JEE Main January 2026

Question Paper With Text Solution

24 January | Shift-2

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN JANUARY 2026 | 24TH JANUARY SHIFT-2

SECTION – A

Question ID : 444792604

1. Let $P = [p_{ij}]$ and $Q = [q_{ij}]$ be two square matrices of order 3 such that $q_{ij} = 2^{(i+j-1)}p_{ij}$ and $\det(Q) = 2^{10}$. Then the value of det(adj(adj P)) is :

(1) 16 (2) 124 (3) 81 (4) 32

Ans. Official answer NTA (1)

Sol. $P = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix}$

$Q = \begin{bmatrix} 2p_{11} & 2^2p_{12} & 2^3p_{13} \\ 2^2p_{21} & 2^3p_{22} & 2^4p_{23} \\ 2^3p_{31} & 2^4p_{32} & 2^5p_{32} \end{bmatrix}$

$|Q| = 2^9|P|$

$2^{10} = 2^9|8|$

$|P| = 2$

$|\text{adj (adj P)}| = |P|^4 = 2^4 = 16$

Question ID : 444792618

2. Let $f(x) = \int \frac{7x^{10} + 9x^8}{(1 + x^2 + 2x^9)^2}dx, x > 0, \lim_{x \to 0} f(x) = 0$ and $f(1) = \frac{1}{4}$. If $A = \begin{bmatrix} 0 & 0 & 1 \\ \frac{1}{4} & f'(1) & 1 \\ \alpha^2 & 4 & 1 \end{bmatrix}$ and

$B = \text{adj(adj A)}$ be such that $|B| = 81$, then α^2 is equal to :

(1) 2 (2) 1 (3) 3 (4) 4

Ans. Official answer NTA (4)

Sol. $f(x) = \int \frac{7x^{10} + 9x^8}{(1 + x^2 + 2x^9)^2}dx$

$$f(x)=\int \frac{7x^{-8}+9x^{-10}}{\left(\frac{1}{x^9}+\frac{1}{x^7}+2\right)^2}dx$$

$$\frac{1}{x^9}+\frac{1}{x^7}+2=t$$

$$-\left(9x^{-10}+7x^{-8}\right)dx=dt$$

$$\int \frac{-dt}{t^2}=\frac{1}{t}+c$$

$$f(x)=\frac{x^9}{1+x^2+2x^9}+C$$

$$\lim_{x\to 0} f(x)=0 \quad c=0$$

$$f(x)=\frac{x^9}{1+x^2+2x^9}$$

$f'(1)=1$

B = adj (adjA)

$|B| = |A|^4$

$81 = |A|^4$

$|A| = \pm 3$

$1 - \alpha^2 f'(1) = \pm 3$

$1 - \alpha^2 = \pm 3$

$\alpha^2 = 4$

Question ID : 444792606

3. $\left(\frac{1}{3}+\frac{4}{7}\right)+\left(\frac{1}{3^2}+\frac{1}{3}\times\frac{4}{7}+\frac{4^2}{7^2}\right)+\left(\frac{1}{3^3}+\frac{1}{3^2}\times\frac{4}{7}+\frac{1}{3}\times\frac{4^2}{7^2}+\frac{4^3}{7^3}\right)+\ldots$ upto infinite terms, is equal to :

(1) $\frac{7}{4}$ (2) $\frac{6}{5}$ (3) $\frac{5}{2}$ (4) $\frac{4}{3}$

Ans. Official answer NTA (3)

Sol. $\left(\frac{1}{3}+\frac{4}{7}\right)+\left(\frac{1}{3^2}+\frac{1}{3}\cdot\frac{4}{7}+\frac{4^2}{7^2}\right)+\cdots\cdots\infty$

$$(a+b)+\left(a^2+ab+b^2\right)+\left(a^3+a^2b+ab^2+b^3\right)+\ldots\infty$$

$$\frac{(a^2-b^2)+(a^3-b^3)+(a^4-b^4)+\ldots+\infty}{a-b}$$

$$\frac{\left(\frac{a^2}{1-a}\right)-\left(\frac{b^2}{1-b}\right)}{a-b}$$

$$\frac{a^2(1-b)-b^2(1-a)}{(a-b)(1-a)(1-b)}$$

$$\frac{a+b-ab}{1-a-b+ab}$$

$$\frac{\frac{1}{3}+\frac{4}{7}-\frac{4}{21}}{1-\frac{1}{3}-\frac{4}{7}+\frac{4}{21}}=\frac{5}{2}$$

