z+2-6i| = 5$. Then the value of $z^3 + 3z^2 - 15z + 141$ is equal to :

(1) 50 (2) 42 (3) 37 (4) 61

Ans. Official answer NTA (1)

Sol. $|z-6| = 5$ $\quad |z-(-2+6i)| = 5$

Circle $\quad$ Circle

$C_1(6, 0)$ $\quad$ $C_2(-2, 6)$

$r_1 = 5$ $\quad$ $r_2 = 5$

$C_1C_2 = \sqrt{8^2 + 6^2} = 10$

$r_1 + r_2 = 10$

$C_1C_2 = r_1 + r_2$ so both circle wall touch externally

5 : 5

$C_1(6, 0)$ —— Z —— $C_2(-2, 6)$

$Z \equiv (2, 3)$

Now $z^3 + 3z^2 - 15z + 141$

$z = 2 + 3i = \alpha$

$\beta = 2 - 3i$

$\alpha + \beta = 4 \qquad \alpha\beta = 13$

$z^2 - 4z + 13 = 0$

$$\begin{array}{r|l} z^2 - 4z + 13 \;) & z^3 + 3z^2 - 15z + 141 \; (\; z + 7 \\ & \underline{z^3 - 4z^2 + 13z} \\ & \quad - \quad + \quad - \\ & 7z^2 - 28z + 141 \\ & \underline{7z^2 - 28z + 91} \\ & \quad - \quad + \quad - \\ & 50 \end{array}$$

uestion ID : 444792677

8. If $g(x) = 3x^2 + 2x - 3, f(0) = -3$ and $4g(f(x)) = 3x^2 - 32x + 72$, then f(g(2)) is equal to :

(1) $-\frac{7}{2}$ (2) $\frac{7}{2}$ (3) $-\frac{25}{6}$ (4) $\frac{25}{6}$

Ans. Official answer NTA (2)

Sol. $g(x) = 3x^2 + 2x - 3$

$f(0) = -3$

$4g(f(x)) = 3x^2 - 32x + 72$

$4\{3(f(x))^2 + 2f(x) - 3\} = 3x^2 - 32x + 72$

Let f(x) = y

$12y^2 + 8y - 12 - 3x^2 + 32x - 72 = 0$

$12y^2 + 8y + (-3x^2 + 32x - 84) = 0$

$$y = \frac{-8 \pm \sqrt{64 - 48(-3x^2 + 32x - 84)}}{24}$$

$$f(x) = \frac{-8 \pm 4(3x - 16)}{24}$$

$\because f(0) = -3$

$\therefore$ we will take + sign

$$f(x) = \frac{-8 + 4(3x - 16)}{24}$$

Now f(g(x))

$$f(13) = \frac{-8 + 4(3 \times 13 - 16)}{24} = \frac{7}{2}$$

Question ID : 444792689

9. For three unit vectors $\vec{a}, \vec{b}, \vec{c}$ satisfying $|\vec{a}-\vec{b}|^2+|\vec{b}-\vec{c}|^2+|\vec{c}-\vec{a}|^2=9$ and $|2\vec{a}+k\vec{b}+k\vec{c}|=3$, the positive value of k is :

(1) 6 (2) 4 (3) 5 (4) 3

Ans. Official answer NTA (3)

Sol. $\left|\vec{a}+\vec{b}\right|^2+\left|\vec{b}-\vec{c}\right|^2+\left|\vec{c}-\vec{a}\right|^2=9$

$2\left(a^2+b^2+c^2\right)-2\left(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}\right)=9$

$2\times 3-2\left(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}\right)=9$

$3+2\left(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}\right)=0$

$\left(\vec{a}+\vec{b}+\vec{c}\right)^2=0$

So $\vec{a}+\vec{b}+\vec{c}=0$

$\vec{b}+\vec{c}=-\vec{a}$

Now $\left|2\vec{a}+k(\vec{b}+\vec{c})\right|=3$

$\left|(2-k)\vec{a}\right|=3$

$\left|2-k\right|=3$

$2-k=\pm 3$

$k=-1, k=5$

Question ID : 444792687

10. If $\dfrac{\tan(A-B)}{\tan A}+\dfrac{\sin^2 C}{\sin^2 A}=1, A, B, C \in \left(0, \dfrac{\pi}{2}\right)$, then :

(1) tanA, tanC, tanB, are in A.P.

