JEE Main January 2026

Question Paper With Text Solution

23 January | Shift-2

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN JANUARY 2026 | 23TH JANUARY SHIFT-2

SECTION – A

Question ID : 444792457

1. The number of ways, in which 16 oranges can be distributed to four children such that each child gets at least one orange, is :

(1) 455 (2) 429 (3) 403 (4) 384

Ans. Official answer NTA (1)

Sol. 12 coins 4 beggars 3 sticks

$$\Rightarrow \frac{\lfloor 15}{\lfloor 12 \lfloor 3} = \frac{15 \times 14 \times 13}{6} = 455$$

Question ID : 444792451

2. Let A = {0, 1, 2, ..., 9}. Let R be a relation on A defined by $(x, y) \in R$ if and only if $|x - y|$ is a multiple of 3.

Given below are two statements :

Statement-I : n(R) = 36.

Statement-II : R is an equivalence relation.

In the light of the above statements, choose the correct answer from the options given below :

(1) Statement I is incorrect but Statement II is correct

(2) Both Statement I and Statement II are incorrect

(3) Statement I is correct but Statement II is incorrect

(4) Both Statement I and Statement II are correct

Ans. Official answer NTA (1)

Sol. A = {0, 1, 2, ... 9}

$S_1 : \{x, y\} = (3k_1, 3k_2)$ or $(3k_1 + 1, 3k_2 + 1)$

or $(3k_1 + 2, 3k_2 + 2)$

$= 3 \times 3 + 3 \times 3 + 3 \times 3$

= 27 elements

S_1 is correct

as $(x, x) \in R \quad \forall x \in A$

$(x, y) \in R \rightarrow (y, x) \in R$

$(x, y) \in R$ and $(y, z) \in R \Rightarrow (x, z) \in R$

$\therefore$ Equivalence relation

Question ID : 444792453

3. The sum of all the real solutions of the equation $\log_{(x+3)}\left(6x^2 + 28x + 30\right) = 5 - 2\log_{(6x+10)}\left(x^2 + 6x + 9\right)$ is equal to :

(1) 0 (2) 1 (3) 4 (4) 2

Ans. Official answer NTA (1)

Sol. $\log_{x+3}\left(6x^2 + 28x + 30\right) = 5 - 2\log_{6x+10}\left(x^2 + 6x + 9\right)$

$6x^2 + 28x + 30 = 6x^2 + 18x + 10x + 30$

$= 6x(x + 3) + 10(x + 3)$

$= (6x + 10)(x + 3)$

$\log_{x+3}(6x + 10) + 1 = 5 - 4\log_{6x+10} x + 3$

$\frac{1}{\log_{6x+10} x + 3} - 4 = -4\log_{6x+10} x + 3$

$\log_{6x+10} x + 3 = t$

$\frac{1}{t} - 4 = -4t$

$1 - 4t = -4t^2$

$4t^2 - 4t + 1 = 0$

$(2t - 1)^2 = 0$

$t = \frac{1}{2}$

$\log_{6x+10} x + 3 = \frac{1}{2}$

$x + 3 = \sqrt{x + 10}$

$x^2 + 6x + 9 = 6x + 10$

$x^2 = 1$

$x = \pm 1$

sum = 0

Question ID : 444792464

4. Let $\frac{\pi}{2} < \theta < \pi$ and $\cot\theta = -\frac{1}{2\sqrt{2}}$. Then the value of

$\sin\left(\frac{15\theta}{2}\right)(\cos 8\theta + \sin 8\theta) + \cos\left(\frac{15\theta}{2}\right)(\cos 8\theta - \sin 8\theta)$ is equal to :

(1) $\frac{1-\sqrt{2}}{\sqrt{3}}$ (2) $-\frac{\sqrt{2}}{\sqrt{3}}$ (3) $\frac{\sqrt{2}}{\sqrt{3}}$ (4) $\frac{\sqrt{2}-1}{\sqrt{3}}$

Ans. Official answer NTA (1)

Sol. $\cot\theta = -\frac{1}{2\sqrt{2}}$

$$\sin\left(\frac{15\theta}{2}\right)\cos 8\theta - \sin 8\theta \cos\left(\frac{15\theta}{2}\right) + \sin 8\theta \sin\left(\frac{15\theta}{2}\right) + \cos 8\theta \cos\left(\frac{15\theta}{2}\right)$$

$$\sin\left(\frac{15\theta}{2} - 8\theta\right) + \cos\left(8\theta - \frac{15\theta}{2}\right)$$

$$-\sin\left(\frac{\theta}{2}\right) + \cos\left(\frac{\theta}{2}\right) = \cos\left(\frac{\theta}{2}\right) - \sin\left(\frac{\theta}{2}\right)$$

$$\tan\theta = -2\sqrt{2}$$

$$\frac{2\tan\left(\frac{\theta}{2}\right)}{1-\tan^2\left(\frac{\theta}{2}\right)} = -2\sqrt{2}$$

$$t = -\sqrt{2} + \sqrt{2}t^2$$

$$\sqrt{2}t^2 - t - \sqrt{2} = 0$$

$$t = \frac{1 \pm \sqrt{1+8}}{2\sqrt{2}}$$

$$t = \frac{1 \pm 3}{2\sqrt{2}}$$

$$t = \frac{4}{2\sqrt{2}} \text{ or } t = -\frac{2}{2\sqrt{2}}$$

$$t = \sqrt{2} \text{ or } t = -\frac{1}{\sqrt{2}}$$

$\tan\left(\frac{\theta}{2}\right)=\frac{\sqrt{2}}{1}=\frac{P}{B}$

$H=\sqrt{3}$

$\sin\left(\frac{\theta}{2}\right)=\frac{P}{H}=\frac{\sqrt{2}}{\sqrt{3}}$

$\cos\left(\frac{\theta}{2}\right)=\frac{B}{H}=\frac{1}{\sqrt{3}}$

$\cos\left(\frac{\theta}{2}\right)-\sin\left(\frac{\theta}{2}\right)=\frac{1-\sqrt{2}}{\sqrt{3}}$