Question ID : 444792605

4. Let a_1, a_2, a_3, a_4 be an A.P. of four terms such that each term of the A.P. and its common difference l are integers. If $a_1 + a_2 + a_3 + a_4 = 48$ and $a_1\, a_2\, a_3\, a_4 + l^4 = 361$, then the largest term of the A.P. is equal to :

(1) 21 (2) 23 (3) 24 (4) 27

Ans. Official answer NTA (4)

Sol. $a - 3d + a - d + a + d + a + 3d = 48$

$4a = 48$

$a = 12$

$l = 2d$

$(144 - 9d^2)(144 - d^2) + 16d^4 = 361$

$(144)^2 - 1440\, d^2 + 25d^4 = 361$

$d^2 = 25 \Rightarrow d = \pm 5$

Largest term = 27

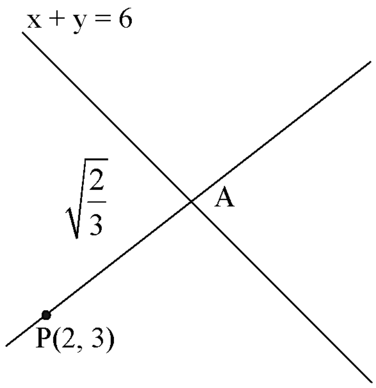
Question ID : 444792612

5. Let the angles made with the positive x-axis by two straight lines drawn from the point P(2, 3) and meeting the line x + y = 6 at a distance $\sqrt{\frac{2}{3}}$ from the point P be θ_1 and θ_2. Then the value of $(\theta_1 + \theta_2)$ is :

(1) $\frac{\pi}{3}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{2}$ (4) $\frac{\pi}{12}$

Ans. Official answer NTA (3)

Sol.



$$A\left(2 \pm \sqrt{\frac{2}{3}}\cos\theta, 3 \pm \sqrt{\frac{2}{3}}\sin\theta\right)$$

$$x + y = 6$$

$$5 \pm \sqrt{\frac{2}{3}}(\cos\theta + \sin\theta) = 6$$

$$\sin\theta + \cos\theta = \pm\sqrt{\frac{3}{2}}$$

$$1 + \sin 2\theta = \frac{3}{2}$$

$$\frac{2\tan\theta}{1 + \tan^2\theta} = \frac{1}{2}$$

$$\tan^2\theta - 4\tan\theta + 1 = 0 \quad \begin{cases} \tan\theta_1 \\ \tan\theta_2 \end{cases}$$

$$\tan(\theta_1 + \theta_2) = \frac{\tan\theta_1 + \tan\theta_2}{1 - \tan\theta_1 \cdot \tan\theta_2}$$

$$\theta_1 + \theta_2 = \tan^{-1}(\infty) = \frac{\pi}{2}$$

Question ID : 444792617

6. Consider the following three statements for the function $f : (0, \infty) \rightarrow R$ definde by $f(x) = |\log_e x| - |x - 1|$

(I) f is differentiable at all $x > 0$.

(II) f is increasing in (0, 1).

(III) f is decreasing in $(1, \infty)$ then :

(1) All (I), (II) and (III) are TRUE
(2) Only (I) is TRUE
(3) Only (II) and (III) are TRUE
(4) Only (I) and (III) are TRUE

Ans. Official answer NTA (4)

Sol. $f : (0, \infty) \rightarrow R$

$f(x) = |\ln x| - |x - 1|$

$$f(x) = \begin{cases} -\ln x + x - 1 & 0 < x < 1 \\ \ln x - x + 1 & x \geq 1 \end{cases}$$

$$f'(x) = \begin{cases} 1 - \dfrac{1}{x} & 0 < x < 1 \\ \dfrac{1}{x} - 1 & x \geq 1 \end{cases}$$

$f'(1^+) = f'(1^-) = 0$ Diff at all $x > 0$

in $x \in (0,1) \Rightarrow f'(x) = 1 - \dfrac{1}{x}$

$f'(x) < 0 \Rightarrow f(x)$ is decreasing

in $x \in (1, \infty) \Rightarrow f'(x) = \dfrac{1}{x} - 1$

$f'(x) < 0 \Rightarrow f(x)$ is decreasing

Question ID : 444792601

7. If the domain of the function $f(x) = \sin^{-1}\left(\dfrac{1}{x^2 - 2x - 2}\right)$, is $(-\infty, \alpha] \cup [\beta, \gamma] \cup [\delta, \infty)$ is equal to $\alpha + \beta + \gamma + \delta$ is equal to :