(2) tanA, tanC, tanB, are in G.P.

(3) tanA, tanB, tanC, are in G.P.

(4) tanA, tanB, tanC, are in A.P.

Ans. Official answer NTA (2)

Sol. $\frac{\tan(A-B)}{\tan A}+\frac{\sin^2 C}{\sin^2 A}=1$

$\frac{\tan A-\tan B}{(1+\tan A\tan B)\tan A}+\frac{\operatorname{cosec}^2 A}{\operatorname{cosec}^2 C}=1$

$\frac{\tan A-\tan B}{(1+\tan A\tan B)\tan A}+\frac{1+\cot^2 A}{1+\cot\ C}=1$

Let $\tan A = x$
$\tan B = y$
$\tan C = z$

$\frac{x-y}{(1+xy)x}+\frac{1+\frac{1}{x^2}}{1+\frac{1}{z^2}}=1$

$\frac{x-y}{x(1+xy)}+\frac{z^2\left(x^2+1\right)}{x^2\left(z^2+1\right)}=1$

after solving we will get

$Z^2 = xy$

So $\tan^2 C = \tan A.\ \tan B$

tanA, tanC, tanB are in G.P.

Question ID : 444792681

11. The common difference of the A.P.: $a_1, a_2, \ldots, a_m$ is 13 more than the common difference of the A.P.: $b_1, b_2, \ldots, b_n$. If $b_{31} = -277, b_{43} = -385$ and $a_{78} = 327$, then a_1 is equal to :

(1) 24 (2) 21 (3) 16 (4) 19

Ans. Official answer NTA (4)

Sol. $a_1, a_2, \ldots, a_m \Rightarrow$ Let common difference = d + 13

$b_1, b_2, \ldots, b_n \Rightarrow$ common difference = d

$b_{31} = -277 = b_1 + 30d$
$b_{43} = -385 = b_1 + 42d$
$\quad + \quad - \quad -$

$108 = -12d$

$d = -9$

$a_{78} \Rightarrow a_1 + 77(d + 13) = 327$

$a_1 + 77 \times 4 = 327$

$a_1 = 327 - 308$

$= 19$

Question ID : 444792692

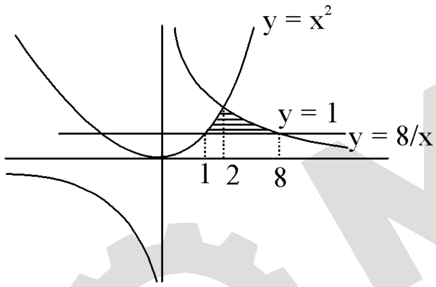
12. The area of the region $R = \{(x, y) : xy \le 8, 1 \le y \le x^2, x \ge 0\}$ is :

(1) $\frac{2}{3}(24\log_e(2) - 7)$ (2) $\frac{1}{3}(40\log_e(2) + 27)$

(3) $\frac{1}{3}(49\log_e(2) - 15)$ (4) $\frac{2}{3}(20\log_e(2) + 9)$

Ans. Official answer NTA (1)

Sol. $xy \le 8, 1 \le y \le x^2, x \ge 0$



$$A = \int_1^2 (x^2 - 1)dx + \int_2^8 \left(\frac{8}{x} - 1\right)dx$$

$$A = \left[\frac{x^3}{3} - x\right]_1^2 + [8\, ln\, x - x]_2^8$$

$$A = \frac{8}{3} - 2 - \frac{1}{3} + 1 + 8\, ln\, 8 - 8\, ln\, 2 + 2$$

$$A = 16\, ln\, 2 - \frac{14}{3}$$

$$= \frac{2}{3}(24\, ln\, 2 - 7)$$

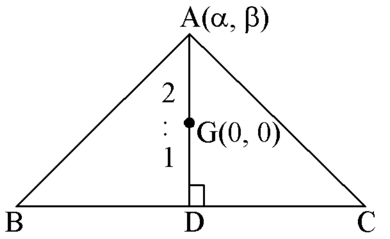
Question ID : 444792685

13. Let ABC be an equilateral triangle with orthocenter at the origin and the side BC on the line $x+2\sqrt{2}y=4$. If the co-ordinates of the vertex A are (α, β), then the greatest integer less than or equal to $\left|\alpha+\sqrt{2}\beta\right|$ is :

(1) 5 (2) 2 (3) 3 (4) 4

Ans. Official answer NTA (4)

Sol.



