JEE Main January 2026

Question Paper With Text Solution

21 January | Shift-2

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN JANUARY 2026 | 21TH JANUARY SHIFT-2

SECTION – A

Question ID : 860654843

1. Let $f : R \to R$ be a twice differentiable function such that $f''(x) > 0$ for all $x \in R$ and $f'(a-1) = 0$, where a is a real number, Let $g(x) = f(\tan^2 x - 2\tan x + a)$, $0 < x < \frac{\pi}{2}$.

Consider the following two statements :

(I) g is increasing in $\left(0, \frac{\pi}{4}\right)$

(II) g is decreasing in $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

Then,

(1) Only (I) is True

(2) Both (I) and (II) are True

(3) Neither (I) nor (II) is True

(4) Only (II) is True

माना दो बार अवकलनीय एक फलन $f : R \to R$ इस प्रकार है कि सभी $x \in R$ के लिए $f''(x) > 0$ है और $f'(a-1) = 0$ है, जहाँ a एक वास्तविक संख्या हैं माना $g(x) = f(\tan^2 x - 2\tan x + a)$, $0 < x < \frac{\pi}{2}$ हैं।

निम्न दो कथनों का विचार कीजिए :

(I) $\left(0, \frac{\pi}{4}\right)$ में g वर्धमान है

(II) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ में g ह्रासमान है

तो,

(1) केवल (I) सही है

(2) (I) और (II) दोनों सही है

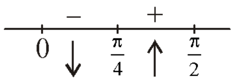
(3) न तो (I) और न ही (II) सही है

(4) केवल (II) सही है

Ans. Official answer NTA (3)

Sol. $g'(x) = f\left((\tan x - 1)^2 + (a-1)\right)$

$g'(x) = f'\left((\tan x - 1)^2 + (a-1)\right)\ 2(\tan x - 1)\sec^2 x$



Question ID : 860654832

2. Let $a_1, \frac{a_2}{2}, \frac{a_3}{2^2}, \ldots, \frac{a_{10}}{2^9}$ be a G.P. of common ratio $\frac{1}{\sqrt{2}}$. If $a_1 + a_2 + \ldots\ldots + a_{10} = 62$, then a_1 is equal to :

माना सार्व अनुपात $\frac{1}{\sqrt{2}}$ की एक G.P. $a_1, \frac{a_2}{2}, \frac{a_3}{2^2}, \ldots, \frac{a_{10}}{2^9}$ है। यदि $a_1 + a_2 + \ldots\ldots + a_{10} = 62$ है, तो a_1 बराबर है :

(1) $\sqrt{2} - 1$ (2) $2 - \sqrt{2}$ (3) $2\left(\sqrt{2} - 1\right)$ (4) $2\left(2 - \sqrt{2}\right)$

Ans. Official answer NTA (3)

Sol. $\frac{a_2}{2a_1} = \frac{1}{\sqrt{2}} \Rightarrow \quad a_2 = \sqrt{2}a_1$

$\frac{a_3}{2^2 \cdot a_2} = \frac{1}{\sqrt{2}}, \quad 2a_3 = \sqrt{2}a_2 = 2a_1$

$a_1 + a_2 + \cdots + a_{10} = 62$

$a.\left(1 + \sqrt{2} + \left(\left(\sqrt{2}\right)^2 + \cdots\right)\right) = 62$

$a.\frac{((\sqrt{2})^{10} - 1)}{(\sqrt{2} - 1)} = 62$

$\Rightarrow \quad a_1 = 2(\sqrt{2} - 1)$

Question ID : 860654833

3. Let $A = \{x : |x^2 - 10| \le 6\}$ and $B = \{x : |x - 2| > 1\}$. Then :

माना $A = \{x : |x^2 - 10| \le 6\}$ और $B = \{x : |x - 2| > 1\}$ हैं। तो :

(1) $A \cap B = [-4, -2] \cup [3, 4]$ है

(2) $A \cup B = (-\infty, 1] \cup (2, \infty)$ है

(3) $A - B = [2, 3)$ है

(4) $B - A = (-\infty, -4) \cup (-2, 1) \cup (4, \infty)$ है

Ans. Official answer NTA (4)

Sol. $A: \quad \left|x^2 - 10\right| \le 6$

$-6 \le x^2 - 10 \le 6$

$\Rightarrow\ 4 \le x^2 \le 16$

$x \in [-4,-2] \cup [2,4]$

$B : |x-2| > 1$

Either $x - 2 < -1$ or $x - 2 > 1$

$x < 1$ $\qquad\qquad$ $x > 3$

$\Rightarrow\ x \in (-\infty, 1) \cup (3, \infty)$

Question ID : 860654831

4. If the system of equations
$3x + y + 4z = 3$
$2x + \alpha y - z = -3$
$x + 2y + z = 4$
has no solution, then the value of α is equal to :

यदि समीकरण निकाय
$3x + y + 4z = 3$
$2x + \alpha y - z = -3$
$x + 2y + z = 4$
का कोई हल नहीं है, तो α का मान बराबर है :

