JEE Main January 2026

Question Paper With Text Solution

24 January | Shift-1

MATHEMATICS

JEE MAIN JANUARY 2026 | 24TH JANUARY SHIFT-1

SECTION – A

Question ID : 444792543

1. Let α, β ∈ R be such that the function $f(x)=\begin{cases}2\alpha(x^2-2)+2\beta x & ,x<1\\(\alpha+3)x+(\alpha-\beta) & ,x\geq 1\end{cases}$ be differentiable at all x ∈ R. Then $34(\alpha+\beta)$ is equal to :

(1) 36 (2) 24 (3) 48 (4) 84

Ans. Official answer NTA (3)

Sol. $f(x)=\begin{cases}2\alpha(x^2-2)+2\beta x & ,x<1\\(\alpha+3)x+(\alpha-\beta) & ,x\geq 1\end{cases}$

differentiable at x = 1

$\Rightarrow$ continuity at x = 1

$\Rightarrow -2\alpha + 2\beta = (\alpha + \beta) + (\alpha - \beta)$

$\Rightarrow -2\alpha + 2\beta = 2\alpha - \beta + 3$

$\Rightarrow 3\beta = 4\alpha + 3$ ....(1)

Now

$f'(x)=\begin{cases}4\alpha x+2\beta & ,x<1\\ \alpha+3 & ,x>1\end{cases}$

$\Rightarrow$ at x = 1

$4\alpha + 2\beta = \alpha + 3$ ....(2)

$\Rightarrow \alpha = \frac{3}{17},\ \beta = \frac{21}{17}$

$\Rightarrow 34(\alpha + \beta) = 48$

Question ID : 444792533

2. The mean and variance of a data of 10 observations are 10 and 2, respectively. If an observations α in this data is replaced by β, then the mean and variance become 10.1 and 1.99 , respectively. Then α + β equals :

(1) 15 (2) 20 (3) 5 (4) 10

Ans. Official answer NTA (2)

Sol. Given $x_1 + x_2 + ... + x_9 + \alpha = 100$ ....(1)

and $2=\dfrac{\sum_{i=1}^{9} x_i^2+\alpha^2}{10}-(10)^2$

$\Rightarrow x_1^2+.........+x_9^2+\alpha^2=1020$ ....(2)

If α replaced by β

$\Rightarrow x_1+..........+x_9+\beta=101$ ....(3)

and $1.99=\dfrac{\sum_{i=1}^{9} x_i^2+\beta^2}{10}-(10.1)^2$

$\Rightarrow x_1^2+x_2^2+.........+x_9^2+\beta^2=1040$ ....(4)

(1) – (3) (2) – (4)

$\alpha-\beta=-1$ $\alpha^2-\beta^2=-20$

$\alpha+\beta=20$

Question ID : 444792532

3. Let $S=\dfrac{1}{25!}+\dfrac{1}{3!23!}+\dfrac{1}{5!21!}+\ldots$ up to 13 terms. If $13S=\dfrac{2^k}{n!}, k\in N$, then $n+k$ is equal to :

(1) 49 (2) 51 (3) 50 (4) 52

Ans. Official answer NTA (1)

Sol. $S=\dfrac{1}{26!}\left({}^{26}C_1+{}^{26}C_3+..........+{}^{26}C_{25}\right)$

$\Rightarrow S=\dfrac{2^{25}}{26!}$

$\Rightarrow 13S=\dfrac{2^{25}\times 13}{(25)!26}$

$\Rightarrow 13S=\dfrac{2^{24}}{(25)!}$

$\Rightarrow n+k=49$

Question ID : 444792527

4. If the domain of the function $f(x)=\log_{(10x^2-17x+7)}\left(18x^2-11x+1\right)$ is $(-\infty,a)\cup(b,c)\cup(d,\infty)-\{e\}$, then $90(a+b+c+d+e)$ equals :

(1) 170 (2) 316 (3) 307 (4) 177

Ans. Official answer NTA (2)

Sol. For domain

$18x^2 - 11x + 1 > 0 \cap 10x^2 - 17x + 7 > 0 \cap 10x^2 - 17x + 7 \neq 1$

$(2x - 1)(ax - 1) > 0 \cap (x - 1)(10x - 7) > 0 \cap 10x^2 - 17x + 6 \neq 0$

$\Rightarrow x \in \left(-\infty, \frac{1}{9}\right) \cup \left(\frac{1}{2}, \infty\right) \cap x \in \left(-\infty, \frac{7}{10}\right) \cup (1, \infty) \cap x \neq \frac{6}{5}, \frac{1}{2}$