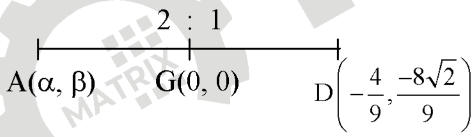
$$x+2\sqrt{2}y+4=0$$

$$=\frac{x-0}{1}=\frac{y-0}{2\sqrt{2}}=\frac{-1(0+0+4)}{9}$$

$$\frac{x}{1}=\frac{y}{2\sqrt{2}}=\frac{-4}{9}$$

$$x=\frac{-4}{9}, y=\frac{-8\sqrt{2}}{9}$$

Now



$$\frac{\alpha-\frac{8}{9}}{3}=0, \frac{\beta-\frac{16\sqrt{2}}{9}}{3}=0$$

$$\alpha=\frac{8}{9} \quad , \quad \beta=\frac{16\sqrt{2}}{9}$$

Now $\left|\alpha+\sqrt{2}\beta\right|$

$$\left|\frac{8}{9}+\frac{32}{9}\right|=\left|\frac{40}{9}\right|=4.4$$

$[4.4]=4$

Question ID : 444792695

14. If $\int\left(\frac{1-5\cos^2 x}{\sin^5 x\cos^2 x}\right)dx = f(x)+C$, where C is the constant of integration, then $f\left(\frac{\pi}{6}\right)-f\left(\frac{\pi}{4}\right)$ is equal to :

(1) $\frac{4}{\sqrt{3}}(8-\sqrt{6})$ (2) $\frac{1}{\sqrt{3}}(26-\sqrt{3})$ (3) $\frac{1}{\sqrt{3}}(26+\sqrt{3})$ (4) $\frac{2}{\sqrt{3}}(4+\sqrt{6})$

Ans. Official answer NTA (1)

Sol. $\int\frac{1-5\cos^2 x}{8m^5 x\cos^2 x}dx = f(x)+c$

$\int\frac{\sec^2 x}{\sin^5 x}-\int\frac{5}{\sin^5 x}dx$

Integration by parts

$\int\underbrace{\sec^2 x}_{II}\underbrace{\frac{1}{\sin 5x}}_{I}dx-\int\frac{5}{\sin^5 x}dx$

$\frac{1}{\sin^5 x}\tan x-\int\frac{-1}{(\sin^5 x)^2}\times 5\sin^4 x\cos x\tan x dx-\int\frac{5}{\sin^5 x}dx+C$

$\frac{\tan x}{\sin^5 x}+\int\frac{5}{\sin^5 x}dx-\int\frac{5}{\sin^5 x}dx+C$

So $f(x)=\frac{\tan x}{\sin 5x}$

Now $f\left(\frac{\pi}{6}\right)-f\left(\frac{\pi}{4}\right)$

$\Rightarrow\frac{\tan\pi/6}{(\sin\pi/6)^5}-\frac{\tan\pi/4}{(\sin\pi/4)^5}$

$\Rightarrow\frac{1}{\sqrt{3}\times\left(\frac{1}{2}\right)^5}-\frac{1}{\left(\frac{1}{\sqrt{2}}\right)^5}$

$\frac{32}{\sqrt{3}}-4\sqrt{2}$

$\frac{4}{\sqrt{3}}\left(8-\sqrt{6}\right)$

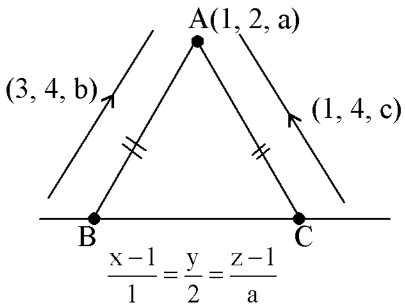
Question ID : 444792688

15. If the distances of the point (1,2, a) from the line $\frac{x-1}{1}=\frac{y}{2}=\frac{z-1}{1}$ along the lines $L_1 : \frac{x-1}{3}=\frac{y-2}{4}=\frac{z-a}{b}$ and $L_2 : \frac{x-1}{1}=\frac{y-2}{4}=\frac{z-a}{c}$ are equal, then a + b + c is equal to :

(1) 4 (2) 6 (3) 5 (4) 7

Ans. Official answer NTA (4)

Sol.