(1) 4 (2) 19 (3) 23 (4) 13

Ans. Official answer NTA (2)

Sol. $D = \begin{vmatrix} 3 & 1 & 4 \\ 2 & \alpha & -1 \\ 1 & 2 & 1 \end{vmatrix} = 3(\alpha+2) + (-3) + 4(4-\alpha)$

$= 19 - \alpha$

$D_1 = \begin{vmatrix} 3 & 1 & 4 \\ -3 & \alpha & -1 \\ 4 & 2 & 1 \end{vmatrix} = 3(\alpha+2) + (-1) + 4(-6-4\alpha)$

$= -13\alpha - 5$

Clearly at $\alpha = 19$, $D = 0$ and $D_1 \ne 0$

Question ID : 860654827

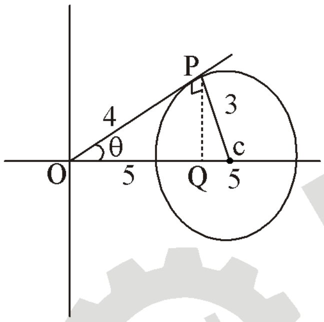
5. Let z be the complex number satisfying $|z-5| \le 3$ and having maximum positive principal argument. Then $34\left|\frac{5z-12}{5iz+16}\right|^2$ is equal to :

माना $|z-5| \le 3$ को संतुष्ट करने वाली और अधिकतम धनात्मक मुख्य आयाम की सम्मिश्र संख्या z है। तो $34\left|\frac{5z-12}{5iz+16}\right|^2$ बराबर है :

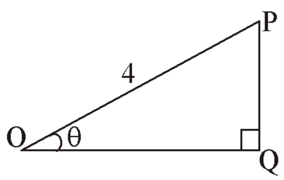
(1) 16 (2) 20 (3) 12 (4) 26

Ans. Official answer NTA (2)

Sol. $|z-5| \le 3$



$\tan\theta = \frac{3}{4}$



$OQ = 4\cos\theta = 4\cdot\frac{4}{5} = \frac{16}{5}$

$PQ = 4\sin\theta = 4\cdot\frac{3}{5} = \frac{12}{5}$

$z = \frac{16}{5} + i\frac{12}{5}$

$\Rightarrow \quad 5z = 16 + 12i$

Now

$$\left|\frac{5z-12}{5iz+16}\right| = \left|\frac{16+12i-12}{16i-12+16}\right| = \left|\frac{4+12i}{16i+4}\right|$$

$$= \left|\frac{1+3i}{1+4i}\right| = \sqrt{\frac{10}{17}}$$

So $34\left|\frac{5z-12}{5iz+16}\right|^2 = \frac{34\cdot 10}{17} = 20$

Question ID : 860654844

6. Let y = y(x) be the solution of the differential equation $\sec x\frac{dy}{dx} - 2y = 2+3\sin x$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, $y(0) = -\frac{7}{4}$.

Then $y\left(\frac{\pi}{6}\right)$ is equal to :

माना अवकल समीकरण $\sec x\frac{dy}{dx} - 2y = 2+3\sin x$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, $y(0) = -\frac{7}{4}$ का हल y = y(x) है। तो $y\left(\frac{\pi}{6}\right)$ बराबर है :

(1) $-3\sqrt{3}-7$ (2) $-\frac{5}{4}$ (3) $-\frac{5}{2}$ (4) $-3\sqrt{2}-7$

Ans. Official answer NTA (3)

Sol. $\frac{dy}{dx} - (2\cos x)y = (2+3\sin x)\cos x$

If $= e^{-\int 2cxdx} = e^{-2\sin x}$

Solution of DE

$y \cdot e^{-2\sin x} = \int (2+3\sin x)e^{-2\sin x} \cdot \cos x\, dx$

let sinx = t

$= \int (2+3t)e^{-2t}dt$

$= -\frac{(2+3t)e^{-2t}}{2} + \frac{3}{2}\int e^{-2t}dt$

$= -\frac{(2+3\sin x)e^{-2\sin x}}{2} - \frac{3}{4}e^{-2\sin x} + C$

$\Rightarrow \quad y = -\frac{(2+3\sin x)}{2} - \frac{3}{4} + ce^{2\sin x}$

Given $y(0) = -\frac{7}{4}$

$\Rightarrow \frac{7}{4} = -1 - \frac{3}{4} + C$

$\Rightarrow c = 0$

$\Rightarrow \quad y = \frac{-(2+3\sin x)}{2} - \frac{3}{4}$

$y(\pi/6) = -\frac{\left(2+\frac{3}{2}\right)}{2} - \frac{3}{4} = -\frac{7}{4} - \frac{3}{4} = -\frac{5}{2}$

Question ID : 860654826

7. Let A = {2, 3, 5, 7, 9}. Let R be the relation on A defined by xRy if and only if $2x \le 3y$. Let l be the number of elements in R, and m be the minimum number of elements required to be added in R to make it a symmetric relation. Then $l + m$ is equal to :