$\Rightarrow x \in \left(-\infty, \frac{1}{9}\right) \cup \left(\frac{1}{2}, \frac{7}{10}\right) \cup (1, \infty) - \left\{\frac{6}{5}\right\}$

$90(a + b + c + d + e) = 316$

Question ID : 444792531

5. Consider an A.P. : $a_1, a_2, \ldots\ldots\ldots, a_n$; $a_1 > 0$. If $a_2 - a_1 = \frac{-3}{4}, a_n = \frac{1}{4}a_1$, and $\sum_{i=1}^{n} a_i = \frac{525}{2}$, then $\sum_{i=1}^{17} a_i$ is equal to :

(1) 238 (2) 952 (3) 136 (4) 476

Ans. Official answer NTA (1)

Sol. $a_2 - a_1 = -3/4, \Rightarrow d = \frac{-3}{4}$

$a_n = \frac{a_1}{4} \Rightarrow a_1 + (n-1)\left(\frac{-3}{4}\right) = \frac{a_1}{4}$

$\Rightarrow \frac{3a_1}{4} = (n-1)\frac{3}{4}$

$\Rightarrow a_1 = n - 1$

$\sum_{i=1}^{n} a_i = \frac{525}{2} \Rightarrow \frac{n}{2}\left[2a_1 + (n-1)\left(\frac{-3}{4}\right)\right] = \frac{525}{2}$

$\Rightarrow n\left[2a_1 + (a_1)\left(\frac{-3}{4}\right)\right] = 525$

$\Rightarrow (a_1 + 1)\left(\frac{5a_1}{4}\right) = 525$

$\Rightarrow a_1(a_1 + 1) = 420$

$\Rightarrow a_1 = 20$ or $a_1 = -21$ (Rejected)

$\Rightarrow \sum_{i=1}^{17} a_i = \frac{17}{2}\left(40 + 16 \times \frac{-3}{4}\right)$

$= \frac{17}{2}(28) = 238$

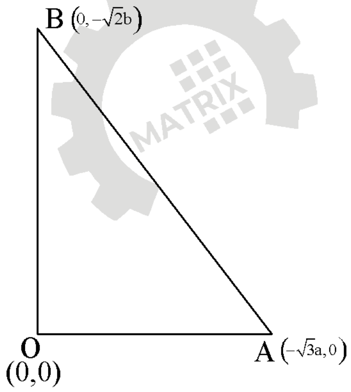
Question ID : 444792536

6. Let a circle of radius 4 pass through the origin O, the points $A(-\sqrt{3}a, 0)$ and $B(0, -\sqrt{2}b)$, where a and b are real parameters and $ab \neq 0$. Then the locus of the centroid of ΔOAB is a circle of radius :

(1) $\frac{7}{3}$ (2) $\frac{11}{3}$ (3) $\frac{8}{3}$ (4) $\frac{5}{3}$

Ans. Official answer NTA (3)

Sol.



Given radius = 4

$\Rightarrow AB = 8$

$\Rightarrow 3a^2 + 2b^2 = 64$

let centroid = (h, k)

$\Rightarrow 3h = -\sqrt{3}a$, $3k = -\sqrt{2}b$

$\Rightarrow\ a = -\sqrt{3}\hbar,\ b = \dfrac{-3k}{\sqrt{2}}$

$\Rightarrow\ 3(3\hbar) + 2\left(\dfrac{9k^2}{2}\right) = 64$

$x^2 + y^2 = \dfrac{64}{9}$

radias = 8/3

Question ID : 444792542

7. If the function $f(x) = \dfrac{e^x\left(e^{\tan x - x} - 1\right) + \log_e(\sec x + \tan x) - x}{\tan x - x}$ is continuous at x = 0, then the value of f(0) is equal to :

(1) $\dfrac{2}{3}$ (2) 2 (3) $\dfrac{3}{2}$ (4) $\dfrac{1}{2}$

Ans. Official answer NTA (3)

Sol. $f(x) = \lim\limits_{x\to 0} \dfrac{e^x\left(e^{\tan x - x} - 1\right) + \log_e(\sec x + \tan x) - x}{\tan x - x}$

$\Rightarrow\ f(0) = \lim\limits_{x\to 0} \dfrac{e^x\left(e^{\tan x - x} - 1\right)}{\tan x - x} + \lim\limits_{x\to 0} \dfrac{\ln(\sec x + \tan x) - x}{\tan x - x}$

