JEE Main January 2026

Question Paper With Text Solution

22 January | Shift-2

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN JANUARY 2026 | 22TH JANUARY SHIFT-2

SECTION – A

Question ID : 860654987

1. Let the locus of the mid-point of the chord through the origin O of the parabola $y^2 = 4x$ be the curve S. Let P be any point on S. Then the locus of the point, which internally divides OP in the ratio 3: 1, is :

(1) $2y^2 = 3x$ (2) $2x^2 = 3y$ (3) $3y^2 = 2x$ (4) $3x^2 = 2y$

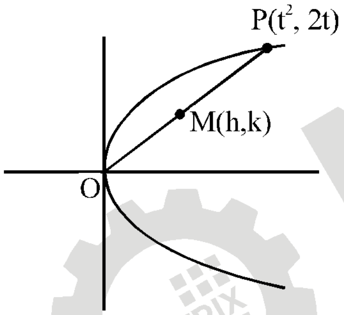
Ans. Official answer NTA (1)

Sol. $y^2 = 4x$

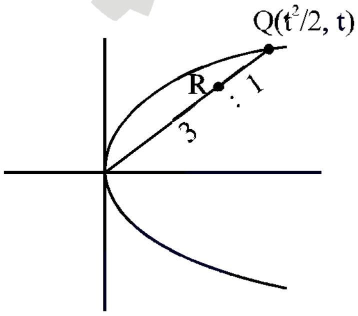
Locus of mid point of OP

$M(h, k) \Rightarrow h = \frac{t^2}{2}, k = t$

$\Rightarrow k^2 = 2h \Rightarrow y^2 = 2x$



S : $y^2 = 2x$



R(h, k)

$$\Rightarrow h = \frac{\frac{3t^2}{2}}{4}, k = \frac{3t}{4}$$

$t^2 = \frac{8h}{3}, t = \frac{4k}{3}$

$\Rightarrow \frac{16k^2}{9} = \frac{8h}{3} \Rightarrow 2k^2 = 3h$

Locus of R : $2y^2 = 3x$

Question ID : 860654988

2. Among the statements

(S1) : If A(5, –1) and B(–2, 3) are two vertices of a triangle, whose orthocentre is (0, 0), then its third vertex is (–4, –7)

and

(S2) : If positive numbers 2a, b, c are three consecutive terms of an A.P., then the lines ax + by + c = 0 are concurrent at (2, –2), :

(1) both are correct (2) only (S1) is correct (3) both are incorrect (4) only (S2) is correct

Ans. Official answer NTA (1)

Sol. Solution of statement -1

$m_{AO}.m_{BC} = -1$

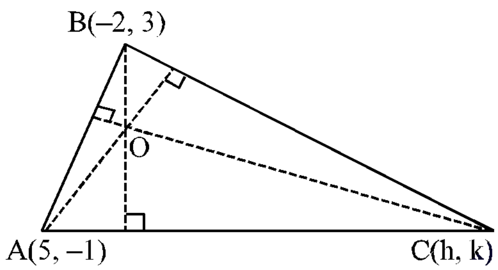
$\Rightarrow 5h - k + 13 = 0$ ....(1)

& $m_{AB}.m_{OC} = -1$

$4k = 7h$ ....(2)

third vertex is (– 4, –7)

$\therefore$ Statement 1 is correct.



Solution of statement -2

2a, b, c $\rightarrow$ A.P.

$b = \frac{2a+c}{2}$

$2a - 2b + c = 0$

$\therefore$ lines ax + by + c = 0 are concurrent then

$\frac{x}{2} = \frac{y}{-2} = \frac{1}{1}$

x = 2 and y = –2

$\therefore$ Point of concurrency is (2, –2)

$\therefore$ Statement 2 is correct.