माना A = {2, 3, 5, 7, 9} है। माना A पर संबंध R, xRy यदि और केवल यदि $2x \le 3y$ है, द्वारा परिभाषित है। माना R में अवयवों की संख्या l है और R को एक सममित संबंध बनाने के लिए इसमें जोड़े जाने वाले आवश्यक अवयवों की न्यूनतम संख्या m है। तो $l + m$ बराबर है :

(1) 23 (2) 21 (3) 27 (4) 25

Ans. Official answer NTA (4)

Sol. $A = \{2, 3, 5, 7, 9\}$

$2x \le 3y$

R = {(2, 2), (2, 3) (2, 5), (2, 7), (2, 9)
(3, 2), (3, 3), (3, 5), (3, 7), (3,9)
(5, 5), (5, 7), (5, 9)
(7, 5), (7, 7), (7, 9)
(9, 5), (9, 9)}

total 18 elements in R

elements to be added to make it symaetric

{(5, 2), (7, 2), (9, 2), (5, 3), (7, 3), (9, 3), (9, 5)}

So l + m = 18 + 7 = 25

Question ID : 860654838

8. If the line $\alpha x + 4y = \sqrt{7}$, where $\alpha \in R$, touches the ellipse $3x^2 + 4y^2 = 1$ at the point P in the first quadrant, then one of the focal distances of P is :

(1) $\frac{1}{\sqrt{3}} - \frac{1}{2\sqrt{5}}$ (2) $\frac{1}{\sqrt{3}} + \frac{1}{2\sqrt{7}}$ (3) $\frac{1}{\sqrt{3}} - \frac{1}{2\sqrt{11}}$ (4) $\frac{1}{\sqrt{3}} + \frac{1}{2\sqrt{5}}$

Ans. Official answer NTA (2)

Sol. Line $\alpha x + 4y = \sqrt{7} \Rightarrow y = \frac{-\alpha x}{4} + \frac{\sqrt{7}}{4}$

Ellipse : $3x^2 + 4y^2 = 1$

Condition of taygency :

$c^2 = a^2 m^2 + b^2$

$$\frac{7}{16} = \frac{1}{3} \cdot \frac{\alpha^2}{16} + \frac{1}{4}$$

$\Rightarrow \; 21 = \alpha^2 + 12 \Rightarrow \alpha^2 = 9 \quad \Rightarrow \alpha = \pm 3$

for first quadrent $\alpha = 3$

Finding point of contact, $P(x_1, y_2)$

targent : $T = 0$

$\Rightarrow (3x_1)x + (4y_1)y = 1$

Compaisuing with $3x + 4y = \sqrt{7}$

$$x_1 = y_1 = \frac{1}{\sqrt{7}} \quad \Rightarrow P\left(\frac{1}{\sqrt{7}}, \frac{1}{\sqrt{7}}\right)$$

Finding focil $e = \sqrt{1 - \frac{3}{4}} = \frac{1}{2}$

$$ae = \frac{1}{2\sqrt{3}}$$

So focil are $s_2\left(-\frac{1}{2\sqrt{3}}, 0\right)$ and $s_1\left(\frac{1}{2\sqrt{3}}, 0\right)$

$$PS_2 = \sqrt{\left(\frac{1}{\sqrt{7}} + \frac{1}{2\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{7}}\right)^2} = \frac{1}{\sqrt{3}} + \frac{1}{2\sqrt{7}}$$

Question ID : 860654839

9. Let the line L pass through the point (–3, 5, 2) and make equal angles with the positive coordinate axes. If the distance of L from the point (–2, r, 1) is $\sqrt{\frac{14}{3}}$, then the sum of all possible values of r is :

(1) 12 (2) 6 (3) 10 (4) 16

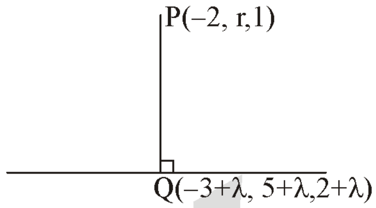
Ans. Official answer NTA (3)

Sol. $\vec{r} = (-3, 5, 2) + \lambda(1, 1, 1)$

$\overrightarrow{PQ} \cdot (1,1,1) = 0$

$\Rightarrow \quad (-3+\lambda+2)+(5+\lambda-r)+(2+\lambda-1)=0$

$\Rightarrow 5 + 3\lambda = r \qquad \qquad ....(1)$



$|\overrightarrow{PQ}| = \sqrt{(\lambda-1)^2+(-2\lambda)^2+(\lambda+1)^2} = \sqrt{\frac{14}{3}}$

$\Rightarrow \quad \lambda^2 - 2\lambda + 1 + 4\lambda^2 + \lambda^2 + 2\lambda + 1 = \frac{14}{3}$