$= 1 + \lim\limits_{x\to 0} 3\,\dfrac{\ln(\sec x + \tan x) - x}{x^3}$

Using LH Rule

$\Rightarrow\ f(0) = 1 + 3.\lim\limits_{x\to 0} \dfrac{\left(\dfrac{\sec x + \tan x + \sec^2 x}{\sec x + \tan x}\right) - 1}{3x^3}$

$f(0) = 1 + \lim\limits_{x\to 0} \dfrac{\sec x - 1}{x^2}$

$= 1 + \dfrac{1}{2} = \dfrac{3}{2}$

Question ID : 444792545

8. Let A_1 be the bounded area enclosed by the curves $y = x^2 + 2$, $x + y = 8$ and y-axis that lies in the first quadrant. Let A_2 be the bounded area enclosed by the curves $y = x^2 + 2$, $y^2 = x$, $x = 2$, and y-axis that lies in the first quadrant. Then $A_1 - A_2$ is equal to :

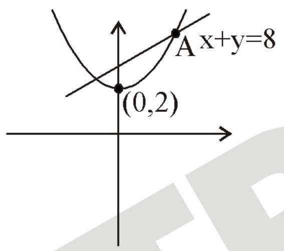
(1) $\frac{2}{3}(3\sqrt{2}+1)$ (2) $\frac{2}{3}(4\sqrt{2}+1)$ (3) $\frac{2}{3}(\sqrt{2}+1)$ (4) $\frac{2}{3}(2\sqrt{2}+1)$

Ans. Official answer NTA (4)

Sol. For A_1

$y = x^2 + 2,\ \ x + y = 8$

Coradinates of $A_1 = (2, 6)$

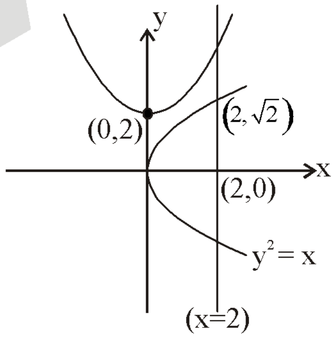


$$A_1 = \int_0^2 (8-x)-(x^2+2)dx$$

$$= 6x - \frac{x^2}{2} - \frac{x^3}{3}\Big|_0^2$$

$$= \frac{22}{3}$$

For A_2



$$A_2 = \int_0^2 (x^2+2) - \sqrt{x}dx$$

$$= \frac{x^3}{3} + 2x - x^{3/2}.\frac{2}{3}\Big|_0^2$$

$$= \frac{20}{3} - \frac{4\sqrt{2}}{3}$$

$$A_1 - A_2 = \frac{2}{3} + \frac{4\sqrt{2}}{3} = \frac{2}{3}(2\sqrt{2}+1)$$

Question ID : 444792539

9. The value of $\frac{\sqrt{3}\operatorname{cosec}20^\circ - \sec 20^\circ}{\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ}$ is equal to :

(1) 64 (2) 16 (3) 12 (4) 32

Ans. Official answer NTA (1)

Sol. $$E = \frac{\dfrac{\sqrt{3}\,\text{cosec}\,20^\circ - \sec 20^\circ}{\cos 20^\circ \cos 40^\circ \cos 80^\circ}}{2}$$

$$= \frac{2\left(\sqrt{3}\cos 20^\circ - \sin 20^\circ\right)}{\sin 20^\circ \cos 20^\circ . \dfrac{\sin\left(8\times 20^\circ\right)}{8\times \sin 20^\circ}}$$

$$= \frac{16\times 2\left(\dfrac{\sqrt{3}}{2}\cos 20^\circ - \dfrac{1}{2}\sin 20^\circ\right)}{\cos 20^\circ \sin 20^\circ}$$

$$= \frac{32\sin\left(40^\circ\right)}{\cos 20^\circ \sin 20^\circ} = 64$$

Question ID : 444792534

10. From a lot containing 10 defective and 90 non-defective bulbs, 8 bulbs are selected one by one with replacement. Then the probability of getting at least 7 defective bulbs is :

(1) $\frac{7}{10^7}$ (2) $\frac{67}{10^8}$ (3) $\frac{81}{10^8}$ (4) $\frac{73}{10^8}$

Ans. Official answer NTA (4)

Sol. Total Sample Space = 100^8.

ways to get atleast 7 defective = exactly 7 defective + exactly 8 defective.