Question ID : 444792466

5. Let $\vec{a},\vec{b},\vec{c}$ be three vectors such that $\vec{a}\times\vec{b}=2(\vec{a}\times\vec{c})$. If $|\vec{a}|=1, |\vec{b}|=4, |\vec{c}|=2$, and the angle between $\vec{b}$ and $\vec{c}$ is 60°, then $|\vec{a}\cdot\vec{c}|$ is equal to :

(1) 4 (2) 1 (3) 0 (4) 2

Ans. Official answer NTA (2)

Sol. $\vec{a}\times\vec{b}=2\left(\vec{a}\times\vec{c}\right)$

$|\vec{a}|=1$ $|\vec{b}|=4$ $|\vec{c}|=2$

$\vec{b}\wedge\vec{c}=\frac{\pi}{3}$

$\vec{a}\times\left(\vec{b}-2\vec{c}\right)=0$

$\vec{a}=\lambda\left(\vec{b}-2\vec{c}\right)$

$|\vec{a}|^2=\lambda^2\left(|\vec{b}|^2+4|\vec{c}|^2-4\vec{b}\cdot\vec{c}\right)$

$1=\lambda^2\left(16+16-4|\vec{b}||\vec{c}|\times\frac{1}{2}\right)$

$1=\lambda^2\left(32-4\times4\times2\times\frac{1}{2}\right)$

$1=\lambda^2(16)$

$\lambda^2=\frac{1}{16}\Rightarrow\lambda=\pm\frac{1}{4}$

$\vec{a} = \pm\frac{1}{4}\left(\vec{b} - 2\vec{c}\right)$

$\vec{a} = \pm\frac{\vec{b}}{4} \mp \frac{\vec{c}}{2}$

$\vec{a} = \frac{\vec{b}}{4} - \frac{\vec{c}}{2}$ or $\vec{a} = -\frac{\vec{b}}{4} + \frac{\vec{c}}{2}$

$\vec{a}\cdot\vec{c} = \frac{\vec{b}\cdot\vec{c}}{4} - \frac{|\vec{c}|^2}{2}$

$\vec{a}\cdot\vec{c} = \frac{|\vec{b}||\vec{c}|}{8} - 2$

$\vec{a}\cdot\vec{c} = \frac{4\times 2}{8} - 2$

$\vec{a}\cdot\vec{c} = -1$

$\vec{a}\cdot\vec{c} = \frac{-\vec{b}\cdot\vec{c}}{4} + 2$

$\vec{a}\cdot\vec{c} = \frac{-4\times 2}{8} + 2$

$\vec{a}\cdot\vec{c} = 1$

Question ID : 444792469

6. Let $I(x) = \int \frac{3dx}{(4x+6)\left(\sqrt{4x^2+8x+3}\right)}$ and $I(0) = \frac{\sqrt{3}}{4} + 20$. If $I\left(\frac{1}{2}\right) = \frac{a\sqrt{2}}{b} + c$, where a, b, c $\in$ N, gcd(a, b) = 1, then a + b + c is equal to :

(1) 30 (2) 31 (3) 28 (4) 29

Ans. Official answer NTA (2)

Sol. $I(x) = \int \frac{3dx}{(4x+6)\sqrt{4x^2+8x+3}}$

$4x + 6 = \frac{1}{t}$

$dx = \frac{-dt}{4t^2}$

$I(t) = -\frac{3}{2}\int \frac{dt}{\sqrt{1-4t}}$

$1 - 4t = u^2$

$dt = \frac{-udu}{2}$

$\Rightarrow -\dfrac{3}{2}\int \dfrac{-udu}{2u} = \dfrac{3u}{4} + c$

$I(x) = \dfrac{3}{4}\sqrt{\dfrac{4x+2}{4x+6}} + c$

$I(0) = \dfrac{3}{4} \times \sqrt{\dfrac{2}{6}} + c$

$c = 20$

$I(x) = \dfrac{3}{4}\sqrt{\dfrac{4x+2}{4x+6}} + 20$

$I\left(\dfrac{1}{2}\right) = \dfrac{3\sqrt{2}}{8} + 20$

$\Rightarrow a = 3 \quad b = 8 \quad c = 20$

$a + b + c = 31$

Question ID : 444792456

7. Let $\sum_{k=1}^{n} a_k = \alpha n^2 + \beta n$. If $a_{10} = 59$ and $a_6 = 7a_1$, then $\alpha + \beta$ is equal to :

(1) 3 (2) 12 (3) 7 (4) 5

Ans. Official answer NTA (4)

Sol. $\sum_{k=1}^{n} a_k = \alpha n^2 + \beta n$

$a_1 + a_2 + \ldots + a_n = \alpha n^2 + \beta n$

$T_n = \alpha n^2 + \beta n - (\alpha(n-1)^2 + \beta(n-1))$

$T_n = \alpha n^2 + \beta n - (\alpha n^2 - 2\alpha n + \alpha + \beta n - \beta)$