$\Rightarrow \quad 6\lambda^2 = \frac{14}{3} - 2 = \frac{8}{3}$

$\Rightarrow \quad 6\left(\frac{r-5}{3}\right)^2 = \frac{8}{3}$

$\Rightarrow \left(\frac{r^2 - 10r + 25}{9}\right) \cdot 6 = \frac{8}{3}$

$\Rightarrow \quad r^2 - 10r + 21 = 0$

$\Rightarrow r = 3, 7$

Sum of all possible values of $r = 10$

Question ID : 860654840

10. Let the line L_1 be parallel to the vector $-3\hat{i}+2\hat{j}+4\hat{k}$ and pass through the point (2, 6, 7), and the line L_2 be parallel to the vector $2\hat{i}+\hat{j}+3\hat{k}$ and pass through the point (4, 3, 5). If the line L_3 is parallel to the vector $-3\hat{i}+5\hat{j}+16\hat{k}$ and intersects the lines L_1 and L_2 at the points C and D , respectively, then $\left|\overrightarrow{CD}\right|^2$ is equal to :

(1) 171 (2) 312 (3) 89 (4) 290

Ans. Official answer NTA (4)

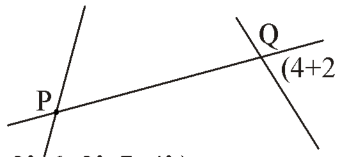
Sol. $\overrightarrow{PQ} = (2\mu+3\lambda+2, \mu-2\lambda-3, 3\mu-4\lambda-2)$

$L_1 : \vec{r} = (2,6,7) + \lambda(-3,2,4)$

$L_2 : \vec{r} = (4,3,5) + \mu(2,1,3)$

direction of L_3 is parallel to (–3, 5, 16)

So $\frac{2\mu+3\lambda+2}{-3}=\frac{\mu-2\lambda-3}{5}=\frac{3\mu-4\lambda-2}{16}$



$\frac{2\mu+3\lambda+2}{-3}=\frac{\mu-2\lambda-3}{5}$

$\Rightarrow 10\mu+15\lambda+10$

$=-3\mu+6\lambda+9$

$\Rightarrow 13\mu+9\lambda+1=0$ ______(1)

$\frac{\mu-2\lambda-3}{5}=\frac{3\mu-4\lambda-2}{16}$

$16\mu-32\lambda-48=18\mu-20\lambda-10$

$\Rightarrow \mu-12\lambda-38=0$ ______(2)

Sloving (1) and (2)

$\lambda=-3$ and $\mu=2$

So C = (11, 0, –5) and D(8, 5,1 1)

$CD^2 = 3^2 + 5^2 + 16^2 = 290$

Question ID : 860654842

11. Let $f(x) = x^3 + x^2 f'(1) + 2xf''(2) + f'''(3)$, $x \in R$. Then the value of $f'(5)$:

(1) $\frac{117}{5}$ (2) $\frac{62}{5}$ (3) $\frac{657}{5}$ (4) $\frac{2}{5}$

Ans. Official answer NTA (1)

Sol. let $f(x)= x^3 + p x^2 + 2qx + r$

$p = f'(1)$

$q = f''(2)$

$r = f'''(3)$

$f'(x) = 3x^2 + 2px + 2q$

$f''(x) = 6x + 2p$

$f'''(x) = 6 = r$

Now $q = f''(2)$

$q = 12 + 2p$ ....(1)

and $p = f'(1) = 3 + 2p + 2q$

$\Rightarrow p + 2q + 3 = 0$ ....(2)

Solving (1) and (2)

$$p = -\frac{27}{5}, \quad q = \frac{6}{5}$$

$$f'(5) = 75 + 2\left(\frac{-27}{5}\right)\cdot 5 + 2\left(\frac{6}{5}\right)$$

$$= \frac{117}{5}$$

Question ID : 860654834

12. The largest $n \in N$, for which 7^n divides 101!, is :

(1) 16 (2) 18 (3) 15 (4) 19

Ans. Official answer NTA (1)

Sol. $\left[\frac{101}{7}\right] + \left[\frac{101}{7^2}\right] + \left[\frac{101}{7^3}\right] = 14 + 2 + 0 = 16$

Question ID : 860654841

13. For a triangle ABC, let $\vec{p} = \overrightarrow{BC}, \vec{q} = \overrightarrow{CA}$ and $\vec{r} = \overrightarrow{BA}$. If $|\vec{p}| = 2\sqrt{3}, |\vec{q}| = 2$ and $\cos\theta = \frac{1}{\sqrt{3}}$, where θ, is the angle between $\vec{p}$ and $\vec{q}$, then $\left|\vec{p}\times(\vec{q} - 3\vec{r})\right|^2 + 3|\vec{r}|^2$ is equal to :

(1) 340 (2) 410 (3) 200 (4) 220

Ans. Official answer NTA (3)

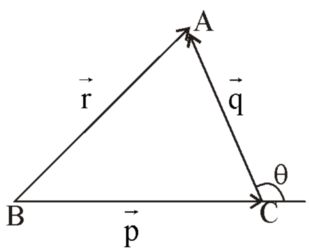
Sol. $\vec{p} + \vec{q} = \vec{r}$

$|\vec{p}| = 2\sqrt{3}, |\vec{q}| = 2, \quad \cos\theta = \frac{1}{3}$

$|\vec{p}\times(\vec{q} - 3\vec{r})|^2 + 3|\vec{r}|^2$

$= |\vec{p}\times(\vec{q} - 3\vec{p} - 3\vec{q})|^2 + 3|\vec{r}|^2$

$= 4|\vec{p}\times\vec{q}|^2 + 3|\vec{r}|^2$ ....(1)