$= {}^8C_7.10^7.90 + {}^8C_8 \times 10^8$

$$\text{Probability} = \frac{8\times 10^7 \times 90 + 1\times 10^8}{10^{16}}$$

$$= \frac{72+1}{10^8} = \frac{73}{10^8}$$

Question ID : 444792537

11. If $\cot x = \frac{5}{12}$ for some $x \in \left(\pi, \frac{3\pi}{2}\right)$, then $\sin 7x\left(\cos\frac{13x}{2} + \sin\frac{13x}{2}\right) + \cos 7x\left(\cos\frac{13x}{2} - \sin\frac{13x}{2}\right)$ is equal to :

(1) $\frac{4}{\sqrt{26}}$ (2) $\frac{1}{\sqrt{13}}$ (3) $\frac{5}{\sqrt{13}}$ (4) $\frac{6}{\sqrt{26}}$

Ans. Official answer NTA (2)

Sol. $\cot x = \frac{5}{12} \qquad x \in \left(\pi, \frac{3\pi}{2}\right)$

$$E = \sin 7x + \cos\frac{13x}{2} + \cos 7x \cos\frac{13x}{2} + \sin 7x \sin\frac{13x}{2} - \cos 7x \sin\frac{13x}{2}$$

$$E = \sin 7x + \cos\frac{13x}{2} - \cos 7x \sin\frac{13x}{2} + \cos 7x \cos\frac{13x}{2} - \sin 7x \sin\frac{13x}{2}$$

$$= \sin\left(7x - \frac{13x}{2}\right) + \cos\left(7x - \frac{13x}{2}\right)$$

$$E = \sin\frac{x}{2} + \cos\left(\frac{x}{2}\right)$$

$$E^2 = 1 + \sin x = 1 - \frac{12}{13} = \frac{1}{13}$$

but $\frac{x}{2} \in \left(\frac{\pi}{2}, \frac{3\pi}{4}\right)$

$\Rightarrow \sin\frac{x}{2} + \cos\frac{x}{2} > 0$

$E = \frac{1}{\sqrt{13}}$

Question ID : 444792529

12. Let 729, 81, 9, 1, ... be a sequence and P_n denote the product of the first n terms of this sequence. If $2\sum_{n=1}^{40}(P_n)^{\frac{1}{n}} = \frac{3^{\alpha}-1}{3^{\beta}}$ and $\gcd(\alpha, \beta) = 1$, then $\alpha + \beta$ is equal to :

(1) 75 (2) 73 (3) 74 (4) 76

Ans. Official answer NTA (2)

Sol. 729, 81, 9, 1,..........

It is GP with a = 729, r = $\frac{1}{9}$

$\Rightarrow a_n = (729)\left(\frac{1}{9}\right)^{n-1} = 9^3 \cdot \frac{1}{9^{n-1}} = 9^{\frac{1}{n-9}}$.

$P_n = a_1 a_2 \ldots\ldots\ldots a_n$

$= 9^3 \cdot 9^2 \cdot 9^1 \cdot 9^0, \ldots\ldots\ldots$

$= 9^{\frac{n}{2}(7-n)}$

$2\sum_{n=1}^{40`} 9^{\frac{7-n}{2}} = \frac{3^\alpha - 1}{3^\beta}$

$\Rightarrow 2\sum_{n=1}^{40`} 3^{7-n} = \frac{3^\alpha - 1}{3^\beta}$

$\Rightarrow 2\sum_{n=1}^{40`} 3^{7-n} = \frac{3^{40} - 1}{3^{33}}$

$\Rightarrow \alpha + \beta = 73$

Question ID : 444792528

13. Let $S = \left\{ z \in C : \left|\frac{z-6i}{z-2i}\right| = 1 \text{ and } \left|\frac{z-8+2i}{z+2i}\right| = \frac{3}{5} \right\}$. Then $\sum_{z \in s} |z|^2$ is equal to :

(1) 398 (2) 413 (3) 423 (4) 385

Ans. Official answer NTA (4)

Sol. $|Z-6i| = |Z-2i|$

$\Rightarrow$ Per Pendicular Bisector of (0, 6) and (0, 2)

$Z = x + 4i \qquad x \in R$

$\Rightarrow 5|z-8+2i| = 3|z+2i|$

$5|(x-8) + 6i| = 3|x + 6i|$

$25((x-8)^2 + 36) = 9(x^2 + 36)$

$\Rightarrow (x-5)(x-20) = -36$

$\Rightarrow x^2 - 25x + 136 = 0.$

$x_1 \quad x_2$

$x_1^2 + x_2^2 = 625 - 2(136) = 353$

$\sum |z|^2 = x_1^2 + 16 + x_2^2 + 16 = 353 + 32 = 385.$

Question ID : 444792526

14. Let R be a relation defined on the set $\{1, 2, 3, 4\} \times \{1, 2, 3, 4\}$ by $R = \{((a, b),(c, d)): 2a + 3b = 3c + 4d\}$. Then the number of elements in R is :