(1) 3
(2) 5
(3) 2
(4) 4

Ans. Official answer NTA (4)

Sol. $f(x)=\sin^{-1}\left(\frac{1}{x^2-2x-2}\right)$

$-1\le\frac{1}{x^2-2x-2}\le 1$

$\frac{x^2-2x-1}{x^2-2x-2}\ge 0$

$\frac{\left(x-1+\sqrt{2}\right)\left(x-1-\sqrt{2}\right)}{\left(x-1+\sqrt{3}\right)\left(x-1-\sqrt{3}\right)}\ge 0$

$x\in\left(-\infty,1-\sqrt{3}\right)\cup\left[1-\sqrt{2},1+\sqrt{2}\right]\cup\left(1+\sqrt{3},\infty\right)$ ....(1)

$\frac{x^2-2x-3}{x^2-2x-2}\ge 0$

$\frac{(x-3)(x+1)}{\left(x-1+\sqrt{3}\right)\left(x-1-\sqrt{3}\right)}\ge 0$

$x\in(-\infty,-1]\cup\left(1-\sqrt{3},1+\sqrt{3}\right)\cup[3,\infty)$ ....(2)

$(1)\cap(2)$

$x\in(-\infty,-1]\cup\left[1-\sqrt{2},1+\sqrt{2}\right]\cup[3,\infty)$

$\alpha=-1,\beta=1-\sqrt{2},\gamma=1+\sqrt{2},\delta=3$

$\alpha+\beta+\gamma+\delta=4$

Question ID : 444792608

8. The letters of the word "UDAYPUR" are written in all possible ways with or without meaning and these words are arranged as in a dictionary. The rank of the word "UDAYPUR" is :

(1) 1581 (2) 1580 (3) 1578 (4) 1579

Ans. Official answer NTA (2)

Sol. ADPRUUY

A.. $=\frac{6!}{2!}$

D.. $=\frac{6!}{2!}$

P.. $= \frac{6!}{2!}$

R...................................... $= \frac{6!}{2!}$

UA............................ = 5!
U D A P ....= 3!
U D A R ....= 3!
U D A U .... = 3!
U D A Y P R .... = 1!
U D A Y P U R .... = 1!

$\boxed{1580}$

Question ID : 444792611

9. Let the image of parabola $x^2 = 4y$, in the line $x - y = 1$ be $(y + a)^2 = b(x - c)$, $a, b, c \in N$. Then $a + b + c$ is equal to :

(1) 6 (2) 12 (3) 4 (4) 8

Ans. Official answer NTA (1)

Sol. $x^2 = 4y$

$P(2t, t^2)$

$x - y = 1$

image $\Rightarrow \frac{x-2t}{1} = \frac{y-t^2}{-1} = \frac{-2\left(2t-t^2-1\right)}{2}$

$x = t^2 + 1,\ y = 2t - 1$

$t = \frac{y+1}{2}$

$x - 1 = \left(\frac{y+1}{2}\right)^2$

$(y + 1)^2 = 4(x - 1)$

$a = 1,\ b = 4,\ c = 1$

$a + b + c = 6$

Question ID : 444792607

10. The largest value of n, for which 40^n divides 60!, is :

(1) 13 (2) 11 (3) 12 (4) 14

Ans. Official answer NTA (4)

Sol. $\frac{60!}{(40)^n} = \frac{60!}{2^{3n} \cdot 5^n}$

$E_5(60!) = \left[\frac{60}{5}\right] + \left[\frac{60}{25}\right] + 0 = 14$

$E_2(60!) = \left[\frac{60}{2}\right] + \left[\frac{60}{4}\right] + \left[\frac{60}{8}\right] + \left[\frac{60}{16}\right] + \left[\frac{60}{32}\right] + 0 = 56$

$\frac{60!}{2^{56} \cdot 5^{14}} \Rightarrow n = 14$

Question ID : 444792609

11. Let $X = \{x \in N : 1 \le x \le 19\}$ and for some a, b $\in$ R, $Y = \{ax + b : x \in X\}$. If the mean and variance of the elements of Y are 30 and 750, respectively, then the sum of all possible values of b is :