$T_n = \alpha n^2 + \beta n - \alpha n^2 + 2\alpha n - \alpha - \beta n + \beta$

$T_n = 2\alpha n - \alpha + \beta$

$a_k = 2\alpha k - \alpha + \beta$

$a_{10} = 20\alpha - \alpha + \beta$

$59 = 19\alpha + \beta$

$a_6 = 12\alpha - \alpha + \beta = 11\alpha + \beta$

$a_1 = \alpha + \beta$

$11\alpha + \beta = 7(\alpha + \beta)$

$11\alpha + \beta = 7\alpha + 7\beta$

$4\alpha = 6\beta \Rightarrow \alpha = \frac{3\beta}{2}$

$59 = 19 \times \frac{3\beta}{2} + \beta$

$59 = \frac{59\beta}{2} \Rightarrow \beta = 2$

$\alpha = \frac{3 \times 2}{2} = 3$

$\alpha + \beta = 5$

Question ID : 444792458

8. Bag A contains 9 white and 8 black balls, while bag B contains 6 white and 4 black balls. One ball is randomly picked up from the bag B and mixed up with the balls in the bag A. Then a ball is randomly drawn from the bag A. If the probability, that the ball drawn is white, is $\frac{p}{q}$, $\gcd(p, q) = 1$, then p + q is equal to :

(1) 22 (2) 23 (3) 24 (4) 21

Ans. Official answer NTA (2)

Sol. $A \rightarrow 9W, 8B$

$B \rightarrow 6W, 4B$

$P = P(W) + P(B)$

$P = \frac{6}{10} \times \frac{10}{18} + \frac{4}{10} \times \frac{9}{18}$

$P = \frac{1}{3} + \frac{1}{5} = \frac{8}{15}$

$p + q = 23$

Question ID : 444792452

9. Consider two sets $A = \{x \in Z : |(|x-3|-3)| \le 1\}$ and $B = \left\{x \in R - \{1,2\} : \frac{(x-2)(x-4)}{x-1} \log_e(|x-2|) = 0\right\}$.

Then the number of onto functions $f : A \rightarrow B$ is equla to :

(1) 79 (2) 32 (3) 62 (4) 81

Ans. Official answer NTA (3)

Sol. $||x-3|-3| \le 1$

$-1 \le |x-3|-3 \le 1$

$2 \le |x-3| \le 4$

$|x-3| \ge 2$ $\qquad$ $|x-3| \le 4$

$x-3 \le -2,\ x-3 \ge 2$ $\qquad$ $-4 \le x-3 \le 4$

$x \le 1 \quad x \ge 5$ $\qquad$ $-1 \le x \le 7$

$x \in [-1, 1] \cup [5, 7]$

$x = -1, 0, 1, 5, 6, 7$

$$\frac{(x-2)(x-4)}{x-1}\ell n|x-2| = 0$$

$x = 4$ $\qquad$ $\ell n|x-2| = 0$

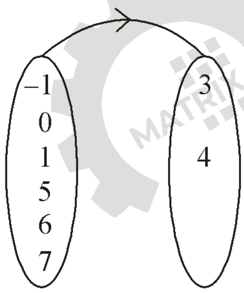
$|x-2| = 1$

$x-2 = \pm 1$

$x = 2 \pm 1$

$x = 3$

$x = 3, 4$



G_1	G_2
1	5
2	4
3	3

$$\frac{\lfloor 6}{\lfloor 1 \lfloor 5} \times \lfloor 2 + \frac{\lfloor 6}{\lfloor 2 \lfloor 4} \times \lfloor 2 + \frac{\lfloor 6}{\lfloor 3 \lfloor 3 \times \lfloor 2} \times \lfloor 2$$

$$12 + 30 + \frac{6 \times 5 \times 4}{6} = 62$$

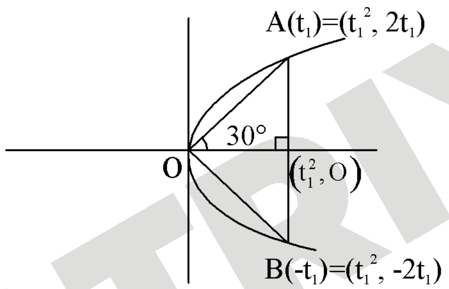
Question ID : 444792461

10. An equilateral triangle OAB is inscribed in the parabola $y^2 = 4x$ with the vertex O at the vertex of the parabola. Then the minimum distance of the circle having AB as a diameter from the origin is :

(1) $2(3+\sqrt{3})$ (2) $4(6+\sqrt{3})$ (3) $2(8-3\sqrt{3})$ (4) $4(3-\sqrt{3})$

Ans. Official answer NTA (4)

Sol. $\tan 30° = \frac{2t_1}{t_1^2}$



$\frac{1}{\sqrt{3}} = \frac{2}{t_1}$

$t_1 = 2\sqrt{3}$

$A(12, 4\sqrt{3})$ $B(12, -4\sqrt{3})$

$(x-12)(x-12)+(y-4\sqrt{3})(y+4\sqrt{3}) = 0$

$x^2 - 24x + 144 + y^2 - 48 = 0$

$x^2 + y^2 - 24x + 96 = 0$

$C = (12, 0)$ $r = \sqrt{144-96} = \sqrt{48} = 4\sqrt{3}$

Minimum distance = OC – r

$= 12 - 4\sqrt{3}$

$= 4(3-\sqrt{3})$

Question ID : 444792465

11. Let $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{b} = 2\hat{i} + \hat{j} - \hat{k}, \vec{c} = \lambda\hat{i} + \hat{j} + \hat{k}$ and $\vec{v} = \vec{a} \times \vec{b}$. If $\vec{v} \cdot \vec{c} = 11$ and the length of the projection of $\vec{b}$ on $\vec{c}$ is p, then $9p^2$ is equal to :

(1) 12 (2) 6 (3) 4 (4) 9

Ans. Official answer NTA (1)