(1) 6 (2) 18 (3) 15 (4) 12

Ans. Official answer NTA (4)

Sol. $2a + 3b = 3 + 4d$

$\Rightarrow 2(a - 2d) = 3(c - b)$

$\Rightarrow a - 2d$ must be multiple of 3

and $c - b$ must be multiple of 2

$\Rightarrow (a, d) = (1, 2), (2,1), (2, 4), (3, 3), (4, 2)$

$\Rightarrow 3(c - b) = -6, 0, -12, -6, 0$

$\Rightarrow (c - b) = -2, 0, -4, -2, 0$

No of Pairs (b, c): 2, 4, 0, 2, 4

Total = 12

Question ID : 444792541

15. Let the lines $L_1 : \vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k}), \lambda \in R$ and $L_2 : \vec{r} = (4\hat{i} + \hat{j}) + \mu(5\hat{i} + 2\hat{j} + \hat{k}), \mu \in R$, intersect at the point R. Let P and Q be the points lying on lines L_1 and L_2, respectively, such that $|\overrightarrow{PR}| = \sqrt{29}$ and $|\overrightarrow{PQ}| = \sqrt{\frac{47}{3}}$. If the point P lies in the first octant, then $27(QR)^2$ is equal to :

(1) 360 (2) 320 (3) 340 (4) 348

Ans. Official answer NTA (1)

Sol. $\vec{r}_1 = (1,2,3) + \lambda(2,3,4)$

$\vec{r}_2 = (4,1,0) + \mu(5,2,1)$

For interseching R

$1 + 2\lambda = u + 5\mu, 2 + 3\lambda = 1 + 2\mu, 3 + 4\lambda = \mu.$

$\Rightarrow \lambda = \mu = -1$

$R : (-1,-1,-1)$

Now

$|\overrightarrow{PR}| = \sqrt{29} \Rightarrow (2+2\lambda)^2 + (3+3\lambda)^2 + (4+4\lambda)^2 = 29$

$\Rightarrow (\lambda+1)^2(4+9+16) = 29$

$\Rightarrow (\lambda+1)^2 = 1$

$\Rightarrow \lambda + 1 = 1, -1$

$\Rightarrow \lambda = 0, -2$

$\Rightarrow P = (1,2,3)$ or $(-3,-4,-5)$

$\Rightarrow P = (1,2,3)$

and $|\overrightarrow{PQ}| = \sqrt{47/3}$

$\Rightarrow (5\mu+3)^2 + (2\mu-1)^2 + (\mu-3)^2 = \frac{47}{3}$

$\Rightarrow 30\mu^2 + 19 + 20\mu = \frac{47}{3}$

$\Rightarrow 90\mu^2 + 60\mu + 10 = 0$

$\Rightarrow 9\mu^2 + 6\mu + 1 = 0.$

$\Rightarrow \mu = -1/3$

$\Rightarrow Q = \left(7/3, \frac{1}{3} - 1/3\right)$

$= 27(QR)^2 = 360$

Question ID : 444792535

16. Let each of the two ellipses $E_1 : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, (a > b)$ and $E_2 : \frac{x^2}{A^2} + \frac{y^2}{B^2} = 1, (A < B)$ have eccentricity $\frac{4}{5}$. Let the lengths of the latus recta of E_1 and E_2 be l_1 and l_2, respectively, such that $2l_1^2 = 9l_2$. If the distance between the foci of E_1 is 8 , then the distance between the foci of E_2 is :

(1) $\frac{16}{5}$ (2) $\frac{8}{5}$ (3) $\frac{32}{5}$ (4) $\frac{96}{5}$

Ans. Official answer NTA (3)

Sol. $E_1 : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (a > b)$

$E_2 : \frac{x^2}{A^2} + \frac{y^2}{B^2} = 1 \quad (A < B)$

given $b^2 = a^2\left(1-\frac{16}{25}\right)$, $A^2 = B^2\left(1-\frac{16}{25}\right)$

$\Rightarrow b^2 = \frac{9a^2}{25}$, $A^2 = \frac{9B^2}{25}$, and $2ae = 8$.