Let equation of line AB $\vec{r}=(1,2,a)+\lambda(3,4,b)$

So $B(1+3\lambda, 2+4\lambda, a+\lambda b)$

lies on line BC So

$$\frac{1+3\lambda-1}{1}=\frac{2+4\lambda}{2}=\frac{a+\lambda b-1}{a}$$

So $\lambda = 1$ and $a + b - 1 = 3$

equation line AC $\vec{r}=(1,2,a)+\mu(1,4,c)$

So $C(1+\mu, 2+4\mu, a+\mu c)$

Lies on line BC so

$$\frac{1+\mu-1}{1}=\frac{2+4\mu}{2}=\frac{a+\mu c-1}{a}$$

$\mu = -1$ $\quad$ $a - c - 1 = -1$

$a = c$

B(4, 6, 4) $\quad$ C(0, –2, 0)

$AB^2 = AC^2$

$9 + 16 + (a-4)^2 = 1 + 16 + a^2$

a = 3, c = 3, b = 1

a + b + c = 7

Question ID : 444792693

16. The value of $\lim\limits_{x\to 0}\dfrac{\log_e\left(\sec(ex)\cdot\sec\left(e^2x\right)\cdot\ldots\cdot\sec\left(e^{10}x\right)\right)}{e^2-e^{2\cos x}}$ is equal to :

(1) $\dfrac{\left(e^{20}-1\right)}{2\left(e^2-1\right)}$ (2) $\dfrac{\left(e^{10}-1\right)}{2\left(e^2-1\right)}$ (3) $\dfrac{\left(e^{10}-1\right)}{2e^2\left(e^2-1\right)}$ (4) $\dfrac{\left(e^{20}-1\right)}{2e^2\left(e^2-1\right)}$

Ans. Official answer NTA (1)

Sol. $\lim\limits_{x\to 0}\dfrac{\log_e\left(\sec(ex)\sec\left(e^2x\right)\ldots\sec e^{10}x\right)}{e^2-e^{2\cos x}}$

$$\lim_{x\to 0}\frac{\ell n(\sec ex)+\ell n\left(\sec\left(e^2x\right)+\cdots\ell n\left(\sec\left(e^{10}x\right)\right.\right.}{e^{2\cos x}\left(e^{2-2\cos x}-1\right)}s$$

$$\lim_{x\to 0}\frac{\ell n(\sec x)+\ell n\left(\sec e^2x\right)+\cdots\ell n\left(\sec e^{10}x\right)}{e^2\times(2-2\cos x)}$$

$$\lim_{x\to 0}\frac{-\ell n(\cos x)}{e^2(2-2\cos x)}-\frac{\ell n\left(\cos e^2x\right)}{e^2(2-2\cos x)}\cdots\frac{-\ell n\left(\cos e^{10}x\right)}{e^2(2-2\cos x)}$$

$$\lim_{x\to 0}\frac{-(\cos x-1)}{e^2(2(1-\cos x))}-\frac{\left(\cos^2 x-1\right)}{e^2(2(1-\cos x))}\cdots\cdot\frac{-\left(\cos e^{10}x-1\right)}{e^2(2(1-\cos x))}$$

$$\left(\lim_{x\to 0}+\frac{(ex)^2}{2\times 2\left(\frac{x^2}{2}\right)}+\frac{\left(e^2x\right)^2}{2\times 2\left(\frac{x^2}{2}\right)}+\cdots\frac{\left(e^{10}x\right)^2}{2\times 2\left(\frac{x^2}{2}\right)}\right)$$

$$\frac{1}{e^2}\left(\frac{e^2}{2}+\frac{e^4}{2}+\cdots-\frac{e^{20}}{2}\right)$$

$$\frac{1}{2}\frac{1}{e^2}\frac{e^2\left(e^{20}-1\right)}{e^2-1}=\frac{e^{20}-1}{2\left(e^2-1\right)}$$

Question ID : 444792683

17. The mean and variance of 10 observations are 9 and 34.2 , respectively. If 8 of these observations are 2,3,5,10,11,13,15,21, then the mean deviation about the median of all the 10 observations is :

(1) 6 (2) 7 (3) 4 (4) 5

Ans. Official answer NTA (4)