(1) 20 (2) 60 (3) 100 (4) 80

Ans. Official answer NTA (2)

Sol. X {1, 2, 3, ... 19}

$\bar{x} = \frac{1+2+.........+19}{19} = 10$

$\sigma_x = \frac{1^2 + 2^2 + \ldots + 19^2}{19} - (10)^2 = 30$

$\bar{y} = a\bar{x} + b$

30 = 10 a + b ....(1)

$\sigma_y = a^2 \sigma_x$

$750 = a^2(30)$

$a^2 = 25$

$a = \pm 5$

b = 30 – 10a

$\left.\begin{array}{l} a = 5 \Rightarrow b = -20 \\ a = -5 \Rightarrow b = 80 \end{array}\right\} \text{sum} = 60$

Question ID : 444792616

12. Let [t] denote the greatest integer less than or equal to t. If the function

$$f(x)=\begin{cases} b^2\sin\left(\frac{\pi}{2}\left[\frac{\pi}{2}(\cos x+\sin x)\cos x\right]\right), & x<0 \\ \dfrac{\sin x-\frac{1}{2}\sin 2x}{x^3} & ,x>0 \\ a & ,x=0 \end{cases}$$

is continuous at $x = 0$, then $a^2 + b^2$ is equal to :

(1) $\frac{5}{8}$ (2) $\frac{9}{16}$ (3) $\frac{3}{4}$ (4) $\frac{1}{2}$

Ans. Official answer NTA (3)

Sol. for cont. at $x = 0$

$f(0) = f(0^+) = f(0^-)$

$$a=\lim_{x\to 0^+}\frac{2\sin x-\sin 2x}{2x^3}=\lim_{x\to 0^-} b^2\sin\left(\frac{\pi}{2}\left[\frac{\pi}{2}(\sin x+\cos x)\cos x\right]\right)$$

$$a=\lim_{x\to 0^+}\frac{2\left(x-\frac{x^3}{3!}\right)-\left(2x-\frac{8x^3}{3!}\right)}{2x^3}=\lim_{x\to 0^-} b^2\sin\left(\frac{\pi}{2}\right)$$

$$a=\frac{1}{2}=b^2$$

$$a^2+b^2=\frac{1}{4}+\frac{1}{2}=\frac{3}{4}$$

Question ID : 444792603

13. The smallest positive integral value of a, for which all the roots of $x^4 - ax^2 + 9 = 0$ are real and distinct, is equal to :

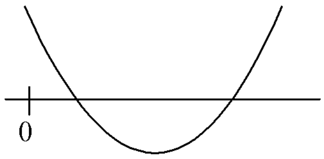
(1) 9 (2) 3 (3) 7 (4) 4

Ans. Official answer NTA (3)

Sol. $x^4 - ax^2 + 9 = 0$

$x^2 = t \in [0, \infty)$

$t^2 - at + 9 = 0$



$D > 0$

$a^2 - 36 > 0$

$a \in (-\infty, -6) \cup (6, \infty)$ ....(1)

$\frac{-b}{2a} > 0$

$\frac{a}{2} > 0 \Rightarrow a > 0$ ....(2)

$f(0) > 0$

$a \in R$ ....(3)

$(1) \cap (2) \cap (3)$

$a \in (6, \infty)$

min = 7

Question ID : 444792610

14. Let the length of the latus rectum of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, (a > b)$, be 30. If its eccentricity is the maximum value of the function $f(t) = -\frac{3}{4} + 2t - t^2$, then $(a^2 + b^2)$ is equal to :

(1) 496 (2) 256 (3) 276 (4) 516

Ans. Official answer NTA (1)

Sol. $\frac{2b^2}{a} = 30$

$b^2 = 15a$

$f(t) = \frac{-3}{4} + 2t - t^2$

$f(t) = \frac{1}{4} - (t-1)^2$

$f(t)_{max} = \frac{1}{4} = e$

$b^2 = a^2(1 - e^2)$

$b^2 = a^2\left(1 - \frac{1}{16}\right)$

$15a = \frac{15}{16}a^2$

$a = 16$

$b^2 = 240$

$a^2 + b^2 = 496$

Question ID : 444792614

15. Let $\vec{a} = 2\hat{i} - \hat{j} - \hat{k}, \vec{b} = \hat{i} + 3\hat{j} - \hat{k}$ and $\vec{c} = 2\hat{i} + \hat{j} + 3\hat{k}$. Let $\vec{v}$ be the vector in the plane of the vectors $\vec{a}$ and $\vec{b}$, such that the length of its projection on the vector $\vec{c}$ is $\frac{1}{\sqrt{14}}$. Then $|\vec{v}|$ is equal to :

(1) $\frac{\sqrt{21}}{2}$ (2) 13 (3) $\frac{\sqrt{35}}{2}$ (4) 7

Ans. Official answer NTA (3)

Ans. By Matrix (Bonus)

Resson : Data is insufficient to find $\vec{v}$.