$l_1 = \frac{2b^2}{a}$, $l_2 = \frac{2A^2}{B}$

$\Rightarrow 2 \times \frac{4b^4}{a^2} = 9.\frac{2A^2}{B}$

$\Rightarrow \frac{4\times 3^4}{25} = \frac{9.9B^2}{25^2\times 12}$

$\Rightarrow B = 4$

distance $= 2\times 4\times \frac{4}{5}$

$= \frac{32}{5}$

Question ID : 444792530

17. The number of the real solutions of the equation : $x|x+3|+|x-1|-2=0$ is :

(1) 5 (2) 4 (3) 3 (4) 2

Ans. Official answer NTA (3)

Sol. $x|x+3|+|x-1|-2=0$

If $x \geq 1$ $\Rightarrow x^2+3x+x-1-2=0$

$\Rightarrow x^2+4x-3=0$

$\Rightarrow x = \frac{-4\pm\sqrt{16+12}}{2}$

$= \frac{-4\pm 2\sqrt{7}}{2}$

$= -2+\sqrt{7}, -2-\sqrt{7}$ Rejected

If $x \in (-3, 1)$

$x^2+3x+1-x-2=0$

$\Rightarrow x^2+2x-1=0 \Rightarrow x = \frac{-2\pm\sqrt{4+4}}{2}$

$= \frac{-2 \pm 2\sqrt{2}}{2}$

$\Rightarrow x = -1+\sqrt{2}, -1-\sqrt{2}$ Both accepted

If $x \le -3$

$-x^2 - 3x - x + 1 - 2 = 0$

$\Rightarrow x^2 + 4x + 1 = 0 \Rightarrow x = \frac{-4 \pm \sqrt{12}}{2}$

$x = \frac{-4 \pm 2\sqrt{3}}{2}$

$x = -2-\sqrt{3}, -2+\sqrt{3}$ (Rejected)

Question ID : 444792538

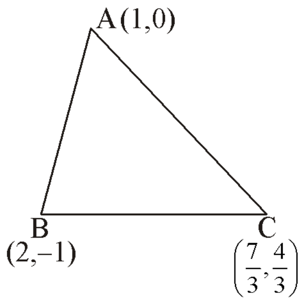
18. Let A(1, 0), B(2, –1) and $C\left(\frac{7}{3}, \frac{4}{3}\right)$ be three points. If the equation of the bisector of the angle ABC is $\alpha x + \beta y = 5$, then the value of $\alpha^2 + \beta^2$ is :

(1) 8 (2) 5 (3) 13 (4) 10

Ans. Official answer NTA (4)

Sol. Equation of AB $x + y - 1 = 0$

Equation of BC : $7x - y - 15 = 0$



Equation of Bisectors

$$\frac{x+y-1}{\sqrt{2}} = \pm \frac{(7x-y-15)}{5\sqrt{2}}$$

$\Rightarrow B_1 : -x + 3y + 5 = 0, \quad B_2 : 3x + y - 5 = 0$

Now $(1, 0)$ and $\left(\frac{7}{3}, \frac{4}{3}\right)$ must lie on opp. side of

Bisector $\Rightarrow B_1 \Rightarrow -x + 3y + 5 = 0$

$\alpha = 1, \beta = -3 \Rightarrow \alpha^2 + \beta^2 = 10$

Question ID : 444792540

19. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}, \vec{b} = \hat{i} + \hat{j}$ and $\vec{c} = \vec{a} \times \vec{b}$. Let $\vec{d}$ be a vector such that $|\vec{d} - \vec{a}| = \sqrt{11}, |\vec{c} \times \vec{d}| = 3$ and the

angle between $\vec{c}$ and $\vec{d}$ is $\frac{\pi}{4}$. Then $\vec{a}\cdot\vec{d}$ is equal to :

(1) 11 (2) 1 (3) 3 (4) 0

Ans. Official answer NTA (4)

Sol. $\vec{a}=(2,1,-2), \vec{b}=(1,1,0), \vec{c}=\vec{a}\times\vec{b}$

Now $|\vec{c}\times\vec{d}|=3$

and $\vec{c}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 1 & -2\\ 1 & 1 & 0\end{vmatrix}$

$=(2,-2,1)$

$|\vec{c}||\vec{d}|\sin\pi/4=3$

$\Rightarrow 3|\vec{d}|\frac{1}{\sqrt{2}}=3$

$\Rightarrow |\vec{d}|=\sqrt{2}$

Now $|\vec{d}-\vec{a}|=\sqrt{11} \Rightarrow |\vec{d}|^2+|\vec{a}|^2-2\vec{a}\cdot\vec{d}=11$