Sol. 2,3,5,10,11,13,15,21, x, y

$$\bar{x} = \frac{2+3+5+10+11+13+15+21+x+y}{10} = 9$$

$x + y = 10$

$$\sigma^2 = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2 = \frac{342}{10}$$

$$\frac{2^2+3^2+5^2+x^2+y^2}{10} - (9)^2 = \frac{342}{10}$$

$$\frac{1094+x^2+y^2}{10} = \frac{342+810}{10}$$

$x^2 + y^2 = 58$

So x = 7 and y = 3

Now data $\Rightarrow$ 2, 3, 3, 5, 7, 10, 11, 13, 15, 21

$$\text{Median} = \frac{7+10}{2} = 8.5$$

$$\text{Mean Deviation} = \frac{|8.5-2|+|8.5-3|+\cdots+|8.5-21|}{10} = 5$$

Question ID : 444792678

18. If α, β, where $\alpha < \beta$, are the roots of the equation $\lambda x^2 - (\lambda+3)x + 3 = 0$ such that $\frac{1}{\alpha} - \frac{1}{\beta} = \frac{1}{3}$, then the sum of all possible values of λ is :

(1) 4 (2) 2 (3) 6 (4) 8

Ans. Official answer NTA (3)

Sol. $\lambda x^2 - (\lambda+3)x + 3 = 0$

$$\alpha+\beta = \frac{\lambda+3}{\lambda} \quad \alpha\beta = \frac{3}{\lambda}$$

$$\frac{1}{\alpha}-\frac{1}{\beta}=\frac{1}{3}$$

$$\frac{\beta-\alpha}{\alpha\beta}=\frac{1}{3}$$

$$\frac{\sqrt{(\lambda+3)^2-12\lambda}}{|\lambda|\times\frac{3}{\lambda}}=\frac{1}{3}$$

Sq. $(\lambda+3)^2-12\lambda=1$

$\lambda^2+9+6\lambda-12\lambda=1$

$\lambda^2-6\lambda+8=0$

sum $=6$

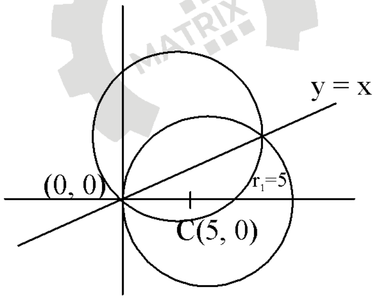
Question ID : 444792686

19. Let $y = x$ be the equation of a chord of the circle C_1 (in the closed half-plane $x \ge 0$) of diameter 10 passing through the origin. Let C_2 be another circle described on the given chord as its diameter. If the equation of the chord of the circle C_2, which passes through the point (2, 3) and is farthest from the center of C_2, is $x + ay + b = 0$, then $a - b$ is equal to :

(1) –6 (2) –2 (3) 6 (4) 10

Ans. Official answer NTA (2)

Sol.



$C_1 : (x-5)^2+y^2=25$

$C_1 : x^2+y^2-10x=0$

$y = x$

$2x^2 - 10x = 0$

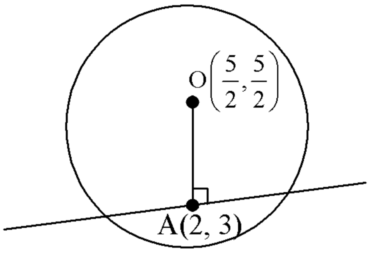
$2x(x - 5) = 0$

$x = 0$ OR $x = 5$

(0, 0) and (5, 5)

$C_2 \equiv (x - 0)(x - 5) + (y - 0)(y - 5) = 0$

$x^2 + y^2 - 5x - 5y = 0$



$$M_{OA} = \frac{5/2-3}{\frac{5}{2}-2} \Rightarrow \frac{-1/2}{1/2} = -1$$

So slope of chord = 1

equation y – 3 = 1(x – 2)

x – y + 1 = 0

a = – 1 b = 1

a – b = –2

Question ID : 444792676

20. Let $S = \{x^3 + ax^2 + bx + c : a, b, c \in N \text{ and } a, b, c \le 20\}$ be a set of polynomials. Then the number of polynomials in S, which are divisible by $x^2 + 2$, is :

(1) 10 (2) 120 (3) 20 (4) 6

Ans. Official answer NTA (1)

Sol. $x^3 + ax^2 + bx + c$ is divisible by $x^2 + 2$

So $x^3 + ax^2 + bx + 2 = x(x^2 + 2) + a(x^2 + 2) + (b – 2)x + (c – 2a)$ is divisible by $x^2 + 2$

So b – 2 = 0 and c – 2a = 0

b = 2 and c = 2a

a	b	c
1	2	2
2	2	4
3	2	6
⋮	⋮	⋮
10	2	20

So total polynomials = 10

SECTION – B

Question ID : 444792696

21. In a G.P., if the product of the first three terms is 27 and the set of all possible values for the sum of its first three terms is $\mathbb{R}-(a,b)$, then a^2+b^2 is equal to _________.