Sol. $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$

$\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$

$\vec{c} = \lambda\hat{i} + \hat{j} + \hat{k}$

$\vec{v} = \vec{a} \times \vec{b}$

$\vec{v} \cdot \vec{c} = 11$

$$P = \frac{\vec{b}\cdot\vec{c}}{|\vec{c}|}$$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 2 & 1 & -1 \end{vmatrix}$$

$$= \hat{i}(-1) - \hat{j}(-7) + \hat{k}(5)$$

$$\vec{a}\times\vec{b} = -\hat{i} + 7\hat{j} + 5\hat{k}$$

$$\vec{v} = -\hat{i} + 7\hat{j} + 5\hat{k}$$

$$\vec{v}\cdot\vec{c} = 11$$

$$-\lambda + 7 + 5 = 11$$

$$-\lambda = -1$$

$$\lambda = 1$$

$$\vec{c} = \hat{i} + \hat{j} + \hat{k}$$

$$P = \frac{2}{\sqrt{3}}$$

$$9P^2 = 9\times\frac{4}{3} = 12$$

Question ID : 444792455

12. The system of linear equations

$x + y + z = 6$

$2x + 5y + az = 36$

$x + 2y + 3z = b$ has :

(1) unique solution for $a = 8$ and $b = 16$

(2) unique solution for $a = 8$ and $b = 14$

(3) infinitely many solutions for $a = 8$ and $b = 14$

(4) infinitely many solutions for $a = 8$ and $b = 16$

Ans. Official answer NTA (3)

Sol. $x + y + z = 6$

$2x + 5y + az = 36$

$x + 2y + 3z = b$

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 5 & a \\ 1 & 2 & 3 \end{vmatrix} = 1(15 - 2a) - 1(6 - a) + 1(-1)$$

$= 15 - 2a - 6 + a - 1$
$= 8 - a$

$$D_x = \begin{vmatrix} 6 & 1 & 1 \\ 36 & 5 & a \\ b & 2 & 3 \end{vmatrix} = 6\,(15 - 2a) - (108 - ab) + 1(72 - 5b)$$

$= 90 - 12a - 108 + ab + 72 - 5b$
$= 54 - 12a - 5b + ab$

$$D_y = \begin{vmatrix} 1 & 6 & 1 \\ 2 & 36 & a \\ 1 & b & 3 \end{vmatrix} = 1(108 - ab) - 6(6 - a) + 1(2b - 36)$$

$= 108 - ab - 36 + 6a + 2b - 36$
$= 36 - ab + 6a + 2b$

$$D_z = \begin{vmatrix} 1 & 1 & 6 \\ 2 & 5 & 36 \\ 1 & 2 & b \end{vmatrix} = 1(5b - 72) - 1(2b - 36) + 6(-1)$$

$= 5b - 72 - 2b + 36 - 6$

$= 3b - 42$

Question ID : 444792454

13. If $z = \frac{\sqrt{3}}{2} + \frac{i}{2}, i = \sqrt{-1}$, then $(z^{201} - i)^8$ is equla to :

(1) 0 (2) 256 (3) –1 (4) 1

Ans. Official answer NTA (2)

Sol. $z = \frac{\sqrt{3}}{2} + \frac{i}{2}$

$$\arg(z) = \tan^{\pi}\left(\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) = \frac{\pi}{6}$$

$z = e^{\frac{i\pi}{6}}$

$z^{201} = e^{i\frac{201\pi}{6}}$

$= \cos\left(\frac{201\pi}{6}\right) + i\sin\left(\frac{201\pi}{6}\right)$

$= \cos\left(\frac{67\pi}{2}\right) + i\sin\left(\frac{67\pi}{2}\right)$

$= 0 + i\sin\left(33\pi + \frac{\pi}{2}\right)$

$= -i\sin\frac{\pi}{2} = -i$

$(-i - i)^8 = (-2i)^8$

$= (-2)^8 (i)^8$

$= 2^8 = 256$

Question ID : 444792470

14. The area of the region enclosed between the circles $x^2 + y^2 = 4$ and $x^2 + (y - 2)^2 = 4$ is :

(1) $\frac{4}{3}\left(2\pi - \sqrt{3}\right)$ (2) $\frac{2}{3}\left(2\pi - 3\sqrt{3}\right)$ (3) $\frac{4}{3}\left(2\pi - 3\sqrt{3}\right)$ (4) $\frac{2}{3}\left(4\pi - 3\sqrt{3}\right)$

Ans. Official answer NTA (4)

Sol. $x^2 + y^2 - 4y + 4 = 4$

$4 = 4y$

$y = 1$

$x^2 = 3$

$x = \pm\sqrt{3}$

$(y - 2)^2 = 4 - x^2$

$y - 2 = \pm\sqrt{4 - x^2}$

$y = 2 \pm \sqrt{4 - x^2}$

$$A = \int_{-\sqrt{3}}^{\sqrt{3}} \left(\sqrt{4 - x^2} - \left(2 - \sqrt{4 - x^2}\right)\right)dx$$

$$A = \int_{-\sqrt{3}}^{\sqrt{3}} \left(2\sqrt{4 - x^2} - 2\right)dx$$

$$A = 2\left(\frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right)\Bigg|_{-\sqrt{3}}^{\sqrt{3}} - 2\left(2\sqrt{3}\right)$$

$$A = x\sqrt{4-x^2} + 4\sin^{-1}\left(\frac{x}{2}\right)\Big|_{-\sqrt{3}}^{\sqrt{3}} - 4\sqrt{3}$$

$$A = \sqrt{3}\sqrt{4-3} + 4\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \left(-\sqrt{3}\sqrt{4-1} + 4\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right) - 4\sqrt{3}$$

$$A = \sqrt{3} + 4\times\frac{\pi}{3} - \left(-\sqrt{3} - 4\times\frac{\pi}{3}\right) - 4\sqrt{3}$$