$\vec{r} = \vec{p} + \vec{q}$

$\Rightarrow r^2 = p^2 + q^2 + 2pq\cos\theta$

$= 12 + 4 + 2\cdot 2\sqrt{3}\cdot 2\cdot\frac{1}{\sqrt{3}} = 24$

and $|\vec{p}\times\vec{q}|^2 = p^2q^2\sin^2\theta = 12\cdot 4\cdot\frac{8}{9} = 32$

So from equation (1)

$4|\vec{p}\times\vec{q}|^2 + 3|\vec{r}|^2$

$= 4 \times 32 + 3 \times 24 = 128 + 72 = 200$

Question ID : 860654829

14. Let α and β be the roots of the equation $x^2 + 2ax + (3a + 10) = 0$ such that $\alpha < 1 < \beta$. Then the set of all possible values of α is :

(1) $(-\infty, -3)$
(2) $(-\infty, -2) \cup (5, \infty)$
(3) $\left(-\infty, -\frac{11}{5}\right) \cup (5, \infty)$
(4) $\left(-\infty, -\frac{11}{5}\right)$

Ans. Official answer NTA (4)

Sol. $f(1) < 0$

$\Rightarrow 1 + 2a + 3a + 10 < 0$

$\Rightarrow \quad a < -\frac{11}{5}$

Question ID : 860654836

15. Let one end of a focal chord of the parabola $y^2 = 16x$ be (16, 16). If $P(\alpha, \beta)$ divides this focal chord internally in the ratio 5 : 2, then the minimum value of $\alpha + \beta$ is equal to :

(1) 22 (2) 7 (3) 5 (4) 16

Ans. Official answer NTA (2)

Sol. $y^2 = 16x$

$4a = 16 \Rightarrow a = 4$

$R(at^2, 2at_1) = (4t_1^2, 8t_1) = (16,16)$

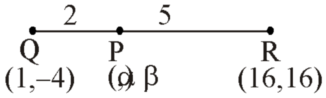
$\Rightarrow t_1 = 2$

Now $t_1 t_2 = -1$ (focal chord)

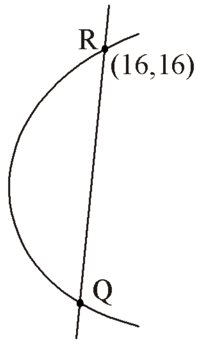
$\Rightarrow t_2 = -\frac{1}{2}$

So co-ordinates of Q = (1,–4)

(for $\alpha + \beta$ to be minimum p should be close to Q)



$$P = \left(\frac{37}{7}, \frac{12}{7}\right)$$



Question ID : 860654828

16. The positive integer n, for which the solutions of the equation $x(x+2)+(x+2)(x+4)+ \ldots\ldots +(x+2n-2)(x+2n) = = \frac{8n}{3}$ are two consecutive even integers, is :

(1) 6 (2) 9 (3) 12 (4) 3

Ans. Official answer NTA (4)

Sol. replace x by 2x

(now roots will be two consecutive integers)

$$2x(2x+2)+(2x+2)(2x+4)+\cdots+(2x+2x-2)(2x+2x)=\frac{8n}{3}$$

$$\Rightarrow x(x+1)+(x+1)(x+2)+\cdots+(x+(n-1))(x+n)=\frac{2n}{3}$$

$$\Rightarrow \sum_{x=1}^{n}(x+k)(x+(k-1))=\frac{2n}{3}$$

$$\Rightarrow \sum_{k=1}^{n}\left(x^2+(2k-1)x+k(k-1)\right)=\frac{2n}{3}$$

$$\Rightarrow nx^2+n^2x+\frac{n\left(n^2-1\right)}{3}=\frac{2n}{3}$$

$$\Rightarrow x^2+nx+\frac{n^2-3}{3}=0$$

difference of roots should be 1

$$\frac{\sqrt{D}}{1}=1 \Rightarrow D=1$$

$$\Rightarrow n^2-4\left(\frac{n^2-3}{3}\right)=1$$

$\Rightarrow -n^2 + 12 = 3$

$\Rightarrow n^2 = 9$

$n = 3$

Question ID : 860654835

17. A random variable X takes values 0, 1, 2, 3 with probabilities $\frac{2a+1}{30}, \frac{8a-1}{30}, \frac{4a+1}{30}$, b respectively, where a, b $\in$ R. Let μ and σ respectively be the mean and standard deviation of X such that $\sigma^2 + \mu^2 = 2$. Then $\frac{a}{b}$ is equal to :

(1) 60 (2) 3 (3) 12 (4) 30

Ans. Official answer NTA (1)

Sol.

x_i	0	1	2	3
p_i	$\frac{2a+1}{30}$	$\frac{8a-1}{30}$	$\frac{4a+1}{30}$	b

$\sum p_i = 1$

$\Rightarrow \frac{14a+1}{30} + b = 1$

$\Rightarrow 14a + 1 + 30b = 30$

$\Rightarrow 14a + 30b = 29$ ....(1)