Ans. Official answer NTA (90)

Sol. Let 3 term be $\frac{a}{r}, a, ar$

$\frac{a}{r}\times a\times ar=27$

$a^3 = 27$

$a = 3$

$\text{Sum} = \frac{3}{r}+3+3r$

$\Rightarrow 3\left(r+\frac{1}{r}+1\right)$

Range of $r+\frac{1}{R}\in(-\infty,-2]\cup[2,\infty)$

$r+\frac{1}{r}+1\in(-\infty,-1]\cup(3,\infty)$

$3\left(r+\frac{1}{r}+1\right)\in(-\infty,-3]\cup[9,\infty)$

so $3\left(r+\frac{1}{r}+1\right)\in R-(-3,9)$

$a = -3$

$b = 9$

$a^2 + b^2 = 9 + 81 = 90$

Question ID : 444792700

22. For some $\theta \in \left(0, \frac{\pi}{2}\right)$, let the eccentricity and the length of the latus rectum of the hyperbola $x^2 - y^2 \sec^2\theta = 8$ be e_1 and l_1, respectively, and let the eccentricity and the length of the latus rectum of the ellipse $x^2\sec^2\theta + y^2 = 6$ be e_2 and l_2, respectively. If $e_1^2 = e_2^2\left(\sec^2\theta + 1\right)$, then $\left(\frac{l_1 l_2}{e_1 e_2}\right)\tan^2\theta$ is equal to ________.

Ans. Official answer NTA (8)

Sol. $x^2 - y^2\sec^2\theta = 8$

$$\frac{x^2}{8} - \frac{y^2}{8\cos^2\theta} = 1$$

$$e_1 = \sqrt{1 + \frac{8\cos^2\theta}{8}} = \sqrt{1+\cos^2\theta}$$

$$\ell_1 = \frac{2b^2}{a} = \frac{2\times 8\cos^2\theta}{2\sqrt{2}} = 4\sqrt{2}\cos^2\theta$$

$$\frac{x^2}{6\cos^2\theta} + \frac{y^2}{6} = 1$$

$$e_2 = \sqrt{1 - \frac{6\cos^2\theta}{6}} = \sqrt{1-\cos^2\theta}$$

$$\ell_2 = \frac{2\times 6\cos^2\theta}{\sqrt{6}} = 2\sqrt{6}\cos^2\theta$$

Now $e_1^2 = e_2^2\left(\sec^2\theta + 1\right)$

$$1+\cos^2\theta = \left(1-\cos^2\theta\right)\left(\frac{1+\cos^2\theta}{\cos^2\theta}\right)$$

$$\cos^2\theta = 1 - \cos^2\theta$$

$$\cos^2\theta = \frac{1}{2}$$

So, $e_1 = \sqrt{\frac{3}{2}}, \ell_1 = 2\sqrt{2}, e_2 = \sqrt{\frac{1}{2}}, \ell_2 = \sqrt{6}$

$$\Rightarrow \left(\frac{\ell_1\ell_2}{e_1e_2}\right)\tan^2\theta$$

$$\Rightarrow \frac{2\sqrt{2}\times\sqrt{6}}{\sqrt{\frac{3}{2}}\times\sqrt{\frac{1}{2}}}\times 1$$

$$\Rightarrow 4 \times 2 = 8$$

Question ID : 444792699

23. The value of $\sum_{r=1}^{20}\left(\left|\sqrt{\pi\left(\int_0^r x\,|\sin \pi x|\,dx\right)}\right|\right)$ is ________.