Question ID : 860654991

3. Let [•] denote the greatest integer function, and let f(x) = $f(x) = \min\left\{\sqrt{2}x, x^2\right\}$. Let S = {x ∈ (–2, 2) : the function g(x) = |x|[x²] is discontinuous at x}. Then $\sum_{x \in S} f(x)$ equals :

(1) $1-\sqrt{2}$ (2) $2-\sqrt{2}$ (3) $2\sqrt{6}-3\sqrt{2}$ (4) $\sqrt{6}-2\sqrt{2}$

Ans. Official answer NTA (1)

Sol. $g(x) = |x|[x^2]$

points of discontinuity of g(x) in (–2, 2) are

$\left(\pm 1, \pm\sqrt{2}, \pm\sqrt{3}\right)$

$\therefore S = \left\{-1, 1, -\sqrt{2}, \sqrt{2}, -\sqrt{3}, \sqrt{3}\right\}$

$\because f(x) = \min\left\{\sqrt{2}x, x^2\right\}$

$\therefore \sum_{x \in S} f(x) = -\sqrt{2} + 1 - 2 + 2 - \sqrt{6} + \sqrt{6}$

$= 1 - \sqrt{2}$

Question ID : 860654986

4. Let $P\left(10, 2\sqrt{15}\right)$ be a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, whose foci are S and S'. If the length of its latus rectum is 8, then the square of the area of ΔPSS' is equal to :

(1) 4200 (2) 2700 (3) 1462 (4) 900

Ans. Official answer NTA (2)

Sol. $P(10, 2\sqrt{15})$ lies on $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\therefore \frac{100}{a^2} - \frac{60}{b^2} = 1$ ....(1)

$\therefore$ length of latus rectum = 8

$\frac{2 \cdot b^2}{a} = 8 \Rightarrow \frac{b^2}{a} = 4$ ....(2)

From (1) & (2)

$\frac{100}{a^2} - \frac{60}{4a} = 1$

$400 - 60a = 4a^2$

$4a^2 + 60a - 400 = 0$

$a^2 + 15a - 100 = 0$

$a = 5$ & -20 (rejected)

$\Rightarrow b = \sqrt{20}$

$\therefore$ Hyperbola is $\frac{x^2}{25} - \frac{y^2}{20} = 1$

$\therefore$ Focal length $S_1S_2 = 2ae = 2.5 \cdot \left(\sqrt{1 + \frac{4}{5}}\right) = 6\sqrt{5}$

$\therefore$ Area of $PS_1S_2 = \frac{1}{2} \cdot 6\sqrt{5} \cdot 2\sqrt{15} = 30\sqrt{3} = A$

$\therefore A^2 = 2700$

Question ID : 860654978

5. Let α, β be the roots of the quadratic equation $12x^2 - 20x + 3\lambda = 0$, $\lambda \in Z$. If $\frac{1}{2} \le |\beta - \alpha| \le \frac{3}{2}$, then the sum of all possible values of λ is :

(1) 6 (2) 3 (3) 1 (4) 4

Ans. Official answer NTA (2)

Sol. $\frac{1}{2} \le |\alpha - \beta| \le \frac{3}{2}$

$\frac{1}{4} \le |\alpha - \beta|^2 \le \frac{9}{4}$

$\frac{1}{4} \le (\alpha + \beta)^2 - 4\alpha\beta \le \frac{9}{4}$

$\frac{1}{4} \le \frac{25}{9} - 4 \times \frac{\lambda}{4} \le \frac{9}{4}$

$-\frac{91}{36} \le -\lambda \le \frac{-19}{36}$

$\frac{19}{36} \le \lambda \le \frac{91}{36}$

$\lambda = 1, 2$

Sum = 3

Question ID : 860654981

6. If $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ is a solution of the system of equations AX = B, where adj $A \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$, then

$|x + y + z|$ is equal to :

(1) 1 (2) 2 (3) 3 (4) $\frac{3}{2}$

Ans. Official answer NTA (2)

Sol. $X = A^{-1}B = \left(\frac{\text{adj } A}{|A|}\right)B$

$= \pm\frac{1}{10}\begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{pmatrix}\begin{pmatrix} 4 \\ 0 \\ 2 \end{pmatrix}$