$\sigma^2 + \mu^2 = \sum p_i x_i^2 - \left(\sum p_i x_i\right)^2 + \left(\sum p_i x_i\right)^2$

$= \sum p_i x_i^2$

$= \frac{8a-1}{30} + \left(\frac{4a+1}{30}\right) \cdot 4 + 9b$

$= \frac{24a+3}{30} + 9 \cdot \left(\frac{29-14a}{30}\right)$

$= \frac{264-102a}{30}$

Now $\sigma^2 + \mu^2 = 2$

$\Rightarrow \frac{264-102a}{30} = 2 \Rightarrow a = 2$

$b = \frac{1}{30}$

So $\frac{a}{b} = \frac{2}{1/30} = 60$

Question ID : 860654837

18. Let $y^2 = 12x$ be the parabola with its vertex at O . Let P be a point on the parabola and A be a point on the x-axis such that $\angle OPA = 90°$. Then the locus of the centroid of such triangles OPA is :

(1) $y^2 - 6x + 4 = 0$ (2) $y^2 - 4x + 8 = 0$ (3) $y^2 - 2x + 8 = 0$ (4) $y^2 - 9x + 6 = 0$

Ans. Official answer NTA (3)

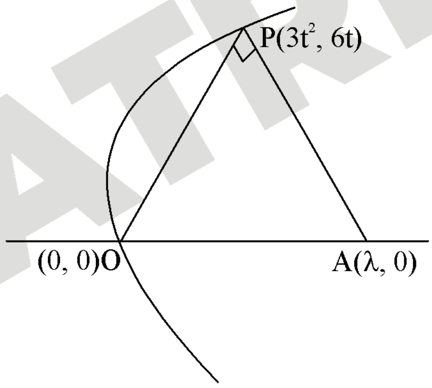
Sol. $y^2 = 12x$

$\Rightarrow 4a = 12 \Rightarrow a = 3$

$m_1.m_2 = -1$

$\Rightarrow \frac{2}{t}\cdot\left(\frac{-6t}{\lambda - 3t^2}\right) = -1$

$\Rightarrow 12 = \lambda - 3t^2 \Rightarrow \lambda = 12 + 3t^2$



Let centroid is (h, k)

$h = \frac{\lambda + 3t^2}{3}$

$\Rightarrow 3h = 12 + 6t^2$ ....(1)

and $k = \frac{6t}{3} = 2t$ ....(2)

from (1) and (2)

$3h = 12 + \frac{6\cdot k^2}{4}$

$\Rightarrow 3x = 12 + \frac{3y^2}{2}$

$\Rightarrow 6x = 24 + 3y^2$

$\Rightarrow y^2 - 2x + 8 = 0$

Question ID : 860654830

19. For the matrices $A=\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}$ and $B=\begin{bmatrix}-29 & 49\\ -13 & 18\end{bmatrix}$, if $(A^{15}+B)\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$, then among the following which one is true :

(1) $x = 11, y = 2$ (2) $x = 18, y = 11$ (3) $x = 5, y = 7$ (4) $x = 16, y = 3$

Ans. Official answer NTA (1)

Sol. $A=\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}$

$A^2=\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}=\begin{bmatrix}5 & -8\\ 2 & -3\end{bmatrix}$

$A^3=\begin{bmatrix}5 & -8\\ 2 & -3\end{bmatrix}\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}=\begin{bmatrix}7 & -12\\ 3 & -5\end{bmatrix}$

$A^{15}=\begin{bmatrix}31 & -60\\ 15 & -29\end{bmatrix}$

$A^{15}+B=\begin{bmatrix}31 & -60\\ 15 & -29\end{bmatrix}+\begin{bmatrix}-29 & 49\\ -13 & 18\end{bmatrix}$

$=\begin{bmatrix}2 & -11\\ 2 & -11\end{bmatrix}$

Now $\left[A^{15}+B\right]\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$

$\begin{bmatrix}2 & -11\\ 2 & -11\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$

$2x - 11y = 0$

$x = 11, y = 2$ satisfies it

Question ID : 860654845

20. If the area of the region $\{(x, y) : 1 - 2x \le y \le 4 - x^2, x \ge 0, y \ge 0\}$ is $\frac{\alpha}{\beta}, \alpha, \beta \in N$, gcd $(\alpha, \beta) = 1$, then the value of $(\alpha + \beta)$ is :

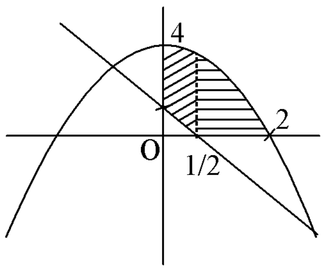
(1) 67 (2) 91 (3) 85 (4) 73

Ans. Official answer NTA (4)