Ans. Official answer NTA (210)

Sol. $\sum_{r=1}^{20}\sqrt{\pi\int_0^r x\,|\sin \pi x|dx}$

Let $I_r = \pi\int_0^r x\,|\sin \pi x|\,dx$ ....(1)

Apply king rule

$Ir = \pi\int_0^r (r-x)\,|\sin \pi x|\,dx$ ....(2)

(1) + (2)

$2I_r = \pi\int_0^r r\,|\sin \pi x|\,dx$

$I_r = \frac{\pi r}{2}\int_0^r |\sin \pi x|\,dx$

Now r = 1

$I_1 = \frac{\pi}{2}\int_0^1 |\sin \pi x|\,dx$

Let $\pi x = t$

$\pi dx = dt$

$I_1 = \frac{1}{2}\int_0^{\pi} |\sin t|\,dt$

$I_1 = \frac{1}{2}\times 2 = 1 = 1^2$

$I_2 = \frac{2\pi}{2}\int_0^2 |\sin \pi x|\,dx$

$\pi x = t$

$\pi dx = dt$

$I_2 = \int_0^{2\pi} |\sin t| dt$

$I_2 = 4 = 2^2$

$I_3 = 3^2$

So $\sum_{r=1}^{20} \left|\sqrt{I_r}\right|$

$= \sqrt{I_1} + \sqrt{I_2} \ldots\ldots\sqrt{I_{20}}$

$= 1 + 2 + \ldots.20$

$= \frac{20 \times 2}{2} = 210$

Question ID : 444792697

24. If $k = \tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\left(\frac{2}{3}\right)\right) + \tan\left(\frac{1}{2}\sin^{-1}\left(\frac{2}{3}\right)\right)$, then the number of solutions of the equation $\sin^{-1}(kx - 1) = \sin^{-1} x - \cos^{-1} x$ is ________.

Ans. Official answer NTA (1)

Sol. $k = \tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\left(\frac{2}{3}\right)\right) + \tan\left(\frac{1}{2}\sin^{-1}\frac{2}{3}\right)$

$= \tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{2}{3}\right) + \tan\left(\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{2}{3}\right)$

Let $\frac{1}{2}\cos^{-1}\frac{2}{3} = \theta$

$\cos 2\theta = \frac{2}{3}$

$\Rightarrow \tan\left(\frac{\pi}{4} + \theta\right) + \tan\left(\frac{\pi}{4} - \theta\right)$

$\frac{1 + \tan\theta}{1 - \tan\theta} + \frac{1 - \tan\theta}{1 + \tan\theta}$

$k = 2\left(\frac{1 + \tan^2\theta}{1 - \tan^2\theta}\right) = \frac{2}{\cos 2\theta} = \frac{2}{2/3} = 3$

$\sin^{-1}(3x - 1) = \sin^{-1} x - \cos^{-1} x$

$\sin^{-1}(3x-1) = 2\sin^{-1}x - \frac{\pi}{2}$

taking sin both side

$\sin\left(\sin^{-1}(3x-1)\right) = \sin\left(2\sin^{-1}x - \frac{\pi}{2}\right)$

$3^{x-1} = -\cos(2\sin^{-1}x)$

Let $\sin^{-1}x = \theta$

$x = \sin\theta$

$3^{x-1} = -\cos 2\theta$

$3^{x-1} = -(1 - 2\sin^2\theta)$

$3^{x-1} = -1 + 2x^2$

$2x^2 - 3x = 0$

$x(2x-3) = 0$

$x = 0$ OR $x = 3/2$

Only 1 solution

Question ID : 444792698

25. Let PQR be a triangle such that $\overrightarrow{PQ} = -2\hat{i} - \hat{j} + 2\hat{k}$ and $\overrightarrow{PR} = a\hat{i} + b\hat{j} - 4\hat{k}, a, b \in Z$. Let S be the point on QR, which is equidistant from the lines PQ and PR. If $|\overrightarrow{PR}| = 9$ and $\overrightarrow{PS} = \hat{i} - 7\hat{j} + 2\hat{k}$, then the value of $3a - 4b$ is ________.

Ans. Official answer NTA (37)

Sol. PS is internal angle bisector of PQ and PR

$$\lambda\left(\frac{\overrightarrow{PQ} \cdot \overrightarrow{PR}}{|\overrightarrow{PQ}||\overrightarrow{PR}|}\right) = \overrightarrow{PS}$$

$a = 7,\qquad b = -4$

But with these values

$\overrightarrow{QS} = \overrightarrow{PS} - \overrightarrow{PQ} = (3, -6, 0)$

$\overrightarrow{SR} = \overrightarrow{PR} - \overrightarrow{PS} = (6, -3, -6)$

$\overrightarrow{QS}$ and $\overrightarrow{SR}$ are not collinear vectors which contradicts the information given in question