$= \pm\frac{1}{10}\begin{pmatrix} 20 \\ -10 \\ 10 \end{pmatrix} = \pm\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$

$\therefore |x + y + z| = 2$

Question ID : 860654995

7. If y = y(x) satisfies the differential equation $16\left(\sqrt{x + 9\sqrt{x}}\right)\left(4 + \sqrt{9 + \sqrt{x}}\right)\cos y\, dy = (1 + 2\sin y)dx, x > 0$

and $y(256) = \frac{\pi}{2}$, $y(49) = \alpha$, then :

(1) $\sqrt{2} - 1$ (2) $2\left(\sqrt{2} - 1\right)$ (3) $3\left(\sqrt{2} - 1\right)$ (4) $2\sqrt{2} - 1$

Ans. Official answer NTA (4)

Ans. By Matrix answer (Bons/Drop)

Sol.

Question ID : 860654982

8. Let n be the number obtained on rolling a fair die. If the probability that the system

$x - ny + z = 6$

$x + (n-2)y + (n+1)z = 8$

$(n-1)y + z = 1$

has a unique solution is $\frac{k}{6}$, then the sum of k and all possible values of n is :

(1) 21 (2) 20 (3) 22 (4) 24

Ans. Official answer NTA (3)

Sol. $x - ny + z = 6$

$x + (n-2)y + (n+1)z = 8$

$(n-1)y + z = 1$

$$\begin{vmatrix} 1 & -n & 1 \\ 1 & (n-2) & n+1 \\ 0 & n-1 & 1 \end{vmatrix} \neq 0$$

$n^2 - 3n + 2 \neq 0$

$n \neq 1, 2$

for unique solution n = 3, 4, 5, 6

Now

P (probability when system of equations has

unique solution) = $\frac{4}{6}$

So k = 4

Now required sum = 4 + (3 + 4 + 5 + 6) = 22

Question ID : 860654984

9. Let C_r denote the coefficient of x^r in the binomial expansion of $(1 + x)^n$, $n \in N$, $0 \le r \le n$. If

$P_n = C_0 - C_1 + \frac{2^2}{3}C_2 - \frac{2^3}{4}C_3 + \ldots\ldots + \frac{(-2)^n}{n+1}C_n$, then the value of $\sum_{n=1}^{25}\frac{1}{P_{2n}}$ equals. :

(1) 675 (2) 650 (3) 580 (4) 525

Ans. Official answer NTA (1)

Sol. $P_n = \sum_{r=0}^{n}\frac{{}^nC_r(-2)^r}{r+1} = \sum_{r=0}^{n}\frac{1}{(n+1)}\,{}^{n+1}C_{r+1}(-2)^r$

$= \frac{-1}{2(n+1)}\sum_{r=0}^{n}{}^{n+1}C_{r+1}(-2)^{r+1}$

$= \frac{-1}{2(n+1)}\left[(1-2)^{n+1} - 1\right]$

$P_n = \frac{1}{2(n+1)}\left[1-(-1)^{n+1}\right]$

$P_{2n} = \frac{1}{2(2n+1)}\left[1-(-1)^{2n+1}\right]$

$P_{2n} = \frac{1}{2n+1}$

$\sum_{n=1}^{25}\frac{1}{P_{2n}} = \sum_{n=1}^{25}(2n+1)$

$= 3 + 5 + \ldots + 51$

$= \frac{25}{2}[51+3]$

$25 \times 27 = 675$

Question ID : 860654985

10. Let S and S' be the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$ and $P(\alpha, \beta)$ be a point on the ellipse in the first quadrant. If $(SP)^2 + (S'P)^2 - SP \cdot S'P = 37$, then $\alpha^2 + \beta^2$ is equal to :

(1) 11 (2) 13 (3) 15 (4) 17

Ans. Official answer NTA (2)

