JEE Main January 2026

Question Paper With Text Solution

22 January | Shift-2

MATHEMATICS

JEE MAIN JANUARY 2026 | 22ᵀᴴ JANUARY SHIFT-2

SECTION – A

Question ID : 860654987

1. Let the locus of the mid-point of the chord through the origin O of the parabola $y^2 = 4x$ be the curve S. Let P be any point on S. Then the locus of the point, which internally divides OP in the ratio 3 : 1, is :

   (1) $2y^2 = 3x$ (2) $2x^2 = 3y$ (3) $3y^2 = 2x$ (4) $3x^2 = 2y$

Ans. Official answer NTA (1)

Sol.

Question ID : 860654988

2. Among the statements

   (S1) : If A(5, –1) and B(–2, 3) are two vertices of a triangle, whose orthocentre is (0, 0), then its third vertex is (–4, –7)

   and

   (S2) : If positive numbers 2a, b, c are three consecutive terms of an A.P., then the lines $ax + by + c = 0$ are concurrent at (2, –2), :

   (1) both are correct   (2) only (S1) is correct (3) both are incorrect   (4) only (S2) is correct

Ans. Official answer NTA (1)

Sol.

Question ID : 860654991

3. Let [•] denote the greatest integer function, and let $f(x) = f(x) = \min\left\{\sqrt{2}x, x^2\right\}$. Let S = {x ∈ (–2, 2) : the function $g(x) = |x|[x^2]$ is discontinuous at x}. Then $\sum_{x \in S} f(x)$ equals :

   (1) $1 - \sqrt{2}$ (2) $2 - \sqrt{2}$ (3) $2\sqrt{6} - 3\sqrt{2}$ (4) $\sqrt{6} - 2\sqrt{2}$

Ans. Official answer NTA (1)

Sol.

Question ID : 860654986

4. Let $P\left(10, 2\sqrt{15}\right)$ be a point on the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$, whose foci are S and S'. If the length of its latus

rectum is 8, then the square of the area of $\Delta PSS'$ is equal to :

(1) 4200   (2) 2700   (3) 1462   (4) 900

Ans.   Official answer NTA (2)

Sol.

Question ID : 860654978

5. Let α, β be the roots of the quadratic equation $12x^2 - 20x + 3\lambda = 0$, $\lambda \in Z$. If $\dfrac{1}{2} \le |\beta - \alpha| \le \dfrac{3}{2}$, then the sum

of all possible values of λ is :

(1) 6   (2) 3   (3) 1   (4) 4

Ans.   Official answer NTA (2)

Sol.

Question ID : 860654981

6. If $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ is a solution of the system of equations AX = B, where adj $A \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$, then

$|x + y + z|$ is equal to :

(1) 1   (2) 2   (3) 3   (4) $\dfrac{3}{2}$

Ans.   Official answer NTA (2)

Sol.

Question ID : 860654995

7. If y = y(x) satisfies the differential equation $16\left(\sqrt{x + 9\sqrt{x}}\right)\left(4 + \sqrt{9 + \sqrt{x}}\right)\cos y \, dy = (1 + 2\sin y)dx, x > 0$

and $y\left(256\right) = \dfrac{\pi}{2}$, y(49) = α, then :

(1) $\sqrt{2} - 1$ (2) $2\left(\sqrt{2} - 1\right)$ (3) $3\left(\sqrt{2} - 1\right)$ (4) $2\sqrt{2} - 1$

Ans. Official answer NTA (4)

Sol.

Question ID : 860654982

8. Let n be the number obtained on rolling a fair die. If the probability that the system

$x - ny + z = 6$

$x + (n - 2)y + (n + 1)z = 8$

$(n - 1)y + z = 1$

has a unique solution is $\dfrac{k}{6}$, then the sum of k and all possible values of n is :

(1) 21   (2) 20   (3) 22   (4) 24

Ans. Official answer NTA (3)

Sol.