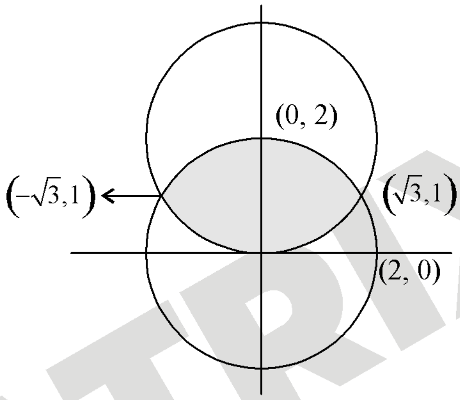
$$A = \sqrt{3} + \frac{4\pi}{3} + \sqrt{3} + \frac{4\pi}{3} - 4\sqrt{3}$$

$$A = 2\sqrt{3} + \frac{8\pi}{3} - 4\sqrt{3}$$

$$A = \frac{8\pi}{3} - 2\sqrt{3}$$

$$A = \frac{8\pi - 6\sqrt{3}}{3}$$

$$A = \frac{2}{3}\left(4\pi - 3\sqrt{3}\right)$$



Question ID : 444792460

15. If the points of intersection of the ellipses $x^2 + 2y^2 - 6x - 12y + 23 = 0$ and $4x^2 + 2y^2 - 20x - 12y + 35 = 0$ lie on a circle of radius r and centre (a, b), then the value of $ab + 18r^2$ is :

(1) 52 (2) 55 (3) 51 (4) 53

Ans. Official answer NTA (2)

Sol. $S_1 + \lambda S_2 = 0$

$x^2 + 2y^2 - 6x - 12y + 23 + \lambda(4x^2 + 2y^2 - 20x - 12y + 35) = 0$

$(1 + 4\lambda)x^2 + (2\lambda + 2)y^2 - (6 + 20\lambda)x - (12 + 12\lambda)y + 23 + 35\lambda = 0$

$1 + 4\lambda = 2\lambda + 2$

$2\lambda = 1$

$\lambda = \frac{1}{2}$

$(1 + 2)x^2 + (1 + 2)y^2 - (6 + 10)x - (12 + 6)y + 23 + \frac{35}{2} = 0$

$3x^2 + 3y^2 - 16x - 18y + \frac{81}{2} = 0$

$x^2 + y^2 - \frac{16x}{3} - 6y + \frac{27}{2} = 0$

$c = \left(\frac{8}{3}, 3\right)$

$r = \sqrt{\frac{64}{9} + 9 - \frac{27}{2}}$

$r = \sqrt{\frac{128 + 9 \times 18 - 27 \times 9}{18}}$

$r = \sqrt{\frac{128 + 162 - 243}{18}}$

$r = \sqrt{\frac{47}{18}}$

$ab + 18r^2 = \frac{8}{3} \times 3 + 47 = 55$

Question ID : 444792459

16. If the mean and the variance of the data

Class	4 – 8	8 – 12	12 – 16	16 – 20
Frequency	3	λ	4	7

are μ and 19 respectively, then the value of $\lambda + \mu$ is :

(1) 20 (2) 18 (3) 19 (4) 21

Ans. Official answer NTA (3)

Sol.

Class	x_i	f_i
4–8	6	3
8–12	10	λ
12–16	14	4
16–20	18	7

$\text{Mean} = \frac{\sum x_i f_i}{\sum f_i}$

$$\mu = \frac{18 + 10\lambda + 56 + 126}{14 + \lambda}$$

$$\mu = \frac{200 + 10\lambda}{14 + \lambda} \quad \text{\_\_\_\_\_\_(1)}$$

$$19 = \frac{\sum f_i x_i^2}{\sum f_i} - \mu^2$$

$$19 = \frac{3160 + 100\lambda}{14 + \lambda} - \left(\frac{200 + 10\lambda}{14 + \lambda}\right)^2 \quad \text{\_\_\_\_\_\_(2)}$$

from (1) and (2)

$\lambda = 6,\ \mu = 13$

$\lambda + \mu = 19$

Question ID : 444792468

17. The least value of $(\cos^2\theta - 6\sin\theta\cos\theta + 3\sin^2\theta + 2)$ is :

(1) $4 - \sqrt{10}$ (2) $4 + \sqrt{10}$ (3) 1 (4) -1

Ans. Official answer NTA (1)

Sol. $\frac{1 + \cos 2\theta}{2} - 3\sin 2\theta + 3\left(\frac{1 - \cos 2\theta}{2}\right) + 2$

$\frac{1}{2} + \frac{\cos 2\theta}{2} - 3\sin 2\theta + \frac{3}{2} - \frac{3}{2}\cos 2\theta + 2$

$-\cos 2\theta - 3\sin 2\theta + 4$

$-\cos 2\theta - 3\sin 2\theta \in \left[-\sqrt{10}, \sqrt{10}\right]$

$-\cos 2\theta - 3\sin 2\theta + 4 \in \left[4 - \sqrt{10}, 4 + \sqrt{10}\right]$

Question ID : 444792467

18. If $f(x) = \begin{cases} \frac{a|x| + x^2 - 2(\sin|x|)(\cos|x|)}{x} & , x \neq 0 \\ b & , x = 0 \end{cases}$ is continuous at $x = 0$, then $a + b$ is equal to :

(1) 1 (2) 2 (3) 0 (4) 4

Ans. Official answer NTA (2)