Sol. $\frac{\alpha^2}{25} + \frac{\beta^2}{9} = 1$

$SP = e\left(\frac{a}{e} - \alpha\right) = a - e\alpha$

$S'P = a + e\alpha$

$2a^2 + 2e^2\alpha^2 - \left(a^2 - e^2\alpha^2\right) = 37$

$a^2 + 3e^2\alpha^2 = 37$

$25 + \frac{418}{25}\alpha^2 = 37$

$625 + 48\alpha^2 = 37 \times 25$

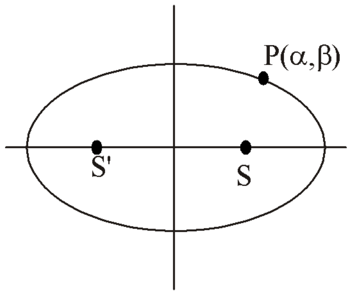
$48\alpha^2 = 25 \cdot 12$

$\alpha^2 = 25/4$

$\frac{\beta^2}{9} = \frac{3}{4}$

$\beta^2 = \frac{27}{4}$

$\alpha^2 + \beta^2 = \frac{52}{4} = 13$



Question ID : 860654979

11. Let the domain of the function $f(x) = \log_3 \log_5 \left(7 - \log_2 \left(x^2 - 10x + 85\right)\right) + \sin^{-1}\left(\left|\frac{3x-7}{17-x}\right|\right)$ be (α, β]. Then α + β is euqal to :

(1) 10 (2) 12 (3) 9 (4) 8

Ans. Official answer NTA (3)

Sol. $\log_5\left(7 - \ln\left(x^2 - 10x + 85\right)\right) > 0$

$7 - \ln\left(x^2 - 10x + 85\right) > 1$

$\ln\left(x^2 - 10x + 85\right) < 6$

$0 < x^2 - 10x + 85 < 64$

$x^2 - 10x + 21 < 0$

$x \in (3, 7)$

$-1 \le \frac{3x-7}{17-x}$ & $\frac{3x-7}{17-x} \le 1$

$\frac{2(x+5)}{17-x} \geq 0$ & $\frac{4x\text{-}24}{17\text{-}x} \leq 0$

$\frac{x+5}{x-17} \leq 0$ & $\frac{x-6}{x-17} \geq 0$

$x \in [-5,17)6\ x \in (-\infty,6) \cup (17,\infty)$

$x \in [-\infty, 6]$

$x \in (3, 6]$

Question ID : 860654993

12. Let $f(x) = [x]^2 - [x+3] - 3, x \in R$, where $[\cdot]$ is the greatest integer funtion. Then :

(1) $f(x) = 0$ for finitely many values of x

(2) $f(x) > 0$ only for $x \in [4, \infty)$

(3) $f(x) < 0$ only for $x \in [-1, 3)$

(4) $\int_0^2 f(x)\,dx = -6$

Ans. Official answer NTA (3)

Sol. $f(x) = [x]^2 - [x] - 6$

$f(x) = ([x]-3)([x]+2)$

(1) $f(x) = 0$ has infinite solution (Wrong)

(2) $(x) < -2$ or $[x] > 3$

$x \in (-\infty, -2) \cup [4, \infty)$ Wrong

(3) $[x] \in (-2, 3)$

$x \in [-1, 3)$ Right

(4) $\int_0^1 -6dx + \int_1^2 (-6)dx = (-12)$ (Wrong)

Question ID : 860654977

13. Let f and g be functions satisfying $f(x+y) = f(x)g(y)$, $f(1) = 7$ and $g(x+y) = g(xy)$, $g(1) = 1$, for all $x, y \in N$.