Question ID : 860654984

9. Let C_r denote the coefficient of x^r in the binomial expansion of $(1 + x)^n$, $n \in N$, $0 \le r \le n$. If

$P_n = C_0 - C_1 + \dfrac{2^2}{3}C_2 - \dfrac{2^3}{4}C_3 + \ldots + \dfrac{(-2)^n}{n+1}C_n$, then the value of $\displaystyle\sum_{n=1}^{25} \dfrac{1}{P_{2n}}$ equals. :

(1) 675   (2) 650   (3) 580   (4) 525

Ans. Official answer NTA (1)

Sol.

Question ID : 860654985

10. Let S and S' be the foci of the ellipse $\dfrac{x^2}{25} + \dfrac{y^2}{9} = 1$ and P(α, β) be a point on the ellipse in the first quadrant. If

$(SP)^2 + (S'P)^2 - SP \cdot S'P = 37$, then $\alpha^2 + \beta^2$ is equal to :

(1) 11   (2) 13   (3) 15   (4) 17

Ans. Official answer NTA (2)

Sol.

Question ID : 860654979

11.   Let the domain of the function $f(x) = \log_3 \log_5 \left(7 - \log_2 \left(x^2 - 10x + 85 \right) \right) + \sin^{-1} \left(\left| \dfrac{3x - 7}{17 - x} \right| \right)$ be $(\alpha, \beta]$. Then $\alpha + \beta$ is euqal to :

   (1) 10   (2) 12   (3) 9   (4) 8

Ans.   Official answer NTA (3)

Sol.

Question ID : 860654993

12.   Let $f(x) = [x]^2 - [x + 3] - 3$, $x \in R$, where $[\cdot]$ is the greatest integer funtion. Then :

   (1) $f(x) = 0$ for finitely many values of x   (2) $f(x) > 0$ only for $x \in [4, \infty)$

   (3) $f(x) < 0$ only for $x \in [-1, 3)$ (4) $\displaystyle\int_0^2 f(x)\,dx = -6$

Ans.   Official answer NTA (3)

Sol.

Question ID : 860654977

13.   Let f and g be functions satisfying $f(x + y) = f(x)g(y)$, $f(1) = 7$ and $g(x + y) = g(xy)$, $g(1) = 1$, for all $x, y \in N$. If $\displaystyle\sum_{x=1}^{n} \left(\dfrac{f(x)}{g(x)} \right) = 19607$, then n is equal to :

   (1) 6   (2) 4   (3) 7   (4) 5

Ans.   Official answer NTA (4)

Sol.

Question ID : 860654992

14.   If $\displaystyle\lim_{x \to 0} \dfrac{e^{(a-1)x} + 2\cos bx + (c-2)e^{-x}}{x \cos x - \log_e(1+x)} = 2$, then $a^2 + b^2 + c^2$ is equal to :

   (1) 7   (2) 3   (3) 5   (4) 9

Ans. Official answer NTA (1)

Sol.

Question ID : 860654976

15. The number of elements in the relation $R = \left\{ (x, y) : 4x^2 + y^2 < 52, x, y \in \mathbf{Z} \right\}$ is :

(1) 77   (2) 89   (3) 86   (4) 67

Ans. Official answer NTA (1)

Sol.

Question ID : 860654980

16. Let $S = \left\{ z \in \mathbb{C} : 4z^2 + \overline{z} = 0 \right\}$. Then $\sum_{z \in S} |z|^2$ is equal to :

(1) $\dfrac{5}{64}$ (2) $\dfrac{3}{16}$ (3) $\dfrac{7}{64}$ (4) $\dfrac{1}{16}$

Ans. Official answer NTA (2)

Sol.