Sol. $$f(x)=\begin{cases}\dfrac{-ax+x^2+2\sin x\cos x}{x} & x<0\\ b & x=0\\ \dfrac{ax+x^2-2\sin x\cos x}{x} & x>0\end{cases}$$

$$\lim_{x\to 0^-}\frac{-ax+x^2+\sin 2x}{x}$$

$$\lim_{x\to 0^-}\frac{-ax+x^2+2x-\frac{8x^3}{6}}{x}$$

$$\lim_{x\to 0^-}-a+x+2-\frac{4x^2}{3}=2-a$$

$$\lim_{x\to 0^+}\frac{ax+x^2-2x+\frac{8x^3}{6}}{x}=\lim_{x\to 0^+}a+x-2+\frac{4x^2}{3}=a-2$$

$2-a=a-2$

$4=2a\Rightarrow a=2\quad b=0$

Question ID : 444792462

19. Let PQ be a chord of the hyperbola $\frac{x^2}{4}-\frac{y^2}{b^2}=1$, perpendicular to the x-axis such that OPQ is an equilateral triangle, O being the centre of the hyperbola. If the eccentricity of the hyperbola is $\sqrt{3}$, then the area of the triangle OPQ is :

(1) $\frac{9}{5}$ (2) $\frac{11}{5}$ (3) $\frac{8\sqrt{3}}{5}$ (4) $2\sqrt{3}$

Ans. Official answer NTA (3)

Sol. $\frac{x^2}{4}-\frac{y^2}{b^2}=1$

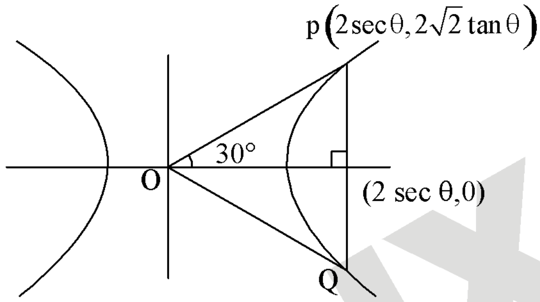
$e^2=1+\frac{b^2}{a^2}$

$3=1+\frac{b^2}{4}$

$2=\frac{b^2}{4}\Rightarrow b^2=8\Rightarrow b=2\sqrt{2}$

$H:\frac{x^2}{4}-\frac{y^2}{8}=1$

$\tan 30°=\frac{2\sqrt{2}\tan\theta}{2\sec\theta}$



$\frac{1}{\sqrt{3}}=\frac{\frac{\sqrt{2}\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}}$

$\sin\theta=\frac{1}{\sqrt{6}}=\frac{P}{H}$

$B=\sqrt{6-1}=\sqrt{5}$

$\cos\theta=\frac{\sqrt{5}}{\sqrt{6}}$ $\qquad$ $\sec\theta=\frac{\sqrt{6}}{\sqrt{5}}$

$P\left(\frac{2\sqrt{6}}{\sqrt{5}},\frac{2\sqrt{2}}{\sqrt{5}}\right)$

$OP=\sqrt{\frac{24}{5}+\frac{8}{5}}=\sqrt{\frac{32}{5}}$

$\text{Area}=\frac{\sqrt{3}}{4}\times\frac{32}{5}=\frac{8\sqrt{3}}{5}$

Question ID : 444792463

20. Let A(1, 2) and C(–3, –6) be two diagonally opposite vertices of a rhombus, whose sides AD and BC are parallel to the line $7x-y=14$. If $B(\alpha,\beta)$ and $D(\gamma,\delta)$ are the other two vertices, then $|\alpha+\beta+\gamma+\delta|$ is equal to :

(1) 6 (2) 1 (3) 3 (4) 9

Ans. Official answer NTA (1)

Sol. AD : $7x-y+\lambda=0$

Passes through A (1,2)

$7-2+\lambda=0$

$\lambda = -5$

AD : $7x - y - 5 = 0$

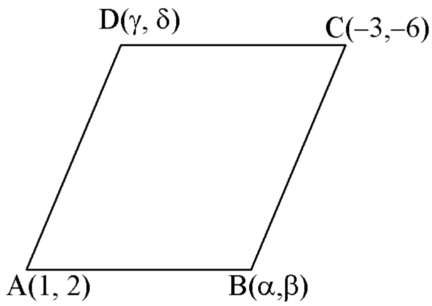
BC : $7x - y + \lambda = 0$

Passes through $(-3, -6)$

$-21 + 6 + \lambda = 0$

$\lambda = 15$

BC : $7x - y + 15 = 0$

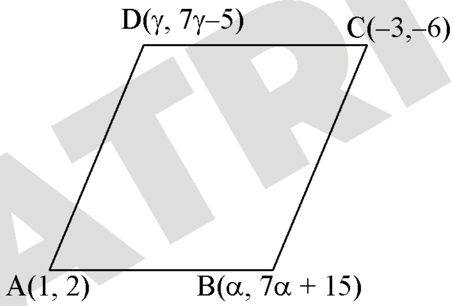


$7\gamma - \delta - 5 = 0$

$7\gamma - 5 = \delta$

$7\alpha - \beta + 15$

$\beta = 7\alpha + 15$



Mid-point of AC = mid-point of BD

$$(-1,-2) = \left(\frac{\alpha+\gamma}{2}, \frac{7\gamma - 5 + 7\alpha + 15}{2}\right)$$

$\alpha + \gamma = -2$

$$-2 = \frac{7\alpha + 7\gamma + 10}{2}$$

$-4 = 7\alpha + 7\gamma + 10$

$7\alpha + 7\gamma + 14 = 0$

$\alpha + \gamma = -2$

$|\alpha + 7\alpha + 15 + \gamma + 7\gamma - 5|$

$|8\alpha + 8\gamma + 10| = |8(-2) + 10|$

$= 6$

SECTION – B

Question ID : 444792472

21. Let S denote the set of 4-digit numbers abcd such that $a > b > c > d$ and P denote the set of 5-digit numbers having product of its digits equal to 20. Then $n(S) + n(P)$ is equal to ________.