If $\sum_{x=1}^{n} \left(\frac{f(x)}{g(x)}\right) = 19607$, then n is equal to :

(1) 6 (2) 4 (3) 7 (4) 5

Ans. Official answer NTA (4)

Sol. $f(x) = 7^x$

$x = y = 1$

$g(x+y) = g(xy)$

Put $y = 1 \Rightarrow g(x+1) = g(x)$

$\Rightarrow g(1) = g(2) = g(3) = ...$

$g(x) = 1$

$$\sum_{x=1}^{n}\left[\frac{f(x)}{g(x)}\right] = \sum_{x=1}^{n} 7^x = 19607$$

$$\frac{7\left(7^x - 1\right)}{6} = 19607$$

$7^x - 1 = 16806$

$7^x = 16807$

$x = 5$

Question ID : 860654992

14. If $\lim\limits_{x\to 0}\dfrac{e^{(a-1)x} + 2\cos bx + (c-2)e^{-x}}{x\cos x - \log_e(1+x)} = 2$, then $a^2 + b^2 + c^2$ is equal to :

(1) 7 (2) 3 (3) 5 (4) 9

Ans. Official answer NTA (1)

Sol. $$\lim_{x\to 0}\frac{\left(1+(a-1)x+\frac{(a-1)^2}{2}x^2\right)+2\left(1-\frac{b^2x^2}{2!}\right)+(c-2)\left(1-x+\frac{x^2}{2!}\right)}{x\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}\cdots\right)-\left(x-\frac{x^2}{2}+\frac{x^3}{3}+\cdots\right)} = 2$$

$1 + 2 + c - 2 = 0 \Rightarrow c = -1$

$a - 1 - (-2) = 0 \Rightarrow a = c - 1 = -2$

$$\frac{\frac{(a-1)^2}{2} - b^2 + \frac{(c-2)}{2!}}{1/2} = 2$$

$(a-1)^2 - 2b^2 + (c-2) = 2$

$b^2 = 2$

$a^2 + b^2 + c^2 = 4 + 2 + 1 = 7$

Question ID : 860654976

15. The number of elements in the relation $R = \{(x, y) : 4x^2 + y^2 < 52, x, y \in \mathbf{Z}\}$ is :

(1) 77 (2) 89 (3) 86 (4) 67

Ans. Official answer NTA (1)

Sol. $\frac{x^2}{13} + \frac{y^2}{52} < 1$

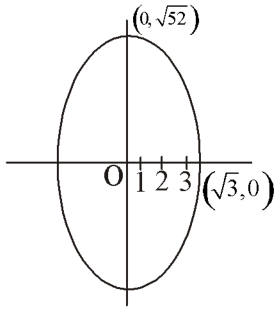
$x = 0) \quad y^2 < 52 \Rightarrow y \in \{-7, -6, -0, -7\}$

$x=-1)$ or $x=1)$ $y^2 < 48 \Rightarrow y \in \{-6, -5, -0........6\}$

$x=-2)$ or $x=2)$ $y^2 < 36 \Rightarrow y \in \{-5, -4 ......5\}$

$x=-3)$ or $x=3)$ $y^2 < 16,\ y \Rightarrow \{-3, -2, -2, 3\}$

Total = 14 + 22 + 26 + 15 = 77



Question ID : 860654980

16. Let $S = \{z \in \mathbb{C} : 4z^2 + \bar{z} = 0\}$. Then $\sum_{z \in S} |z|^2$ is equal to :

(1) $\frac{5}{64}$ (2) $\frac{3}{16}$ (3) $\frac{7}{64}$ (4) $\frac{1}{16}$

Ans. Official answer NTA (2)

Sol. $4z^2 = -\bar{z}$

$|z|^2 = \frac{3}{16}$

$|z| = 0 \quad |z| = 1/4$

$z = 0 \quad |z|^2 = \frac{1}{16}$

$z\bar{z} = \frac{1}{16}$

$4z^2 = \frac{1}{16z}$

$z^3 = \frac{1}{64} \Rightarrow$ Total 3 solution

$\sum_{z \in s} |z|^2 = \frac{3}{16}$

Question ID : 860654989

17. Let $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \lambda\hat{j} + 2\hat{k}, \lambda \in Z$ be two vectors. Let $\vec{c} = \vec{a} \times \vec{b}$ and $\vec{d}$ be a vector of magnitude 2 in yz-plane. If $|\vec{c}| = \sqrt{53}$, then the maximum possible value of $\left(\vec{c} \cdot \vec{d}\right)^2$ is equal to :