$\Rightarrow 2+9-2\vec{a}\cdot\vec{d}=11$

$\Rightarrow \vec{a}\cdot\vec{d}=0$

Question ID : 444792544

20. Let $f(t)=\int\left(\frac{1-\sin(\log_e t)}{1-\cos(\log_e t)}\right)dt, t>1$. If $f\left(e^{\pi/2}\right)=-e^{\pi/2}$ and $f\left(e^{\pi/4}\right)=\alpha e^{\pi/4}$, then α equals :

(1) $-1-2\sqrt{2}$ (2) $-1-\sqrt{2}$ (3) $1+\sqrt{2}$ (4) $-1+\sqrt{2}$

Ans. Official answer NTA (2)

Sol. Consider $\frac{1-\sin\theta}{1-\cos\theta}=\frac{1-\sin\theta}{2\sin^2\theta/2}$

$=\frac{1}{2}\text{cosec}^2\theta/2-\cot\theta/2$

$\Rightarrow \int\left(\frac{1}{2}\text{cosec}^2\left(\frac{\ln t}{2}\right)-\cot\frac{(\ln t)}{2}\right)dt$

$f(t)=-t\cot\left(\frac{\ln t}{2}\right)+c$

Put $t = e^{\pi/2} \Rightarrow \ln t = \pi/2$

$-e^{+\pi/2} = -e^{\pi/2} \times 1 + c \Rightarrow c = 0$

$f\left(e^{\pi/4}\right) = -e^{\pi/4} \cot\left(\frac{\pi}{8}\right) = -\left(\sqrt{2}+1\right)e^{\pi/4}$

$\Rightarrow \alpha = -\left(\sqrt{2}+1\right)$

SECTION – B

Question ID : 444792546

21. The number of 3×2 matrices A, which can be formed using the elements of the set $\{-2, -1, 0, 1, 2\}$ such that the sum of all the diagonal elements of $A^T A$ is 5 , is ________.

Ans. Official answer NTA (312)

Sol. $A = \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}$

$\Rightarrow A^T A = \begin{bmatrix} a & c & e \\ b & d & f \end{bmatrix} \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}$

$= \begin{bmatrix} \Sigma a^2 & \Sigma ab \\ \Sigma ab & \Sigma b^2 \end{bmatrix}$

$\Rightarrow a^2 + b^2 + c^2 + d^2 + e^2 + f^2 = 5$

C – 1: $1+1+1+1+1+0 = {}^6C_5 \times 2 = 192$

C – 2: $4+1+0+0+0+0 = {}^6C_1 \times 2 \times {}^5C_1 \times 2 = 120$

Ans = 312

Question ID : 444792548

22. Let a line L passing through the point P(1, 1, 1) be perpendicular to the lines $\frac{x-4}{4} = \frac{y-1}{1} = \frac{z-1}{1}$ and $\frac{x-17}{1} = \frac{y-71}{1} = \frac{z}{0}$. Let the line L intersect the yz-plane at the point Q. Another line parallel to L and passing

through the point S(1, 0, –1) intersects the yz-plane at the point R. Then the square of the area of the parallelogram PQRS is equal to ________.

Ans. Official answer NTA (6)

Sol. Direction ratio of given line $= \begin{vmatrix} \hat{n} & \hat{j} & \hat{k} \\ 4 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix} = (-1, 1, 3)$

Equation of line L: $\dfrac{x-1}{-1} = \dfrac{y-1}{1} = \dfrac{z-1}{3} = \lambda$

$\Rightarrow (1-\lambda, 1+\lambda, 1+3\lambda)$ any point on L

For yz plane $\Rightarrow x = 0 \Rightarrow \lambda = 1$

Q : (0, 2, 4)

Equation of other line : $(1-\delta, 0+\delta, -1+3\delta)$

For R : Put $x = 0 \Rightarrow \delta = 1$

R : (0, 1, 2)

Vector area of PQRS

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -1 & -2 \\ 1 & -1 & -3 \end{vmatrix}$

= (1, –2, 1)

Area = $\sqrt{6}$

Question ID : 444792549

23. Let (2α, α) be the largest interval in which the function $f(t) = \dfrac{|t+1|}{t^2}, t < 0$, is strictly decreasing. Then the local maximum value of the function g(x) = 2 $\log_e (x-2) + \alpha x^2 + 4x - \alpha, x > 2$, is ________.