Sol. $1-2x \le y \le 4-x^2$

$y \ge 1-2x$ and $y \le 4-x^2$

$$\text{Area} = \int_0^{1/2}\left(\left(4-x^2\right)-(1-2x)\right)dx + \int_{1/2}^{2}\left(4-x^2\right)dx$$



$$= \int_0^{1/2}\left(3+2x-x^2\right)dx + \left[4x-\frac{x^3}{3}\right]_{1/2}^{2}$$

$$= \left[3x+x^2-\frac{x^3}{3}\right]_0^{1/2} + \left[4x-\frac{x^3}{3}\right]_{1/2}^{2}$$

$$= \frac{3}{2}+\frac{1}{4}-\frac{1}{2/4}+8-\frac{8}{3}-2+\frac{1}{24}$$

$$= 6+\frac{3}{2}+\frac{1}{4}-\frac{8}{3} = \frac{72+18+3-32}{12}$$

$$= \frac{61}{12}$$

So $\alpha + \beta = 73$

SECTION – B

Question ID : 860654849

21. Let [•] denote the greatest integer function and $f(x) = \lim_{n\to\infty}\frac{1}{n^3}\sum_{k=1}^{n}\left[\frac{k^2}{3^x}\right]$. Then $12\sum_{j=1}^{\infty} f(j)$ is equal to ________.

Ans. Official answer NTA (2)

Sol. $f(x) = \lim_{n\to\infty}\frac{1}{n^3}\sum_{k=1}^{n}\left[\frac{k^2}{3^x}\right]$

$$= \lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{n^3}\left(\frac{k^2}{3^x}\right) - \lim_{n\to\infty}\sum_{k=1}^{n}\frac{\left\{\frac{k^2}{3^x}\right\}}{n^3}$$

$$=\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{n}\cdot\frac{k^2}{n^2}\cdot\frac{1}{3^x}-0$$

$$=\frac{1}{3^x}\int_0^1 x^2dx$$

$$=\frac{1}{3}\cdot\frac{1}{3^x}$$

$$\text{so } 12\sum_{j=1}^{\infty}f(j)=12\cdot\frac{1}{3}\cdot\left(\frac{1}{3}+\frac{1}{3^2}+\cdots\infty\right)$$

$$=12\cdot\frac{1}{3}\cdot\frac{1/3}{1-1/3}$$

$$=12\cdot\frac{1}{3}\cdot\frac{1}{2}$$

$$=2$$

Question ID : 860654846

22. If $\left(\frac{1}{{}^{15}C_0}+\frac{1}{{}^{15}C_1}\right)\left(\frac{1}{{}^{15}C_1}+\frac{1}{{}^{15}C_2}\right)\cdots\left(\frac{1}{{}^{15}C_{12}}+\frac{1}{{}^{15}C_{13}}\right)=\frac{\alpha^{13}}{{}^{14}C_0\,{}^{14}C_1\cdots{}^{14}C_{12}}$, then 30α is equal to _________.

Ans. Official answer NTA (32)

Sol. $$\left(\frac{1}{{}^{15}C_0}+\frac{1}{{}^{15}C_1}\right)\left(\frac{1}{{}^{15}C_1}+\frac{1}{{}^{15}C_2}\right)\cdots\cdots\cdots\left(\frac{1}{{}^{15}C_{12}}+\frac{1}{{}^{15}C_{13}}\right)$$

$$=\frac{{}^{16}C_1\cdot{}^{16}C_2\cdot{}^{16}C_3\cdots\cdots\cdots{}^{16}C_{13}}{{}^{15}C_0\cdot\left({}^{15}C_1\right)^2\left({}^{15}C_2\right)^2\cdots\cdots\cdots\left({}^{15}C_{12}\right)^2\cdot{}^{15}C_{13}}$$

$$=\frac{\left(\frac{16}{1}\cdot\frac{16}{2}\cdot\frac{16}{3}\cdots\cdots\cdots\frac{16}{13}\right)\left({}^{15}C_0\cdot{}^{15}C_1\cdot{}^{15}C_2\cdots\cdots\cdots{}^{15}C_{12}\right)}{\left(\frac{15}{1}\frac{15}{2}\cdots\cdots\cdots\frac{15}{13}\right)\left({}^{14}C_0{}^{14}C_1\cdots\cdots\cdots{}^{14}C_{12}\right)\left({}^{15}C_0\cdot{}^{15}C_1\cdots\cdots\cdots{}^{15}C_{12}\right)}$$

$$=\frac{\left(\frac{16}{15}\right)^{13}}{{}^{14}C_0\cdot{}^{14}C_1\cdots\cdots\cdots{}^{14}C_{12}}$$

$$\text{So } \alpha=\frac{16}{15}$$

$$\Rightarrow 30\alpha=32$$

Question ID : 860654848

23. Let the maximum value of $\left(\sin^{-1}x\right)^2+\left(\cos^{-1}x\right)^2$ for $x\in\left[-\frac{\sqrt{3}}{2},\frac{1}{\sqrt{2}}\right]$ be $\frac{m}{n}\pi^2$, where gcd (m, n) = 1.

There m + n is equal to ________.