(1) 104 (2) 208 (3) 26 (4) 52

Ans. Official answer NTA (2)

Sol. Let $\vec{d} = xj + yk$

$$\left|\vec{d}\right|^2 = x^2 + y^2 = 4$$

$x = 2\cos\theta$
$y = 2\sin\theta$

$$\vec{c} = \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 0 & \lambda & 2 \end{vmatrix} = -(2+\lambda)\hat{i} + -4j + 2\lambda\hat{k}$$

$$\vec{c} = 4 + 4\lambda + \lambda^2 + 16 + 4\lambda^2 = 53$$

$$5\lambda^2 + 4\lambda - 33 = 0$$

$$(\lambda + 3)(5\lambda - 11) = 0$$

$$\lambda = -3$$

$$\vec{c} = i - 4j - 6k$$

$$\vec{d} = xj + yk$$

$$(\vec{\tau} \cdot \vec{d})^2 = (-4x - 6y)^2$$

$$= 4(2x - 3y)^2$$

$$= 16(2\cos\theta - 3\sin\theta)^2$$

$$\left(\vec{\tau}.\vec{d}\right)^2_{max} = 16.13 = 208$$

Question ID : 860654990

18. Let L be the line $\frac{x+1}{2} = \frac{y+1}{3} = \frac{z+3}{6}$ and let S be the set of all points (a, b, c) on L, whose distance from the line $\frac{x+1}{2} = \frac{y+1}{3} = \frac{z-9}{0}$ along the line L is 7. Then $\sum_{(a,b,c)\in S} (a + b + c)$ is equal to :

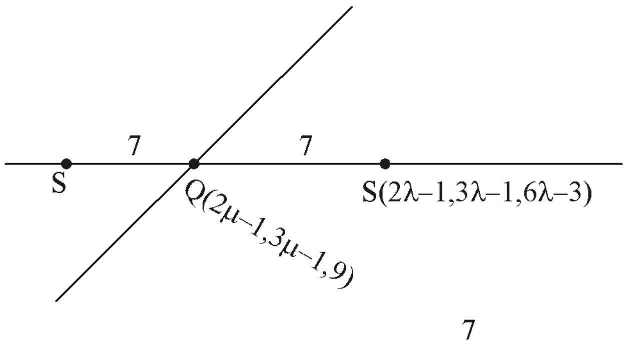
(1) 34 (2) 6 (3) 28 (4) 40

Ans. Official answer NTA (1)

Sol. $6\lambda - 3 = 9 \Rightarrow \lambda = 2$

$\Rightarrow Q = (3, 5, 9)$

$\mu = 2$



For S : $\frac{x-3}{2} = \frac{y-5}{3} = \frac{z-9}{6} = \pm 7$

$(x, y, z) = (17, 26, 51)$ or $(-11, -16, -33)$

$\sum (a+b+c) = 34$

Question ID : 860654983

19. If the mean deviation about the median of the numbers k, 2k, 3k, ..., 1000k is 500, then k^2 is equal to :

(1) 4 (2) 1 (3) 9 (4) 16

Ans. Official answer NTA (1)

Sol. $k, 2k, 3k, ..., 1000k$

$\Rightarrow$ Median = 500.5K

$\Rightarrow \frac{2(0.5+1.5+2.5+\cdots+499.5)k}{1000} = 500$

$\frac{2 \times \frac{500}{2} \times 500k}{1000} = 500$

$|k| = 2 \Rightarrow k^2 = 4$

Question ID : 860654994

20. The area of the region $A = \{(x, y) : 4x^2 + y^2 \le 8 \text{ and } y^2 \le 4x\}$ is :

(1) $\frac{\pi}{2} + \frac{1}{3}$ (2) $\frac{\pi}{2} + 2$ (3) $\pi + 4$ (4) $\pi + \frac{2}{3}$

Ans. Official answer NTA (4)