Ans. Official answer NTA (260)

Sol. $a > b > c > d$

$$S = {}^{10}C_4 = \frac{\lfloor 10}{\lfloor 4 \lfloor 6} = \frac{10 \times 9 \times 8 \times 7}{24} = 210$$

For P :

$$\underline{1}\ \underline{1}\ \underline{1}\ \underline{4}\ \underline{5} \Rightarrow \frac{\lfloor 5}{\lfloor 3} = 20$$

$$\underline{1}\ \underline{1}\ \underline{2}\ \underline{2}\ \underline{5} \Rightarrow \frac{\lfloor 5}{\lfloor 2 \lfloor 2} = 30$$

$n(S) + n(P) = 210 + 50 = 260$

Question ID : 444792475

22. If the solution curve $y = f(x)$ of the differential equation $(x^2 - 4)y' - 2xy + 2x(4 - x^2)^2 = 0$, $x > 2$, passes through the point (3, 15), then the local maximum value of f is ________.

Ans. Official answer NTA (16)

Sol. $y = f(x)$

$$\left(x^2 - 4\right)\frac{dy}{dx} - 2xy + 2x\left(4 - x^2\right)^2 = 0$$

$$\frac{dy}{dx} - \left(\frac{2x}{x^2 - 4}\right)y + 2x\left(x^2 - 4\right) = 0$$

$$\frac{dy}{dx} - \left(\frac{2x}{x^2 - 4}\right)y = -2x\left(x^2 - 4\right)$$

$$\text{I.F.} = e^{\int \frac{-2x}{x^2 - 4}dx}$$

$$= e^{-\ell n\left(x^2 - 4\right)}$$

$$= e^{\ell n\left(\frac{1}{x^2 - 4}\right)} = \frac{1}{x^2 - 4}$$

$$\frac{y}{x^2 - 4} = -\int 2x\left(x^2 - 4\right) \times \frac{1}{x^2 - 4}dx$$

$$\frac{y}{x^2 - 4} = -\frac{2x^2}{2} + c$$

$$\frac{F(x)}{x^2 - 4} = -x^2 + c$$

$\frac{15}{5} = -9 + c$

c = 12

$\frac{F(x)}{x^2 - 4} = -x^2 + 12$

$F(x) = (x^2 - 4)(12 - x^2)$

$F'(x) = (x^2 - 4)(-2x) + (12 - x^2)(2x)$

$= 2x(12 - x^2 - x^2 + 4)$

$= 2x(16 - 2x^2)$

$= 4x(8 - x^2)$

$= 4x\left(2\sqrt{2} - x\right)\left(2\sqrt{2} + x\right)$

$\underset{-2\sqrt{2}}{+ \quad -} \quad \underset{O}{\quad} \quad \underset{2\sqrt{2}}{+ \quad -}$

$F\left(2\sqrt{2}\right) = \left(\left(2\sqrt{2}\right)^2 - 4\right)\left(12 - \left(2\sqrt{2}\right)^2\right)$

$= 4 \times 4 = 16$

Question ID : 444792471

23. Let $A = \begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & 1 \\ 3 & -1 & 0 \end{bmatrix}$ and B be a matrix such that $B(I - A) = I + A$. Then the sum of the diagonal elements of B^TB is equal to _________.

Ans. Official answer NTA (3)

Sol. $A = \begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & 1 \\ 3 & -1 & 0 \end{bmatrix}$

$A^T = -A$

$I - A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 1 & -1 \\ -3 & 1 & 1 \end{bmatrix}$

$$I + A = \begin{bmatrix} 1 & 2 & -3 \\ -2 & 1 & 1 \\ 3 & -1 & 1 \end{bmatrix}$$

$$B\begin{bmatrix} 1 & -2 & 3 \\ 2 & 1 & -1 \\ -3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & -3 \\ -2 & 1 & 1 \\ 3 & -1 & 1 \end{bmatrix}$$

$|I - A| = 1(2) + 2(2 - 3) + 3(5)$
$= 2 - 2 + 15 = 15$

$$(I - A)^{-1} = \frac{1}{15}(\text{Matrix of co-factors})^{T}$$

$$\text{Maxtrix of co-factors} = \begin{bmatrix} 2 & 1 & 5 \\ 5 & 10 & 5 \\ -1 & 7 & 5 \end{bmatrix}^{T}$$

$$(I - A)^{-1} = \frac{1}{15}\begin{bmatrix} 2 & 5 & -1 \\ 1 & 10 & 7 \\ 5 & 5 & 5 \end{bmatrix}$$

$$B = \frac{1}{15}\begin{bmatrix} 1 & 2 & -3 \\ -2 & 1 & 1 \\ 3 & -1 & 1 \end{bmatrix}\begin{bmatrix} 2 & 5 & -1 \\ 1 & 10 & 7 \\ 5 & 5 & 5 \end{bmatrix}$$

$$B = \frac{1}{15}\begin{bmatrix} -11 & 10 & -2 \\ 2 & 5 & 14 \\ 10 & 10 & -5 \end{bmatrix}$$

$$B^{-1}B = \frac{1}{225}\begin{bmatrix} -11 & 2 & 10 \\ 10 & 5 & 10 \\ -2 & 14 & -5 \end{bmatrix}\begin{bmatrix} -11 & 10 & -2 \\ 2 & 5 & 14 \\ 10 & 10 & -5 \end{bmatrix}$$

$$= \frac{1}{225}\begin{bmatrix} 121+4+100 & & \\ & 100+25+100 & \\ & & 4+196+25 \end{bmatrix}$$

$$\text{Sum} = \frac{1}{225}(225 + 225 + 225) = 3$$

Question ID : 444792474

24. The number of elements in the set $S = \left\{x : x \in [0,100] \text{ and } \int_0^x t^2 \sin(x-t)dt = x^2\right\}$ is ________.