Ans. Official answer NTA (65)

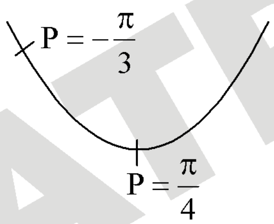
Sol. Let $y = (\sin^{-1}x)^2 + (\cos^{-1}x)^2$

and $\sin^{-1}x = P$ also $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

$y = p^2 + \left(\frac{\pi}{2} - p\right)^2$

$= p^2 + p^2 + \frac{\pi^2}{4} - p\pi$

$= 2p^2 - p\pi + \frac{\pi^2}{4}$



as $x \leftarrow \left[\frac{-\sqrt{3}}{2},\frac{1}{\sqrt{2}}\right]$

so $\sin^{-1}x \in \left[-\frac{\pi}{3},\frac{\pi}{4}\right]$

Consider $y = 2p^2 - p\pi + \frac{\pi^2}{4}$

So maximum at $p = -\pi/3$

$y_{max} = \frac{2\cdot\pi^2}{4} + \frac{\pi^2}{3} + \frac{\pi^2}{4}$

$= \frac{8\pi^2 + 12\pi^2 + 9\pi^2}{36} = \frac{29\pi^2}{36}$

So m + n = 65

Question ID : 860654850

24. If $\int_0^1 4\cot^{-1}\left(1-2x+4x^2\right)dx = a\tan^{-1}(2) - b\log_e(5)$, where a, b ∈ N, then (2a + b) is equal to ________.

Ans. Official answer NTA (9)

Sol. Let $I = \int_0^1 4\cot^{-1}\left(1-2x+4x^2\right)dx$

$$= 4\int_0^1 \tan^{-1}\left(\frac{1}{1+2x(2x-1)}\right)dt$$

$$I = 4\int_0^1\left(\tan^{-1}(2x)-\tan^{-1}(2x-1)\right)dx$$

Let $I_1 = \int_0^1 \tan^{-1}2x\,dx$

$$= \left(\tan^{-1}2x.x\right)_0^1 - \frac{1}{4}\int\frac{4.2x}{1+4x^2}dx$$

$$= \tan^{-1}2 - \frac{1}{4}\left(x\left(1+4x^2\right)\right)_0^1$$

$$I_1 = \tan^{-1}2 - \frac{1}{4}ln \quad ....(1)$$

$$I_2 = \int_0^1 \tan^{-1}(2x-1)dx$$

$2x - 1 = t$

$$= \int_{-1}^1 \frac{1}{2}\tan^{-1}t\cdot dt = 0 \text{ (odd function)}$$

So $I = 4I_1$

$= 4\tan^{-1}2 - ln 5$

So a = 4, b = 1

2a + b = 9

Question ID : 860654847

25. If P is a point on the circle $x^2 + y^2 = 4$, Q is a point on the straight line $5x + y + 2 = 0$ and $x - y + 1 = 0$ is the perpendicular bisector of PQ, then 13 times the sum of abscissas of all such points P is _________.

Ans. Official answer NTA (2)

Sol. Slope of PQ = –1

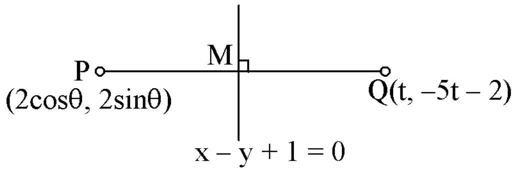
$$\Rightarrow \frac{2\sin\theta+5t+2}{2\cos\theta-t} = -1$$

$$\Rightarrow 2\sin\theta+5t+2 = -2\cos\theta+t$$

$$\Rightarrow 2(\sin\theta+\cos\theta) = -4t-2 \quad ....(1)$$

Mid point $M\left(\frac{2\cos\theta+t}{2}, \frac{2\sin\theta-5t-2}{2}\right)$ will satisty $x - y + 1 = 0$

$$\frac{2\cos\theta+t}{2}-\left(\frac{2\sin\theta-5t-2}{2}\right)+1=0$$



$\Rightarrow \quad 2(\cos\theta-\sin\theta)+6t+4=0$

$\Rightarrow \quad \sin\theta-\cos\theta=3t+2 \quad$ ....(2)

from (1) & (2)

$$\frac{\sin\theta+\cos\theta+1}{-2}=\frac{\sin\theta-\cos\theta-2}{3}$$

$\Rightarrow \quad 3\sin\theta+3\cos\theta+3=-2\sin\theta+2\cos\theta+4$

$\Rightarrow \quad 5\sin\theta+\cos\theta=1$

$\Rightarrow \quad 5\sin\theta=1-\cos\theta$

$\Rightarrow 25\left(1-\cos^2\theta\right)=\cos^2\theta-2\cos\theta+1$

$\Rightarrow \quad 26\cos^2\theta-2\cos\theta-24=0$

$\Rightarrow \quad 13\cos^2\theta-\cos\theta-12=0$

So if these are two such points $P(\theta_1)$ and $Q(\theta_2)$ then $\cos\theta_1+\cos\theta_2=\frac{1}{13}$

So sum of assissas of P = $2(\cos\theta_1+\cos\theta_2)$

$=\frac{2}{13}$