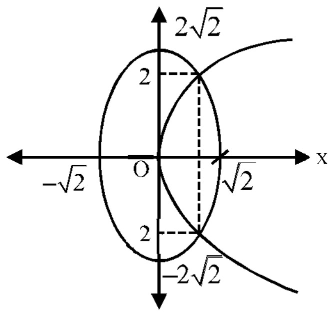
Sol. $4x^2 + 4x = 8$

$x^2 + x - 2 = 0$

$(x+2)(x-1)=0$

$A = 2\left(\int_0^1 2\sqrt{x}dx + \int_1^{\sqrt{2}} \sqrt{8-4x^2}dx\right)$

$A = 4\cdot\frac{2}{3} + 4\cdot\left[\frac{x}{2}\sqrt{2-x^2} + \frac{2}{2}\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right]_1^{\sqrt{2}}$



$A = \frac{8}{3} + 4\left[\frac{\pi}{2} - \frac{1}{2} - \frac{\pi}{4}\right]$

$A = \frac{8}{3} + \pi - 2$

$A = \pi + \frac{2}{3}$

SECTION – B

Question ID : 860654999

21. Let a vector $\vec{a} = \sqrt{2}\hat{i} - \hat{j} + \lambda\hat{k}, \lambda > 0$, make an obtuse angle with the vector $\vec{b} = -\lambda^2\hat{i} + 4\sqrt{2}\hat{j} + 4\sqrt{2}\hat{k}$ and an angle $\theta, \frac{\pi}{6} < \theta < \frac{\pi}{2}$, with the positive z-axis. If the set of all possible values of λ is $(\alpha, \beta) - \{\lambda\}$, then $\alpha + \beta + \gamma$ is equal to _________.

Ans. Official answer NTA (5)

Sol. $-\sqrt{2}\lambda^2 - 4\sqrt{2} + 4\sqrt{2}\lambda < 0$

$\lambda^2 - 4\lambda + 4 > 0$

$(\lambda-2)^2 > 0 \ (\lambda \neq 2)$

$0 < \frac{\lambda}{\sqrt{2+1+\lambda^2}} < \frac{\sqrt{3}}{2}$

$\frac{\lambda^2}{3+\lambda^2} < \frac{\sqrt{3}}{4}$

$4\lambda^2 < 3\lambda^2 + 9$

$\lambda^2 < 9$

$\lambda \in (0,3) - \{2\}$

$\alpha + \beta + \gamma = 5$

Question ID : 860654997

22. Let S be the set of the first 11 natural numbers. Then the number of elements in $A = \{B \subseteq S : n(B) \geq 2$ and the product of all elements of B is even$\}$ is ________.

Ans. Official answer NTA (1979)

Sol. $S = \{1, 2, 3, \ldots, 11\}$

$= \left({}^{11}C_2 - {}^{6}C_2\right) + \left({}^{11}C_3 - {}^{6}C_3\right) + \cdots + {}^{11}C_{11}$

$= \left({}^{11}C_2 + {}^{11}C_3 + \ldots\ldots {}^{11}C_{11}\right) - \left({}^{6}C_2 + {}^{6}C_3 + \ldots\ldots + {}^{6}C_6\right)$

$= \left(2^{11} - (1+11)\right) - \left(2^{6} - (1+6)\right)$

$= 2048 - 12 - 64 + 7 = 1979$

Question ID : 860654996

23. Suppose a, b, c are in A.P. and a^2, $2b^2$, c^2 are in G.P. If $a < b < c$ and $a + b + c = 1$, then $9(a^2 + b^2 + c^2)$ is equal to ________.