Sol. Data insufficient

Question ID : 444792613

16. The sum of all values of α, for which the shortest distance between the lines $\frac{x+1}{\alpha} = \frac{y-2}{-1} = \frac{z-4}{-\alpha}$ and $\frac{x}{\alpha} = \frac{y-1}{2} = \frac{z-1}{2\alpha}$ is $\sqrt{2}$, is :

(1) –6 (2) 8 (3) 6 (4) –8

Ans. Official answer NTA (1)

Sol. $\vec{r} = (-1, 2, 4) + \lambda(\alpha, -1, -\alpha)$

$\vec{r} = (0, 1, 1) + \mu(\alpha, 2, 2\alpha)$

$\overrightarrow{AB} = \hat{i} - \hat{j} - 3\hat{k}$

$$\vec{P}\times\vec{q}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \alpha & -1 & -\alpha\\ \alpha & 2 & 2\alpha\end{vmatrix}=-3\alpha^2\hat{j}+3\alpha\hat{k}$$

$$S\cdot D=\left|\frac{(\hat{i}-\hat{j}-3\hat{k})\cdot\left(-3\alpha^2\hat{j}+3\alpha\hat{k}\right)}{\left|-3\alpha^2\hat{j}+3\alpha\hat{k}\right|}\right|=\sqrt{2}$$

$$\frac{3\alpha^2-9\alpha}{3\sqrt{\alpha^4+\alpha^2}}=\sqrt{2}$$

$$\left(\alpha^2-3\alpha\right)^2=2\left(\alpha^4+\alpha^2\right)$$

$$\alpha^4-6\alpha^3+9\alpha^2=2\alpha^4+2\alpha^2$$

$$\alpha^4+6\alpha^3-7\alpha^2=0$$

$$\alpha^2\left(\alpha^2+6\alpha-7\right)=0$$

$$\alpha^2(\alpha+7)(\alpha-1)=0$$

$$\alpha=-7,1\Rightarrow \text{ Sum }=-6$$

Question ID : 444792602

17. Let f be a function such that $3f(x)+2f\left(\frac{m}{19x}\right)=5x, x\neq 0$, where $m=\sum_{i=1}^{9}(i)^2$. Then f(5) – f(2) is equal to :

(1) –9 (2) 36 (3) 18 (4) 9

Ans. Official answer NTA (3)

Sol. $3f(x)+2f\left(\frac{m}{19x}\right)=5x$

$$m=1^2+2^2+\cdots+9^2=\frac{9\times10\times19}{6}=15\times19$$

$$\frac{m}{19}=15$$

$$3f(x)+2f\left(\frac{15}{x}\right)=5x$$

$$\left.\begin{aligned}3f(3)+2f(5)=15\\ 3f(5)+2f(3)=25\end{aligned}\right\}f(5)=9$$

$$\left.\begin{aligned}3f(2)+2f\left(\frac{15}{2}\right)=10\\ 3f\left(\frac{15}{2}\right)+2f(2)=\frac{75}{2}\end{aligned}\right\}f(2)=-9$$

$f(5)-f(2)=18$

Question ID : 444792620

18. Let $y=y(x)$ be a differentiable function in the interval $(0,\infty)$ such that $y(1)=2$, and $\lim\limits_{t\to x}\left(\frac{t^2y(x)-x^2y(t)}{x-t}\right)=3$ for each $x>0$. Then $2y(2)$ is equal to :

(1) 12 (2) 23 (3) 27 (4) 18

Ans. Official answer NTA (2)