Question ID : 860654989

17. Let $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \lambda\hat{j} + 2\hat{k}, \lambda \in Z$ be two vectors. Let $\vec{c} = \vec{a} \times \vec{b}$ and $\vec{d}$ be a vector of magnitude 2 in yz-plane. If $|\vec{c}| = \sqrt{53}$, then the maximum possible value of $\left(\vec{c} \cdot \vec{d} \right)^2$ is equal to :

(1) 104   (2) 208   (3) 26   (4) 52

Ans. Official answer NTA (2)

Sol.

Question ID : 860654990

18. Let L be the line $\dfrac{x+1}{2} = \dfrac{y+1}{3} = \dfrac{z+3}{6}$ and let S be the set of all points (a, b, c) on L, whose distance from the line $\dfrac{x+1}{2} = \dfrac{y+1}{3} = \dfrac{z-9}{0}$ along the line L is 7. Then $\sum_{(a,b,c) \in S} (a + b + c)$ is equal to :

(1) 34   (2) 6   (3) 28   (4) 40

Ans. Official answer NTA (1)

Sol.

Question ID : 860654983

19. If the mean deviation about the median of the numbers k, 2k, 3k, ..., 1000k is 500, then k^2 is equal to :

(1) 4   (2) 1   (3) 9   (4) 16

Ans. Official answer NTA (1)

Sol.

Question ID : 860654994

20. The area of the region $A = \{(x, y) : 4x^2 + y^2 \leq 8 \text{ and } y^2 \leq 4x\}$ is :

(1) $\dfrac{\pi}{2} + \dfrac{1}{3}$ (2) $\dfrac{\pi}{2} + 2$ (3) $\pi + 4$ (4) $\pi + \dfrac{2}{3}$

Ans. Official answer NTA (4)

Sol.

SECTION – B

Question ID : 860654999

21. Let a vector $\vec{a} = \sqrt{2}\hat{i} - \hat{j} + \lambda\hat{k}, \lambda > 0$, make an obtuse angle with the vector $\vec{b} = -\lambda^2\hat{i} + 4\sqrt{2}\hat{j} + 4\sqrt{2}\hat{k}$ and an angle $\theta, \dfrac{\pi}{6} < \theta < \dfrac{\pi}{2}$, with the positive z-axis. If the set of all possible values of λ is $(\alpha, \beta) - \{\lambda\}$, then $\alpha + \beta + \gamma$ is equal to _____.

Ans. Official answer NTA (5)

Sol.

Question ID : 860654997

22. Let S be the set of the first 11 natural numbers. Then the number of elements in $A = \{B \subseteq S : n(B) \geq 2$ and the product of all elements of B is even$\}$ is _____.

Ans. Official answer NTA (1979)

Sol.

Question ID : 860654996

23. Suppose a, b, c are in A.P. and a^2, $2b^2$, c^2 are in G.P. If a < b < c and a + b + c = 1, then $9(a^2 + b^2 + c^2)$ is equal to _____.

Ans. Official answer NTA (9)

Sol.

Question ID : 8606541000

24. Let [.] be the greatest integer function. If $\alpha = \int_0^{64} \left(x^{1/3} - \left[x^{1/3} \right] \right) dx$, then $\dfrac{1}{\pi} \int_0^{\alpha\pi} \left(\dfrac{\sin^2 \theta}{\sin^6 \theta + \cos^6 \theta} \right) d\theta$ is equal to _____.

Ans. Official answer NTA (36)

Sol.

Question ID : 860654998

25. Let $\cos(\alpha + \beta) = -\dfrac{1}{10}$ and $\sin(\alpha - \beta) = \dfrac{3}{8}$, where $0 < \alpha < \dfrac{\pi}{3}$ and $0 < \beta < \dfrac{\pi}{4}$.

If $\tan 2\alpha = \dfrac{3\left(1 - r\sqrt{5}\right)}{\sqrt{11}\left(s + \sqrt{5}\right)}, r, s \in N$, then r + s is equal to _____.

Ans. Official answer NTA (20)

Sol.