Ans. Official answer NTA (4)

Sol. $f(t) = \dfrac{|t+1|}{t^2} \quad t < 0$

$$\Rightarrow f(t) = \begin{cases} \dfrac{-t-1}{t^2}; & t \le -1 \\ \dfrac{t+1}{t^2}; & -1 < t < 0 \end{cases}$$

$$\Rightarrow f'(t)=\begin{cases}\dfrac{t(t+2)}{t^4}; & t<-1\\ -\dfrac{t(t+2)}{t^4}; & -1<t<0\end{cases}$$

If $t\in(-1,0)\Rightarrow f'(t)>0$

If $t\in(-\infty,-2)\Rightarrow f'(f)>0$

If $t\in(-2,-1)\Rightarrow f'(t)<0$

$\Rightarrow \alpha=-1$

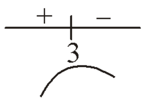
Now

$g(x)=2\ln(x-2)+\alpha x^2+4x-\alpha \quad x>2$

$\Rightarrow\ g(x)=2\ln(x-2)-x^2+4x+1; x>2$

$\Rightarrow\ g'(x)=\dfrac{2}{x-2}-2x+4$

$g'(x)=\dfrac{-2(x-1)}{(x-2)}(x-3) \quad x>2$



$\Rightarrow g(x)$ max at $x=3$

$\Rightarrow g(3)=-9+12+1$

$=4$

Question ID : 444792547

24. The number of numbers greater than 5000, less than 9000 and divisible by 3, that can be formed using the digits 0, 1, 2, 5, 9, if the repetition of the digits is allowed, is _________.

Ans. Official answer NTA (42)

Sol. $N = a\,b\,c\,d$ using 0, 1, 2, 5, 9

0, 9 : gives remainder 0 $\Rightarrow$ 3k type

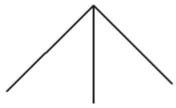
1 : gives remainder 1 $\Rightarrow$ 3k + 1 type

2, 5 : gives remainder 2 $\Rightarrow$ 3k + 2 type

since less than 9000

$\Rightarrow N = 5\,b\,c\,d$ type

$\Rightarrow b+c+d$ gives remaninder 1 when divided by 3 b + c + d = 3k + 1 type



(3k, 3k, 3k +1) (3k, 3k + 2, 3k + 2) (3k + 1, 3k + 1, 3k +2)

$\Rightarrow {}^3C_1 \times 2 \times 2 + {}^3C_1 \times 2 \times 2 \times 2 + {}^3C_1 \times 2 \times 1 \times 1$

= 12 + 24 + 6 = 42

Question ID : 444792550

25. Let a differentiable function f satisfy the equation $\int_0^{36} f\left(\frac{tx}{36}\right)dt = 4\alpha f(x)$. If y = f(x) is a standard parabola passing through the points (2, 1) and (–4, β), then β^α is equal to ________.

Ans. Official answer NTA (64)

Sol. Given

$$\int_0^{36} f\left(\frac{tx}{36}\right)dt = 4\alpha f(x)$$

let $\frac{tx}{36} = z \Rightarrow dt = \frac{36}{x}$

$$\Rightarrow \frac{36}{x}\int_0^x f(z)dz = 4\alpha f(x)$$

$$\Rightarrow 9\int_0^x f(z)dz = \alpha x f(x)$$

Diff. w.r.t x

$$\Rightarrow \quad 9f(x) = \alpha f(x) + \alpha x f'(x)$$

$$\Rightarrow \quad (9-\alpha)f(x) = \alpha x f'(x)$$

$$\Rightarrow \quad (9-\alpha)y = \alpha x \cdot \frac{dy}{dx}$$

$$\Rightarrow \frac{dx}{x} = \frac{\alpha}{9-\alpha}\frac{dy}{y}$$

$$\Rightarrow \ln|x| = \frac{\alpha}{(9-\alpha)}\ln|y| + C$$

passing through (2,1)

$\Rightarrow C = \ln 2$

$\Rightarrow \ln\left|\frac{x}{2}\right| = \frac{\alpha}{(9-\alpha)}\ln|y|$

$\Rightarrow |y| = \left|\frac{x}{2}\right|^{\frac{9-\alpha}{\alpha}}$

$\Rightarrow \frac{\alpha}{9-\alpha} = \frac{1}{2} \Rightarrow \alpha = 3$

passing through $(-4, \beta)$

$\Rightarrow \beta = 4 \Rightarrow \beta^3 = 2^6$