Sol. $\lim\limits_{t\to x}\frac{t^2y(x)-x^2y(t)}{x-t}=3$

$\lim\limits_{t\to x}\frac{2ty(x)-x^2y'(t)}{-1}=3$

$x^2y'(x)-2xy(x)=3$

$\frac{dy}{dx}-\frac{2y}{x}=\frac{3}{x^2}$

$I\cdot F\cdot=e^{2\int-\frac{1}{x}dx}=\frac{1}{x^2}$

$y\cdot\frac{1}{x^2}=\int\frac{3}{x^4}dx$

$\frac{y}{x^2}=\frac{-1}{x^3}+c$

$y(1)=2$

$c=3$

$y=-\frac{1}{x}+3x^2$

$2y(2)=2\left(-\frac{1}{2}+12\right)=23$

Question ID : 444792615

19. Let $\vec{a} = 2\hat{i} - 5\hat{j} + 5\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + 3\hat{k}$. If $\vec{c}$ is a vector such that $2(\vec{a}\times\vec{c}) + 3(\vec{b}\times\vec{c}) = \vec{0}$ and $(\vec{a}-\vec{b})\cdot\vec{c} = -97$, then $|\vec{c}\times\hat{k}|^2$ is equal to :

(1) 233 (2) 205 (3) 218 (4) 193

Ans. Official answer NTA (3)

Sol. $2(\vec{a}\times\vec{c}) + 3(\vec{b}\times\vec{c}) = \vec{0}$

$(2\vec{a}+3\vec{b})\times\vec{c} = \vec{0}$

$\vec{c} = \lambda(2\vec{a}+3\vec{b})$

$(\vec{a}-\vec{b})\cdot\vec{c} = -97$

$\lambda(\vec{a}-\vec{b})\cdot(2\vec{a}+3\vec{b}) = -97$

$\lambda\left\{2|\vec{a}|^2 + \vec{a}\cdot\vec{b} - 3|\vec{b}|^2\right\} = -97$

$\lambda(108+22-33) = -97$

$\lambda = -1$

$\vec{c} = -2\vec{a} - 3\vec{b}$

$\vec{c} = -7\hat{i} + 13\hat{j} - 19\hat{k}$

$$\vec{c}\times\hat{k} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -7 & 13 & -19 \\ 0 & 0 & 1 \end{vmatrix} = 13\hat{i} + 7\hat{j}$$

$|\hat{c}\times\hat{k}|^2 = |169+49 = 218$

Question ID : 444792619

20. Let f(α) denote the area of the region in the first quadrant bounded by x = 0, x = 1, $y^2 = x$ and $y = |\alpha x - 5| - |1-\alpha x| + \alpha x^2$. Then (f(0) + f(1)) is equal to :

(1) 9 (2) 7 (3) 12 (4) 14

Ans. Official answer NTA (2)

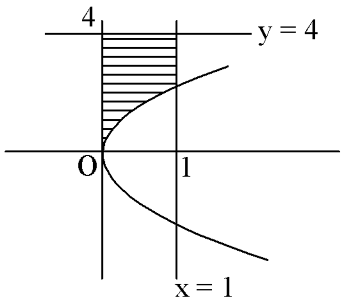
Sol. $x = 0, x = 1, y^2 = x$

$f(0) \Rightarrow \alpha = 0$

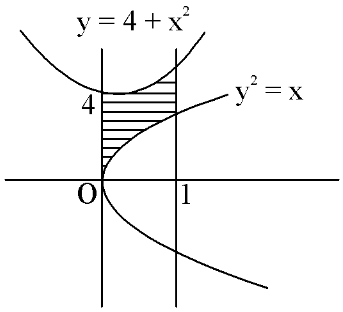
$y = 4$

$$f(0) = 4 - \int_0^1 \sqrt{x}\,dx = 4 - \frac{2}{3}\left(x^{3/2}\right)_0^1 = \frac{10}{3}$$

$f(1) \Rightarrow \alpha = 1$



$y = 4 + x^2$



$f(1) = \int_0^1 \left(4 + x^2 - \sqrt{x}\right) dx$

$f(1) = \left(4x + \frac{x^3}{3} - \frac{2}{3}x^{3/2}\right)_0^1$

$f(1) = 4 + \frac{1}{3} - \frac{2}{3} = \frac{11}{3}$

$f(0) + f(1) = \frac{21}{3} = 7$

SECTION – B

Question ID : 444792623

21. Let (h, k) lie on the circle C : $x^2 + y^2 = 4$ and the point (2h + 1, 3k + 2) lie on an ellipse with eccentricity e. Then the value of $\frac{5}{e^2}$ is equal to _________.