Ans. Official answer NTA (9)

Sol. $a = \frac{1}{3} - d$

$b = \frac{1}{3} \quad = \frac{1}{3} + d$

$4b^4 = a^2c^2$

$\frac{4}{3^4} = \left(\frac{1}{9} - d^2\right)^2$

$\Rightarrow 4 = \left(1 - 9d^2\right)^2$

$1 - 9d^2 = 2 \text{ or } -2$

$9d^2 = -1 \text{ or } 3$

$d^2 = 1/9 \quad \Rightarrow d = 1/\sqrt{3}$

$a = \frac{1}{3} - \frac{1}{\sqrt{3}}$

$b = 1/3$

$c = \frac{1}{3} + \frac{1}{\sqrt{3}}$

$9\left(a^2+b^2+c^2\right)=9(1)=9$

Question ID : 8606541000

24. Let [.] be the greatest integer function. If $\alpha=\int_0^{64}\left(x^{1/3}-\left[x^{1/3}\right]\right)dx$, then $\frac{1}{\pi}\int_0^{\alpha\pi}\left(\frac{\sin^2\theta}{\sin^6\theta+\cos^6\theta}\right)d\theta$ is equal to ________.

Ans. Official answer NTA (36)

Sol. $\alpha=\frac{3}{4}\times(64)^{4/3}-\left[\int_0^1 0dx+\int_1^8 dx+\int_8^{27}2dx+\int_{27}^{64}3dx\right]$

$\alpha=3.64-[7+38+37\times3]$

$\alpha=192-156=36$

$I=\frac{36}{\pi}\times2\int_0^{\pi/2}\frac{s^2}{s^6+c^6}d\theta=\frac{72}{\pi}\int_0^{\pi/2}\frac{s^2}{1-3s^2c^2}d\theta$

King property $I=\frac{72}{\pi}\int_0^{\pi/2}\frac{c^2}{1-3s^2c^2}d\theta$

$2I=\frac{72}{\pi}\int_0^{\pi/2}\frac{1}{1-3s^2c^2}d\theta=\frac{72}{\pi}\int_0^{\pi/2}\frac{\left(1+\tan^2\theta\right)\sec^2\theta}{\left(1+\tan^2\theta\right)^2-3\tan^2\theta}$

$I=\frac{36}{\pi}\int_0^{\infty}\frac{\left(1+t^2\right)dt}{t^4-t^2+1}=\frac{36}{\pi}\int_0^{\infty}\frac{\left(1+\frac{1}{t^2}\right)dt}{\left(t-\frac{1}{t}\right)^2+1}=\frac{36}{\pi}\cdot\left(\tan^{-1}\left(t-\frac{1}{t}\right)\right)_0^{\infty}$

$I=\frac{36}{\pi}\cdot\left[\frac{\pi}{2}+\frac{\pi}{2}\right]$

$I=36$

Question ID : 860654998

25. Let $\cos(\alpha+\beta)=-\frac{1}{10}$ and $\sin(\alpha-\beta)=\frac{3}{8}$, where $0<\alpha<\frac{\pi}{3}$ and $0<\beta<\frac{\pi}{4}$.

If $\tan 2\alpha = \dfrac{3\left(1-r\sqrt{5}\right)}{\sqrt{11}\left(s+\sqrt{5}\right)}$, $r, s \in N$, then $r + s$ is equal to ________.

Ans. Official answer NTA (20)

Sol. $\alpha+\beta \in \left(0, \dfrac{7\pi}{12}\right)$

$$\cos(\alpha+\beta) = -\frac{1}{10}$$

$$\tan(\alpha+\beta) = \frac{-\sqrt{94}}{1} = -3\sqrt{11}$$

$$\alpha-\beta \in (-\pi/4, \pi/3)$$

$$\sin(\alpha-\beta) = 3/8$$

$$\tan(\alpha-\beta) = \frac{3}{\sqrt{55}}$$

$$\tan 2\alpha = \frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)+\tan(\alpha-\beta)}$$

$$= \frac{-3\sqrt{11}+\dfrac{3}{\sqrt{55}}}{1+\dfrac{9}{\sqrt{55}}\cdot\sqrt{11}}$$

$$= \frac{-33\sqrt{5}+3}{\sqrt{11}(\sqrt{5}+9)} = \frac{3(1-11\sqrt{5})}{\sqrt{11}(9+\sqrt{5})}$$

$r = 11$
$s = 9$
$r + s = 20$