Ans. Official answer NTA (16)

Sol. $S = x \in [0, 100]$

$$\int_0^x t^2 \sin(x-t)\,dt = x^2$$

$$\int_0^x (x-t)^2 \sin(t)\,dt = x^2$$

$$\int_0^x (x^2 - 2tx + t^2)\sin t\,dt = x^2$$

$$x^2\int_0^x \sin t\,dt - 2x\int_0^x t\sin t\,dt + \int_0^x t^2 \sin t\,dt = x^2$$

$$-x^2 \cos t\Big|_0^x - 2x\left[-t\cos t - \int 1 \times -\cos t\,dt\right] + \left[-t^2 \cos t - \int 2t \times -\cos t\,dt\right] = x^2$$

$$-x^2(\cos x - 1) - 2x(-t\cos t + \sin t)\Big|_0^x - t^2 \cos t + 2\int t\cos t\,dt = x^2$$

$$x^2(1-\cos x) - 2x(-x\cos x + \sin x) - (x^2 \cos x) + 2\left(t\sin t - \int \sin t\,dt\right) = x^2$$

$$x^2 - x^2 \cos x + 2x^2 \cos x - 2x \sin x - x^2 \cos x + 2(t \sin t + \cos t)\Big|_0^x = x^2$$

$$x^2 - 2x \sin x + 2(x \sin x + \cos x - (1)) = x^2$$

$$x^2 - 2x \sin x + 2x \sin x + 2 \cos x - 2 = x^2$$

$\cos x = 1$

$x = 2n\pi$

$x = 0, 2\pi, 4\pi, 6\pi, \ldots\ldots\ldots 31\pi$

No. of elements = 16

Question ID : 444792473

25. If the image of the point P(a, 2, a) in the line $\frac{x}{2} = \frac{y+a}{1} = \frac{z}{1}$ is Q and the image of Q in the line $\frac{x-2b}{2} = \frac{y-a}{1} = \frac{z+2b}{-5}$ is P, then a + b is equal to ________.

Ans. Official answer NTA (3)

Sol. P(a,2,a)

$L_1 : \vec{r} = (0, -a, 0) + \lambda(2,1,1)$

$R = (2\lambda, \lambda - a, \lambda)$

$\overrightarrow{PR} = (2\lambda - a)\hat{i} + (\lambda - a - 2)\hat{j} + (\lambda - a)\hat{k}$

$\overrightarrow{PR} \cdot (2\hat{i} + \hat{j} + \hat{k}) = 0$

$2(2\lambda - a) + 1(\lambda - a - 2) + 1(\lambda - a) = 0$

$4\lambda - 2a + \lambda - a - 2 + \lambda - a = 0$

$6\lambda - 4a - 2 = 0$

$3\lambda - 2a - 1 = 0$

$\lambda = \frac{2a+1}{3}$

$R = \left(\frac{4a+2}{3}, \frac{2a+1}{3} - a, \frac{2a+1}{3}\right)$

$R = \left(\frac{4a+2}{3}, \frac{1-a}{3}, \frac{2a+1}{3}\right)$

$Q(\alpha, \beta, \gamma)$

$\frac{\alpha + a}{2} = \frac{4a+2}{3}$

$\alpha + a = \frac{8a+4}{3}$

$\alpha = \frac{5a+4}{3}$

$\frac{\beta+2}{2} = \frac{1-a}{3}$

$\beta + 2 = \frac{2-2a}{3}$

$\beta = \frac{2-2a}{3} - 2$

$\beta = \frac{2-2a-6}{3}$

$\beta = \frac{-2a-4}{3}$

$$\frac{\gamma+a}{2}=\frac{2a+1}{3}$$

$$\gamma+a=\frac{4a+2}{3}$$

$$\gamma=\frac{4a+2}{3}-a$$

$$\gamma=\frac{a+2}{3}$$

$$Q\left(\frac{5a+4}{3},\frac{-2a-4}{3},\frac{a+2}{3}\right)$$

$P(a, 2, a)$ $\qquad$ $Q\left(\frac{5a+4}{3},\frac{-2a-4}{3},\frac{a+2}{3}\right)$

$$\overrightarrow{PQ}=\left(\frac{4+2a}{3}\right)\hat{i}-\left(\frac{2a+10}{3}\right)\hat{j}+\left(\frac{2-2a}{3}\right)\hat{k}$$

$$\overrightarrow{PQ}\cdot\left(2\hat{i}+\hat{j}-5\hat{k}\right)=0$$

$$2\left(\frac{4+2a}{3}\right)-\left(\frac{2a+10}{3}\right)-\frac{5(2-2a)}{3}=0$$

$8 + 4a - 2a - 10 - 10 + 10a = 0$

$12a - 12 = 0$

$a = 1$

$Q(3, -2, 1)$

$$L_2 : \vec{r}=(2b,1,-2b)+\lambda(2,1,-5)$$

$$S=(2b+2\lambda,1+\lambda,-2b-5\lambda)$$

$$\frac{1+3}{2}=2b+2\lambda$$

$2 = 2b + 2\lambda$

$b + \lambda = 1$

$$\frac{2-2}{2}=1+\lambda$$

$\lambda = -1$

$b - 1 = 1$

$b = 2$

$a + b = 3$