Ans. Official answer NTA (9)

Sol. $x^2 + y^2 = 4$

$h^2 + k^2 = 4$ ___________(1)

$x = 2h + 1 \Rightarrow h = \dfrac{x-1}{2}$

$y = 3k + 2 \Rightarrow k = \dfrac{y-2}{3}$

from eq. (1)

$\dfrac{(x-1)^2}{4} + \dfrac{(y-2)^2}{9} = 4$

$\dfrac{(x-1)^2}{16} + \dfrac{(y-2)^2}{36} = 1$

$a^2 = b^2\ (1-e^2)$

$16 = 36\ (1-e^2)$

$1 - e^2 = \dfrac{4}{9}$

$e^2 = \dfrac{5}{9}$

$\dfrac{5}{e^2} = 9$

Question ID : 444792624

22. The number of elements in the set $\{x \in [0, 180°]: \tan(x + 100°) = \tan(x + 50°) \tan x \tan(x - 50°)\}$ is ______.

Ans. Official answer NTA (4)

Sol. $\tan(x+100) = \tan(x+50) \tan x \tan(x-50)$

$$\frac{\sin\left(x+100^\circ\right)\cos\left(x-50^\circ\right)}{\cos\left(x+100^\circ\right)\sin\left(x-50^\circ\right)} = \frac{\sin\left(x+50^\circ\right)\sin x}{\cos\left(x+50^\circ\right)\cos x}$$

Apply C & D

$$\frac{\sin\left(2x+50^\circ\right)}{\sin\left(150^\circ\right)} = \frac{\cos\left(50^\circ\right)}{-\cos\left(2x+50^\circ\right)}$$

$$\sin\left(4x+100^\circ\right) = -\cos\left(50^\circ\right)$$

$$\sin\left(4x+100^\circ\right) = \sin\left(-40^\circ\right)$$

$$4x+100 = n\pi - (-1)^n\left(40^\circ\right)$$

$$x = 30^\circ, 55^\circ, 120^\circ, 145^\circ$$

Question ID : 444792621

23. Let $z = (1 + i)(1 + 2i)(1 + 3i) \ldots\ldots (1 + ni)$, where $i = \sqrt{-1}$. If $|z|^2 = 44200$, then n is equal to _______.

Ans. Official answer NTA (5)

Sol. $z = (1 + i)(1 + 2i)(1 + 3i) \ldots\ldots\ldots (1+ni)$

$|z|^2 = (1 + 1)(1 + 4)(1 + 9)(1 + 16) \ldots\ldots\ldots (1 + n^2)$

$44200 = (2)(5)(10)(17)(26)$

$n = 5$

Question ID : 444792622

24. Let S be a set of 5 elements and P(S) denote the power set of S. Let E be an event of choosing an ordered pair (A, B) from the set P(S) × P(S) such that $A \cap B = \varnothing$. If the probability of the event E is $\frac{3^p}{2^q}$, where $p, q \in N$, then p + q is equal to _________.

Ans. Official answer NTA (15)

Sol. $n(S) = 5$

$n(P(S)) = 32$

tota; cases $= 32 \times 32$

fav. cases $= \sum_{r=0}^{5} {}^5c_r \cdot 2^{5-r} = 3^5$

Prob. $= \frac{3^5}{2^{10}}$

$p = 5,\ q = 10$

$p + q = 15$

Question ID : 444792625

25. If f(x) satisfies the relation $f(x) = e^x + \int_0^1 \left(y + xe^x\right) f(y)dy$, then e + f(0) is equal to _________.

Ans. Official answer NTA (2)

Sol. $f(x) = e^x + \int_0^1 \left(y + xe^x\right) f(y)dy$

$f(x) = e^x + xe^x \int_0^1 f(y)dy + \int_0^1 yf(y)dy$

$f(x) = e^x + \lambda xe^x + \mu$

$\lambda = \int_0^1 f(y)dy$

$\lambda = \int_0^1 \left(e^y + \lambda y e^y + \mu\right)dy$

$\lambda = \left(e^y + \lambda\left(ye^y - e^y\right) + \mu y\right)_0^1$

$\lambda = (e + \mu) - (1 - \lambda)$

$\mu = 1 - e$

$f(0) = e^0 + 0 + 1 - e$

$f(0) = 2 - e$

$f(0) + e = 2$